A Bound on Expected Stock Returns

Ohad Kadan and Xiaoxiao Tang

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Stock Options and Expected Returns

- Option prices can teach us a great deal about the volatility of underlying assets (implied volatility).
- Breeden and Litzenberger (1978): option prices can be used to extract risk neutral probabilities, price assets.
- Puzzle: Can option prices be used to elicit any useful information on the expected returns of the underlying asset?
- Such estimates would be forward looking!!

Stock Options and Expected Returns: Recent Literature

- Ross (2015): **Recovery!!!** Under some stationarity assumptions and a regularity condition on the pricing kernel, option prices can be used to recover the physical probability distribution of asset returns.
- Borovička, Hansen, and Scheinkman (2015): **No recovery!!!** Ross restricts the dynamics of the stochastic discount factor in an unrealistic manner. What you recover is NOT the true probability distribution.

Different Approach: Martin (2015)

- Start with the following identify,

$$\mathrm{E}_t R_{j,t+1} - R_{f,t} = \frac{Var_t^* \left(R_{j,t+1} \right)}{R_{f,t}} - Cov_t \left(M_{t+1} R_{j,t+1}, R_{j,t+1} \right).$$

- Restrict attention to the case in which asset j is the market.
- Explore whether $Cov_t \left(M_{t+1} R_{m,t+1}, R_{m,t+1} \right)$ is weakly negative, where $R_{m,t+1}$ is the return on the market - *Negative Correlation Condition* (NCC).
- Main idea: If NCC holds, then

$$\mathrm{E}_t R_{m,t+1} - R_{f,t} \geq \frac{Var_t^* \left(R_{m,t+1} \right)}{R_{f,t}}.$$

- And, $Var_t^* \left(R_{m,t+1} \right)$ can be readily calculated from option prices.

Does the NCC Hold?

- Martin (2015): For the market - the answer is YES if relative risk aversion is at least 1: $\gamma \geq 1$.
- Our main theoretical result: The NCC holds for any asset j (up to a first-order approximation) for which

$$\gamma \geq \frac{\beta_j}{R_j^2}.$$

- In words: The product of systematic and idiosyncratic risk needs to be lower than relative risk aversion for the NCC to hold.

The NCC and the Lower Bound in the Cross-Section

- The NCC holds for 50%-90% of S&P 500 constituents.
- The range depends on the level of conservatism assumed risk aversion.
- For a large cross-section of stocks we have the following lower bound

$$\mathrm{E}_t R_{j,t+1} - R_{f,t} \geq \frac{Var_t^* \left(R_{j,t+1}\right)}{R_{f,t}}.$$

- The lower bound can be estimated from options on stocks with liquid option trading.
- Forward-looking, high-frequency lower bound on expected stock returns.

Cross-Sectional Fama-MacBeth Analysis

- Consistent with Fama and French (1992):
  - The lower bound is increasing with B/M.
  - The lower bound is decreasing with size.
- Inconsistent with Fama and French (1992):
  - The lower bound is increasing with CAPM beta.
- Inconsistent with Jegadeesh and Titman (1993):
  - The lower bound is decreasing with momentum.

Predictive Value of the Bound

- Does the lower bound on expected excess returns provide a valuable signal on realized future returns?
  - It may be that the market expectations reflected in option prices are systematically wrong.
  - It may well be that the lower bound we obtain is far from being binding.
  - It is likely that even if our lower bound is theoretically justified and it often binds, our estimates are quite noisy and the time period is too short to find something.
- Result: Pretty strong economic significance but almost no statistical significance.

Martin's Argument

- Consider a one-period consumption/investment model.
- Assume the existence of a representative agent with utility function $u(\cdot)$.
- The agent's problem is thus choose portfolio weights $\{w_i\}$ to solve

$$\begin{aligned} &\max_{\{w_i\}} \mathrm{E}_t u(\sum w_i R_{i,T}) \\ &s.t. \sum w_i = 1 \end{aligned} \tag{1}$$

- M_T is proportional to $u'(R_{m,T})$.
- We need $Cov_t\left(u'\left(R_{m,T}\right)R_{m,T}, R_{m,T}\right) \leq 0$.
- If $\gamma\left(R_{m,T}\right) \geq 1$ then $u'\left(R_{m,T}\right)R_{m,T}$ is a decreasing function of $R_{m,T}$

