Asset Pricing with Return Extrapolation

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August 2018

Overview

- there is growing interest in "return extrapolation"
  - the idea that investors' beliefs about an asset's future return are a positive function of the asset's recent past returns

- models with return extrapolation offer two appealing features
  - they are consistent with survey evidence on the beliefs of real-world investors
  - they show promise in matching important asset pricing facts
    - momentum and reversals in the cross-section
    - volatility and predictability in the aggregate market
    - bubbles

Overview, ctd.

- existing models of return extrapolation can only be compared to the data in a *qualitative* way

- early models, such as Cutler, Poterba, and Summers (1990) and De Long, Shleifer, Summers, and Waldmann (1990), highlight the conceptual importance of return extrapolation
  - but they are not designed to match asset pricing facts quantitatively

Overview, ctd.

- Barberis, Greenwood, Jin, and Shleifer (2015) is a dynamic model that tries to make sense of both survey expectations and aggregate stock market prices
  - however, their simplifying assumptions of CARA preferences and a constant interest rate make it difficult to evaluate the model's fit with the empirical facts
  - the ratio-based quantities at the heart of asset pricing (e.g. the P/D ratio) do not have well-defined distributions

Overview, ctd.

- in this paper, we propose a new model of aggregate stock market prices based on return extrapolation that overcomes this limitation
- a representative-agent Lucas-type equilibrium model with Epstein-Zin preferences
- goal of the paper is to
  - see if the model can match important facts about the aggregate stock market
    - when the agent's beliefs are calibrated to match survey expectations of investors
  - compare the model in a quantitative way to rational expectations models of the stock market

Overview, ctd.

- we consider a Lucas economy in continuous time
- three tradeable assets
  - a Lucas tree: a claim to an aggregate consumption process which follows a geometric Brownian motion
  - the stock market: a claim to an aggregate dividend process whose growth rate is positively related with consumption growth
  - a riskfree asset: in zero net supply with its interest rate determined in equilibrium
- a representative agent who has Epstein-Zin preferences and extrapolative beliefs
  - she perceives that the expected growth rate of stock market prices is governed by a switching process between two regimes

Overview, ctd.

- we calibrate the agent's beliefs to match the survey expectations of investors studied in Greenwood and Shleifer (2014)

- the model *quantitatively* matches important facts about stock market prices and returns
  - excess volatility, predictability, equity premium, low interest rate volatility

- the model allows for a direct comparison with rational expectations models
  - the model produces expectations that robustly match survey data
  - the model also generates asset prices that differ from rational expectations models

Overview, ctd.

Model intuition for excess volatility

Suppose the stock market has had high past returns

- return extrapolation leads the agent to forecast high future returns
- under Epstein-Zin preferences, the separation between the elasticity of intertemporal substitution and risk aversion gives rise to a strong intertemporal substitution effect
- so, the agent's forecast of high future returns leads her to push up the current price

Overview, ctd.

Model intuition for the high equity premium

Three factors affect the long-run equity premium *perceived* by the agent

- excess volatility interacts with the agent's risk aversion and causes the perceived equity premium to increase
- return extrapolation gives rise to *perceived* persistence of the aggregate dividend process, which, under Epstein-Zin preferences, is significantly priced
- the separation between the elasticity of intertemporal substitution and the relative risk aversion helps to keep the equilibrium interest rate low

Overview, ctd.

Model intuition for the high equity premium

The true long-run equity premium is significantly *higher* than the perceived long-run equity premium

- the agent's beliefs mean-revert faster than what she perceives
- given this, she underestimates short-term stock market fluctuations and hence the risk associated with the stock market
- that is, an infinitestimal rational agent who enters our economy would have demanded a higher equity premium

Overview, ctd.

- the model also points to some challenges
  - when calibrated to the survey expectations data, the model predicts a persistence of the price-dividend ratio that is lower than its empirical value
  - to match the empirical persistence of the price-dividend ratio, investors need to form beliefs about future returns based on many years of past returns
  - however, the available surveys suggest that they focus on just the past year or two
- after presenting the model, we compare it to the standard rational expectations models
  - we focus on the long-run risks models and document some different implications
- we also compare our model to a model with *fundamental* extrapolation

Related literature

- theories that study the role of belief formation in driving the behavior of asset prices and macroeconomy
  - *fundamental extrapolation*: Fuster, Hubert, and Laibson (2011); Choi and Mertens (2013); Alti and Tetlock (2014); Hirshleifer, Li, and Yu (2015)
  - *return extrapolation*: Barberis et al. (2015)
  - *default extrapolation*: Greenwood, Hanson, and Jin (2016)
  - *availability heuristic*: Jin (2015)
  - *learning about fundamentals and model uncertainty*: Collin-Dufresne, Johannes, and Lochastoer (2016a, b); Bidder and Dew-Becker (2016)
  - *experience effect*: Vanasco, Malmendier, and Pouzo (2015)

- source of stock price movements
  - Cochrane (2008, 2011); Chen, Da, and Zhao (2013); De La O and Myers (2017)

Roadmap

- model setup
- model parameterization
- model implications
- comparison with rational expectations models
- conclusion

The model: Setup

Assets

- a Lucas tree with the aggregate consumption process

$$dC_t/C_t = g_C dt + \sigma_C d\omega_t^C$$

- the aggregate dividend process for the stock market

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t^D$$

  with $\mathbb{E}_t[d\omega_t^C \cdot d\omega_t^D] = \rho dt$

- an instantaneous riskless asset in zero net supply with an equilibrium shadow interest rate r_t

Setup, ctd.

Agent: Preferences

- the representative agent has recursive intertemporal utility

$$U_t = \left[(1 - e^{-\delta dt})C_t^{1-\psi} dt + e^{-\delta dt} \left(\mathbb{E}_t^e[\tilde{U}_{t+dt}^{1-\gamma}]\right)^{(1-\psi)/(1-\gamma)}\right]^{1/(1-\psi)},$$

  where δ is the time discount rate, γ is the coefficient of relative risk aversion, and ψ is the reciprocal of EIS

- the *subjective* Euler equation is

$$\mathbb{E}_t^e \left[e^{-\delta(1-\gamma)dt/(1-\psi)} \left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t+dt}^{(\psi-\gamma)/(1-\psi)} \tilde{R}_{j,t+dt}\right] = 1,$$

  where

$$\tilde{M}_{t+dt} = \frac{\tilde{P}_{t+dt}^C + C_t dt}{P_t^C}$$

Setup, ctd.

Agent: Beliefs about future price growth

- investors believe that the expected growth rate of the stock market price is $(1-\theta)g_D + \theta\tilde{\mu}_{S,t}$, where $\tilde{\mu}_{S,t}$ is a latent variable governed by

$$\begin{array}{cc} & \begin{array}{cc} \tilde{\mu}_{S,t+dt} = \mu_H & \tilde{\mu}_{S,t+dt} = \mu_L \end{array} \\ \begin{array}{c} \tilde{\mu}_{S,t} = \mu_H \\ \tilde{\mu}_{S,t} = \mu_L \end{array} & \begin{pmatrix} 1-\chi dt & \chi dt \\ \lambda dt & 1-\lambda dt \end{pmatrix} \end{array}$$

- the regime-switching learning model implies that investors perceive

$$dP_t^D/P_t^D = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e,$$

where $S_t \equiv \mathbb{E}^e[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$ and

$$\mu_P^{D,e}(S_t) = (1-\theta)g_D + \theta S_t$$

Setup, ctd.

Agent: Beliefs about future price growth

- optimal filtering theory implies

  $$dS_t = [\lambda\mu_H + \chi\mu_L - (\lambda + \chi)S_t]dt + (\sigma^D_{P,t})^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega^e_t$$

  where

  $$d\omega^e_t \equiv [dP^D_t/P^D_t - (1-\theta)g_D dt - \theta S_t dt]/\sigma^D_{P,t}$$

The evolution of S_t captures return extrapolation

- high past realizations of returns dP^D_t/P^D_t push up perceived shocks $d\omega^e_t$
- this leads investors to raise their expectation S_t

Note: optimal filtering does *not* imply rational expectations

Setup, ctd.