NCC for Individual Assets

- Martin's argument cannot be directly applied to individual assets.
- To see this, write $Cov_t\left(R_{j,T}u'(\sum_{i=1}^{N} w_i R_{i,T}), R_{j,T}\right) \leq 0$.
- To overcome the difficulty we use a first-order approximation.
- Define $f : \mathbb{R}^N \rightarrow \mathbb{R}$ by

$$f(R_{1,T}, R_{2,T}, ...R_{N,T}) = R_{j,T}u'(\sum_{i=1}^{N} w_i R_{i,T}). \quad (2)$$

- With a first order approximation of f, $Cov_t\left(u'\left(R_{m,T}\right)R_{j,T}, R_{j,T}\right) \approx u'(\mathrm{E}_t R_{m,T}) Var_t\left(R_{j,T}\right)\gamma\left(\mathrm{E}_t R_{m,T}\right)\left[\frac{1}{\gamma(\mathrm{E}_t R_{m,T})} - \frac{Cov_t(R_{m,T},R_{j,T})}{Var_t(R_{j,T})}\right]$
- Define $\delta_{j,t} \equiv \frac{Var_t(R_{j,T})}{Cov_t(R_{m,T},R_{j,T})}$, we conclude that
- Up to a first order approximation, the NCC holds for asset j whenever $\delta_{j,t} \leq \gamma\left(\mathrm{E}_t R_{m,T}\right)$. For such assets, $\frac{var^*(R_{j,T})}{R_{f,t}}$ serves as a lower bound on the asset's expected excess return between time t and T.

Explaining Delta

- Consider standard one-factor model: $R_{j,t} = \alpha_j + \beta_j R_{m,t} + \varepsilon_{j,t}$,
- OLS estimation: $\hat{\beta}_j = \frac{Cov(R_{j,t}, R_{m,t})}{Var(R_{m,t})}$
- R-Squared: $R_j^2 = \frac{\hat{\beta}_j^2 Var(R_{m,t})}{Var(R_{j,t})} = \frac{\hat{\beta}_j Cov(R_{j,t}, R_{m,t})}{Var(R_{j,t})} = \frac{\hat{\beta}_j}{\hat{\delta}_j}$.
- Combined risk: $\hat{\delta}_j = \frac{\hat{\beta}_j}{R_j^2}$.
- For any asset j, $\hat{\delta}_j \geq \hat{\beta}_j$ with equality occurring only for assets with zero idiosyncratic risk.
- NCC is more likely to hold for assets that have low idiosyncratic risk and/or small beta.

Testing for the NCC

- γ is not directly observable.
- The finance literature has provided a wide range of reasonable values for relative risk aversion.
- Bliss and Panigirtzoglou (2004, Table 7): estimates anywhere between 0 and 55.
- Recent studies in asset pricing typically consider relative risk aversion levels between 1 and 10 as being "reasonable."
  - Mehra and Prescott (1985) argue that relative risk aversion should be lower than 10.
  - Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2011) use a relative risk aversion coefficient of either 7.5 or 10 for their calibrations.
  - Recent estimates: Vissing-Jorgensen and Attanasio (2003): between 5 to 10 for Epstein-Zin Euler equations; Bliss and Panigirtzoglou (2004) between 3 to 10 as implied from option prices with power utility,.

Testing for the NCC

- Estimate $\hat{\delta}_j = \frac{Var(R_{j,t})}{Cov(R_{j,t},R_{m,t})}$
- Data: historical monthly stock returns (obtained from CRSP) for common stocks (share code 10 and 11) for the time period January 1995 to December 2014.
- We restrict attention to S&P 500 constituents starting from the year 2005
- The mean δ in this sample is 6.5 and the median is 5.2.
- $\gamma > 5$, NCC holds for about 50% of S&P constituents.
- More liberal, $\gamma > 10$, then the NCC holds for 90% of these stocks.