Agent: Beliefs about future dividend growth

- the *perceived* dividend process is

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e,$$

  where

$$g_D^e(S_t) = \underbrace{(1-\theta)g_D + \theta S_t}_{\text{expectation of price growth}} \quad \underbrace{-(f'/f)\mu_S^e(S_t)}_{\text{expectation of sentiment evolution}}$$

$$\underbrace{+\sigma_D^2 - \sigma_P^D(S_t)\sigma_D - \tfrac{1}{2}(f''/f)(\sigma_S(S_t))^2}_{\text{Ito correction terms}}$$

- when the agent expects high price growth, she also expects high dividend growth
  - at a pace that *exceeds* her expectations about future price growth

Setup, ctd.

Agent: Beliefs about future dividend growth

Why?

- iterating forward the subjective Euler equation gives

$$\frac{P_t^D}{D_t} = \mathbb{E}_t^e \left[\int_t^\infty e^{-\delta(1-\gamma)(s-t)/(1-\psi)} \left(\frac{\tilde{C}_s}{C_t} \right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t\to s}^{(\psi-\gamma)/(1-\psi)} \left(\frac{\tilde{D}_s}{D_t} \right) ds \right]$$

- the agent's beliefs about future dividend growth are linked to her beliefs about future price growth

- the agent perceives the price-dividend ratio to be mean-reverting

Setup, ctd.

Agent: Beliefs about consumption growth

- the *perceived* consumption process is

$$dC_t/C_t = g_C^e(S_t)dt + \sigma_C \left(\rho \cdot d\omega_t^e + \sqrt{1-\rho^2} \cdot d\omega_t^{\perp}\right),$$

  where

$$g_C^e(S_t) - g_C = \rho \sigma_C \sigma_D^{-1}(g_D^e(S_t) - g_D)$$

- the bias in the agent's beliefs about consumption growth derives only from the bias in her beliefs about dividend growth
  - and it is small

- this is consistent with the lack of empirical evidence that investors have extrapolative beliefs about consumption growth

Parameterization

Parameters	Variable	Value
Asset parameters:		
Expected consumption growth	g_C	1.91%
Standard deviation of consumption growth	σ_C	3.80%
Expected dividend growth	g_D	2.45%
Standard deviation of dividend growth	σ_D	11%
Correlation between dD and dC	ρ	0.2
Utility parameters:		
Relative risk aversion	γ	10
Reciprocal of EIS	ψ	0.9
Subjective discount rate	δ	0.02
Belief parameters:		
Upper bound of sentiment H	μ_H	0.15
Lower bound of sentiment L	μ_L	–0.15
Degree of extrapolation	θ	0.5
Perceived transition intensity from H to L	χ	0.18
Perceived transition intensity from L to H	λ	0.18

Parameterization, ctd.

- we set θ, χ, and λ such that
  - for a regression of the agent's expectations about future returns on past twelve-month returns, the model produces a regression coefficient and a t-statistic that match the empirical estimates
  - the agent's beliefs match the survey evidence on the relative weight investors put on recent versus distant past returns when forming beliefs about future returns

Parameterization, ctd.

Investor expectations

	$\mathbb{E}^e_t[dP^D_t/P^D_t dt]$			
$R^D_{t-12\to t}$	0.04	0.04		
	(8.4)	(12.1)		
$\ell\text{n}(P/D)$			0.12	0.12
			(36.8)	(48.9)
Constant	0.02	0.02	–0.33	–0.34
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.58	0.55	0.99	0.99

- empirical value:
  - for a 5-year long sample of data from the Michigan survey, the regression coefficient is 3.9% with a t-statistic of 1.68
  - for a 15-year long sample of data from the Gallup survey, the regression coefficient is 8% with a t-statistic of 8.81

Parameterization, ctd.