Testing for the NCC

- We divide the stocks into four groups by the empirical estimates of δ_j

$$\text{stock } j \text{ is } \begin{cases} \textit{conservative} & \text{if} \quad 1 \leq \delta_j < 4 \\ \textit{moderate} & \text{if} \quad 4 \leq \delta_j < 7 \\ \textit{liberal} & \text{if} \quad 7 \leq \delta_j \leq 10 \\ \textit{very liberal} & \text{if} \quad \delta_j > 10 \end{cases} . \tag{3}$$

- Our main analysis will be centered around the conservative and moderate stocks
- For very liberal stocks the lower bound is unlikely to be valid.
- instead of a lower bound we will likely be getting an upper bound in this case.

Table 2 Summary Statistics for δ_j by Different Groups

Panel A: Summary Statistics for δ_j

	Obs	Mean	Std.Dev	5%	25%	50%	75%	95%
Conservative	181	3.2625	0.4692	2.3801	2.9910	3.3485	3.6469	3.9139
Moderate	329	5.3151	0.8478	4.1347	4.6232	5.2019	6.0260	6.7395
Liberal	112	8.0600	0.7929	7.0706	7.4126	7.8043	8.5571	9.5958
Very Liberal	70	17.5642	23.3860	10.1268	11.1075	12.6492	16.0356	28.0837

Panel B: Firm Characteristics

	Beta	Size($ billions)	B/M	R^2
	Mean (Std. Dev)	Mean (Std. Dev)	Mean (Std. Dev)	Mean (Std. Dev)
Conservative	0.9978 (0.3500)	31.5082 (52.5945)	0.3802 (0.2076)	0.3097 (0.1141)
Moderate	1.0559 (0.5227)	14.9356 (27.2627)	0.4230 (0.2179)	0.1989 (0.0966)
Liberal	1.4157 (0.7888)	7.0995 (7.4788)	0.4472 (0.3691)	0.1742 (0.0965)
Very Liberal	0.9408 (0.6610)	9.5519 (17.9272)	0.3965 (0.2383)	0.0737 (0.0597)

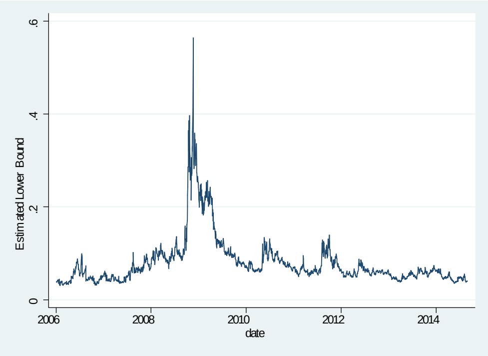
Estimating the Lower Bound

- We follow the methodology in Martin (2015).
- $\frac{1}{R_{f,t}} var_t^* R_{j,T} = \frac{2}{S_{j,t}^2} \left\{ \int_0^{F_{j,t}} \text{put}_{j,t}(K)\, dK + \int_{F_{j,t}}^{\infty} \text{call}_{j,t}(K)\, dK \right\}$
- Three practical limitations:
  - No observation of deep out-of-money options.
  - We do not observe a continuum of strike prices.
  - We only observe prices of American options for individual stocks.

Data

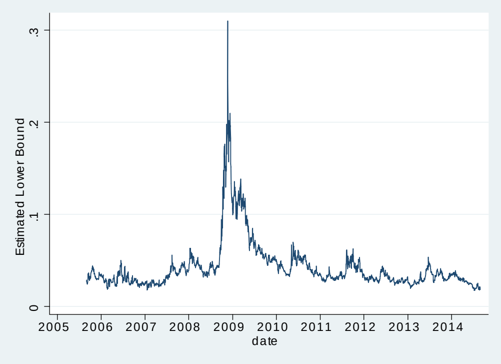
- OptionMetrics: options written on stocks which were included in the S&P 500 index starting from 2005.
- Sample period: 2005/01 to 2014/08.
- Aim: precisely estimate the bound on the sub-sample of assets that have a rich and liquid option market.
- After applying screens, we are left with 54,485,878 observations which yield 862,290 stock/day combinations of 652 distinct stocks.
- Besides the OptionMetrics data we also draw data from CRSP and Compustat to calculate firm characteristics.

Example: Lower Bound on MSFT Expected Excess Return



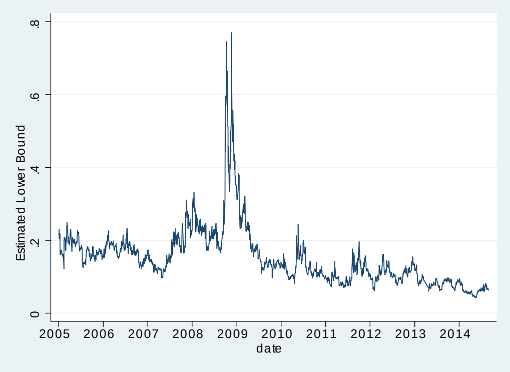
- Average: 8%, average excluding crisis 6%.

Example: Lower Bound on PG Expected Excess Return



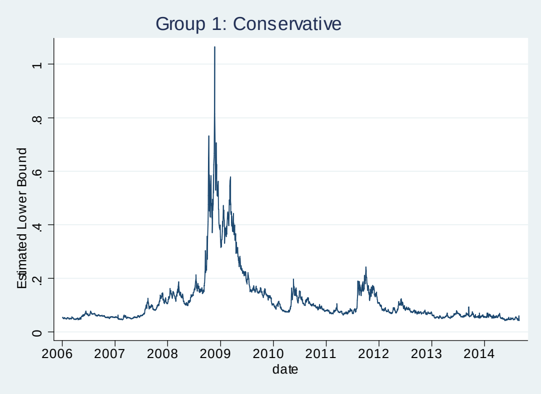
Average: 4.2%, average excluding crisis 3.3%.

Example: Lower Bound on Apple's Expected Excess Return



Average: 16%, average excluding crisis 13.7%.

Lower Bound for "Conservative" Group (181 stocks)



Summary Statistics for the Lower Bounds

Table 3 Summary Statistics for Estimated Lower Bound

Panel A: Entire Sample Period

	Obs	Mean	Std.Dev	5%	10%	25%	50%	75%	90%	95%
Entire Sample	862290	0.1664	0.2184	0.0362	0.0450	0.0671	0.1095	0.1855	0.3187	0.4709
Conservative	247494	0.1223	0.1493	0.0318	0.0379	0.0528	0.0803	0.1335	0.2326	0.3487
Moderate	407349	0.1684	0.2216	0.0382	0.0482	0.0702	0.1113	0.1855	0.3194	0.4744
Liberal	131615	0.2082	0.2457	0.0468	0.0617	0.0943	0.1440	0.2309	0.3905	0.5679
Very Liberal	75832	0.2240	0.2938	0.0414	0.0574	0.0966	0.1549	0.2505	0.4111	0.5795
Market Premium	2424	0.0455	0.0342	0.0192	0.0203	0.0232	0.0366	0.0540	0.0791	0.1221

Panel B: Sample Period Without Years 2008 and 2009

	Obs	Mean	Std.Dev	5%	10%	25%	50%	75%	90%	95%
Entire Sample	663697	0.1192	0.1065	0.0339	0.0413	0.0598	0.0920	0.1440	0.2197	0.2856
Conservative	188086	0.0816	0.0555	0.0301	0.0352	0.0472	0.0672	0.0990	0.1434	0.1808
Moderate	312237	0.1171	0.0880	0.0356	0.0441	0.0630	0.0936	0.1427	0.2140	0.2745
Liberal	104192	0.1640	0.1586	0.0433	0.0560	0.0859	0.1279	0.1900	0.2847	0.3952
Very Liberal	59182	0.1692	0.1452	0.0375	0.0511	0.0860	0.1343	0.2056	0.3088	0.4139
Market Premium	1922	0.0349	0.0175	0.0188	0.0200	0.0217	0.0277	0.0418	0.0590	0.0729