Determinants of investor expectations

	$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$	
ϕ	0.43	0.42
a	0.004	0.004
b	2.04	2.07
Sample size	15 yr.	50 yr.
R^2	0.99	0.98

$$\text{Expectation}_t = a + b \sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\rightarrow(t-(j-1)\Delta t)} + \varepsilon_t$$

$$\text{where } w_j \equiv e^{-\phi(j-1)\Delta t} \Big/ \sum_{l=1}^{n} e^{-\phi(l-1)\Delta t}$$

- empirical value:
  - Barberis et al. (2015) run the same regression using survey data studied in Greenwood and Shleifer (2014) and obtain an empirical estimate of 0.44 for ϕ

Parameterization, ctd.

Remark:

- the literature has not reached consensus on the value of ϕ
  - Greenwood and Shleifer (2014) and Kuchler and Zafar (2016) find that investor expectations depend only on recent returns
  - Malmendier and Nagel (2011, 2013) suggest that distant past events may also play an important role when investors form beliefs

- two possible explanations:
  - investors may simultaneously adopt two mechanisms when forming beliefs: one that focuses on recent past events, the other that focuses on infrequent but salient events
  - the horizon over which investors form expectations may affect how far they look back into the past

Model implications

- basic moments
- return predictability
- correlation between stock market returns and consumption growth
- cash flow expectations
- autocorrelations of returns and price-dividend ratios

Model implications, ctd.

Basic moments

Statistic	Theoretical value	Empirical value
Equity premium	4.88%	3.90%
Return volatility	27.4%	18.0%
Sharpe ratio	0.20	0.22
Interest rate	2.16%	2.92%
Interest rate volatility	0.33%	2.89%
Price-dividend ratio	19.4	21.1

Model implications, ctd.

Basic moments: excess volatility

- the interaction between beliefs and preferences is quantitatively important:
  - without return extrapolation, Epstein-Zin preferences alone with *i.i.d.* dividend growth and consumption growth do not lead to any excess volatility
  - without Epstein-Zin preferences, return extrapolation alone leads to average return volatility of 13.8%

Basic moments: the equity premium

- the *true* equity premium is higher than the perceived one
  - when measured in log excess returns, the true equity premium is 4.9%; the perceived one is 1.6%
  - when measured in excess returns, the true equity premium is 8.6%; the perceived one is 5.1%

Model implications, ctd.

Return predictability

	Model		Empirical value	
Horizon (years)	10× coefficient	Adjusted R-squared	10× coefficient	Adjusted R-squared
1	–7.2	0.13	–1.3	0.04
2	–9.5	0.16	–2.8	0.08
3	–10.1	0.15	–3.5	0.09
5	–10.6	0.13	–6.0	0.18
7	–11.0	0.12	–7.5	0.23

$$r^{D,e}_{t\to t+j} = \alpha_j + \beta_j \ell\mathrm{n}(P_t/D_t) + \varepsilon_{j,t+j}$$

- regressions are based on simulated data from the model with a total length of 10,000 years and with a monthly frequency

Model implications, ctd.

Model intuition for return predictability

- if past returns have been high, return extrapolation causes the current price to increase
  - the same mechanism that generates excess volatility also gives rise to return predictability
- a strong degree of mean reversion in the agent's expectations about stock returns is also an important factor
  - by assumption, the agent believes that her expectations about stock market returns will mean-revert
  - more important, the agent's beliefs are *incorrect*: her return expectations mean-revert faster than what she perceives

Model implications, ctd.