Cross-Sectional Analysis of the Bound

Table 4: Fama MacBeth Analysis of the Bounds

	Obs	β	Size	B/M Ratio	Momentum
Entire Sample	448.8	0.0620 (0.0109)***	-0.0317 (0.0028)***	0.0149 (0.0073)**	-6.5083 (1.9839)***
Conservative $(1 \leq \delta < 4)$	105.7	0.0459 (0.0073)***	-0.0074 (0.0010)***	0.0183 (0.0078)**	-4.0203 (1.7500)**
Moderate $(4 \leq \delta < 7)$	276.9	0.0658 (0.0114)***	-0.0314 (0.0036)***	0.0221 (0.0100)**	-6.6235 (2.0979)***
Liberal $(7 \leq \delta \leq 10)$	51.5	0.0776 (0.0147)***	-0.0264 (0.0035)***	0.1113 (0.0061)*	-7.6483 (2.4033)***
Extremely Liberal $(\delta > 10)$	29.7	0.0643 (0.0151)***	-0.0565 (0.0089)***	-0.0186 (0.0081)**	-4.6244 (2.5542)*

Predictive Value of the Bound

- To evaluate whether the lower bound delivers an informative signal about future returns, in each month t we sort all stocks in our sample based on their monthly average lower bound.
- We then divide the stocks in each month into ten deciles based on their average bound.
- Decile 1 consists of the stocks with the lowest estimates and Decile 10 with the highest estimates.
- We then calculate the equal weighted average of stock realized returns in month $t+1$ for each decile.
- If the lower bound provides an informative signal, then we expect stocks in lower deciles to show lower average realized returns compared to stocks in the higher deciles.

Predictive Value of the Bound

Table 5: Predictive Value of the Bound

Average Future Return of Stocks Grouped by Lower Bound Estimates

Decile	Entire Sample	Conservative ($1 \leq \delta < 4$)	Moderate ($4 \leq \delta < 7$)	Liberal ($7 \leq \delta \leq 10$)	Very Liberal ($\delta > 10$)
1	0.0059	0.0073	0.0058	0.0053	0.0075
2	0.0080	0.0080	0.0085	0.0078	0.0102
3	0.0099	0.0092	0.0071	0.0155	0.0036
4	0.0054	0.0080	0.0058	0.0163	0.0102
5	0.0081	0.0122	0.0079	0.0097	0.0149
6	0.0100	0.0144	0.0111	0.0205	0.0180
7	0.0117	0.0103	0.0076	0.0110	0.0201
8	0.0100	0.0129	0.0107	0.0098	0.0157
9	0.0147	0.0075	0.0157	0.0148	0.0230
10	0.0171	0.0169	0.0164	0.0102	0.0306
10-1	0.0113	0.0095	0.0106	0.0049	0.0230
	(0.0099)	(0.0073)	(0.0108)	(0.0132)	(0.0186)

Precision of Approximations

- A key advantage of our approach is that the form of the utility function need not be known in order to check whether the NCC holds.
- In order to assess the approximation error we consider CRRA functions, which allow us to estimate $cov\left(u'\left(R_m\right)R_j, R_j\right)$ precisely without resorting to an approximation.
- We then check directly whether $cov\left(u'\left(R_m\right)R_j, R_j\right) \leq 0$ holds true.

Table 6: Type I and Type II Errors for Taylor Approximations

γ	Type I Error	Type II Error
2	0	0
3	0.0022	0.1892
4	0.0028	0.1915
5	0.0058	0.1048
6	0.0136	0.1037
7	0.0239	0.0982
8	0.0230	0.0753
9	0.0403	0.0663
10	0.0778	0.0570

The Use of American Options in the Calculation

- The calculation of risk-neutral variance of stock returns relies on the prices of European options.
- All options on individual stocks in the U.S. are of the American style, introducing a potential upward bias in our lower bound estimation due to the early exercise premium (EEP).
- Options we are using are mostly out-of-the-money, a case in which the EEP is known to be relatively small.
- To this end, we follow the framework in MacMillan (1986) and Barone-Adesi and Whaley (1987), who offer an analytic approximation for the EEP of American options in the Black-Scholes framework.
- Our estimates suggest that the use of American options inflates the lower bound by about 2% for the conservative group and by about 5% for the other groups.
- None of the conclusions in the cross sectional analysis changes.

Conclusions

- We present a sufficient condition under which the NCC holds for individual stocks.
- We estimate the lower bound empirically for constituents of the S&P 500 index.
- We find that the bound increases with beta and book-to-market and decreases with size and momentum.
- The bound also provides a noisy signal on future realized stock returns.