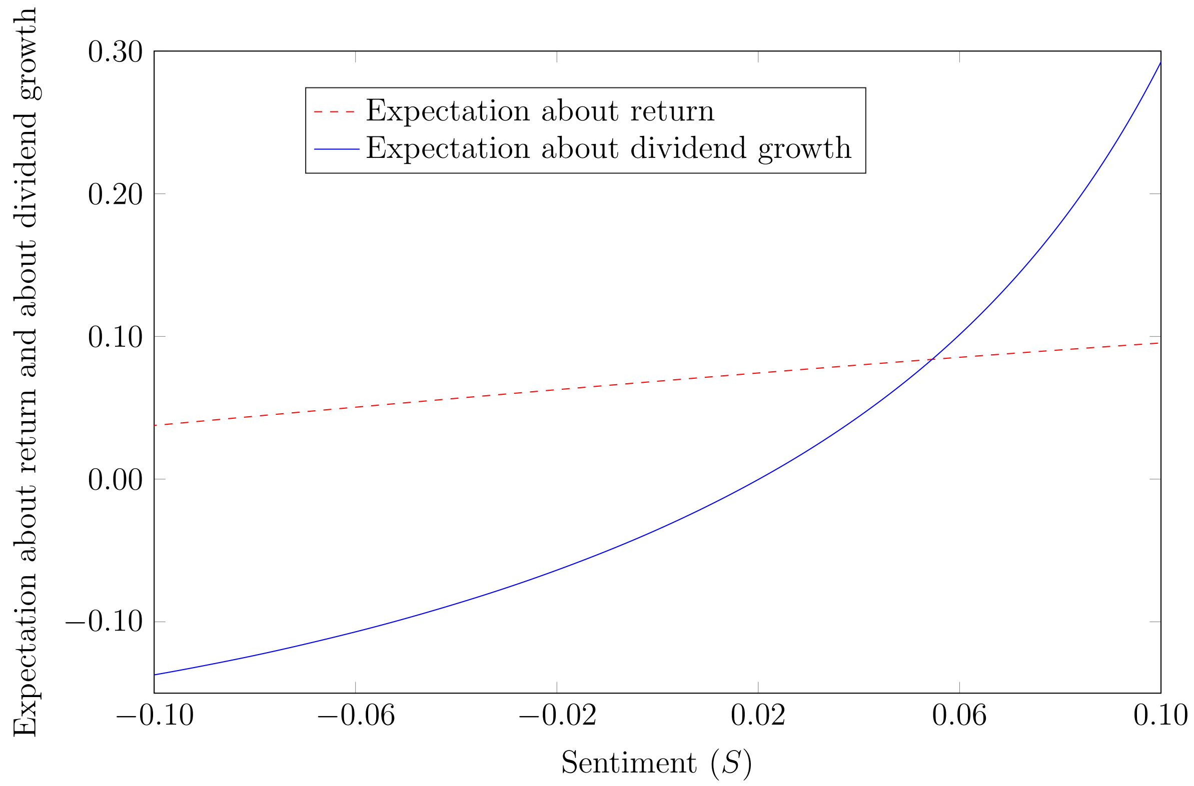
Correlation between stock market returns and consumption growth

Correlation	Model			Empirical value
	monthly	quarterly	annual	annual
$\mathbb{C}\mathrm{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\mathrm{n}(C_{t-1}/C_{t-2}))$	–0.01	–0.02	–0.02	–0.05
$\mathbb{C}\mathrm{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\mathrm{n}(C_{t}/C_{t-1}))$	–0.01	–0.03	–0.06	–0.08
$\mathbb{C}\mathrm{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\mathrm{n}(C_{t+1}/C_{t}))$	0.20	0.20	0.19	0.09
$\mathbb{C}\mathrm{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\mathrm{n}(C_{t+2}/C_{t+1}))$	0.00	–0.00	0.01	0.49
$\mathbb{C}\mathrm{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\mathrm{n}(C_{t+3}/C_{t+2}))$	–0.00	–0.00	0.01	0.05

- correlations in Campbell and Cochrane (1999) are 0.79 and 0.40 at a monthly and annual frequency, respectively
- the observed correlation between consumption growth and dividend growth is low, and the bias in the agent's beliefs about consumption growth is small
  - so the agent's beliefs about stock returns are not significantly affected by fluctuations in consumption growth

Model implications, ctd.

Cash flow expectations



- the agent's expectation about price growth is less responsive to changes in sentiment than her expectation about dividend growth

Model implications, ctd.

Cash flow expectations

- the Campbell-Shiller decomposition of the log price-dividend ratio is

$$\ell\mathrm{n}(P_t^D / D_t) \approx \sum_{j=0}^{\infty} \xi^j \left(\Delta d_{(t+j\Delta t)\rightarrow(t+(j+1)\Delta t)} - \Delta r^D_{(t+j\Delta t)\rightarrow(t+(j+1)\Delta t)} \right) + \text{constant}$$

- two approaches to empirically compute the relative importance of the "cash flow news" and "discount rate news" components

$$1 \approx \underbrace{\frac{\mathbb{C}\mathrm{ov}\left(\sum_{j=0}^{\infty} \xi^j \Delta d_{(t+j\Delta t)\rightarrow(t+(j+1)\Delta t)}, \ell\mathrm{n}(P_t^D / D_t)\right)}{\mathbb{V}\mathrm{ar}\left(\ell\mathrm{n}(P_t^D / D_t)\right)}}_{CF_{objective}}$$

$$+ \underbrace{\frac{\mathbb{C}\mathrm{ov}\left(-\sum_{j=0}^{\infty} \xi^j r^D_{(t+j\Delta t)\rightarrow(t+(j+1)\Delta t)}, \ell\mathrm{n}(P_t^D / D_t)\right)}{\mathbb{V}\mathrm{ar}\left(\ell\mathrm{n}(P_t^D / D_t)\right)}}_{DR_{objective}}$$

Model implications, ctd.

Cash flow expectations

$$1 \approx \underbrace{\frac{\mathbb{C}\text{ov}\left(\mathbb{E}_t^e[\sum_{j=0}^{\infty} \xi^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}], \ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{CF_{subjective}}$$

$$+ \underbrace{\frac{\mathbb{C}\text{ov}\left(-\mathbb{E}_t^e[\sum_{j=0}^{\infty} \xi^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}], \ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{DR_{subjective}}$$

- using realized dividend growth and returns or the subjective expectations of these quantities, Campbell-Shiller decomposition delivers very different results
  - $DR_{objective} = 0.98$, $CF_{objective} = 0.02$, $DR_{subjective} = -0.08$, and $CF_{subjective} = 1.08$

Model implications, ctd.

Autocorrelations of price-dividend ratios and returns

Lag (years)	Model		Empirical value	
	$\ell\text{n}(P/D)$	$r^{D,e}$	$\ell\text{n}(P/D)$	$r^{D,e}$
1	0.33	–0.28	0.78	0.05
2	0.11	–0.09	0.57	–0.21
3	0.05	–0.02	0.50	–0.08
5	0.00	–0.01	0.32	–0.14
7	–0.02	–0.01	0.29	0.11

- when calibrated to available survey expectations, the model predicts a persistence for P/D that is lower than its empirical value

Model comparison

- we compare our model to models with rational expectations
  - our model does a better job in matching survey expectations data
  - these models generate different implications for asset prices

Comparison with long-run risks models

- our model produces an equity premium that does *not* vary significantly with *EIS*
  - this differs from the long-run risks models

Reasons:

- sentiment is our model is less persistent than the stochastic growth component in the long-run risks models
- the perceived dividend growth in our model depends more strongly on the agent's beliefs about price growth
  - dividend growth in Bansal and Yaron (2004) depends less on the stochastic growth component
- the true equity premium in our model is above the perceived one

Conclusion and future research

Conclusion

- we build a new return extrapolation model that allows for a quantitative comparison with the data on asset prices
- with the agent's beliefs calibrated to fit the survey expectations, the model matches important facts about the aggregate stock market
- we compare our model with rational expectations models and document their different implications

Future research

- better understand the formation of investor expectations
- interaction between behavioral agents and rational agents