Effort, Efficiency and Welfare

Xuezhong (Tony) He

Lei Shi & Marco Tolotti

University of Technology Sydney
Macquarie University
Ca' Foscari University of Venice

2018 IFS, SWUFE International Conference
July 21, 2018, Chengdu

Information efficiency is impossible

in a competitive economy **when information is costly**

- **Grossman and Stiglitz** (1980):

  > *In a competitive economy [...] prices reflect the information of informed individuals but only partially [...] How informative the price system is depends on the number of individuals who are informed but the number of individuals who are informed is itself an endogenous variable.*

- Information efficiency of a price system depends on the number of individual who are informed.

- **Price becomes more efficient with low information cost**

Information is detrimental for social welfare in exchange economies

- Information has **social value** if it allows everyone to be made better off
- Allen (1984): In exchange economies, information is detrimental for social welfare (in line with Hirshleifer (1971))
- Kurlat & Veldkamp (2015): *Should we regulate financial information?*
  - By making payoffs less uncertain, information reduces risk and therefore reduces returns.
  - **With exponential utility and normally distributed payoffs, the latter effect always dominates**
  - Each investor is better off with the information, but collectively investors are better off if they are all uninformed.

... apparently, there is no room for good news: market efficienty (higher information) and welfare are not *compatible* in a GS economy.

... apparently, there is no room for good news: market efficienty (higher information) and welfare are not *compatible* in a GS economy.

Is it possible to prevent the *Hirshleifer effect*?

... apparently, there is no room for good news: market efficienty (higher information) and welfare are not *compatible* in a GS economy.

Is it possible to prevent the *Hirshleifer effect*?

We show that there is a room for "compatibility" when investors are uncertain to be informed.

Uncertain, Effort, and Trade-off Mechanism

- Information is multidimensional and uncertain...**being (fully) informed is uncertain**; instead of certainty with a constant cost in the Grossman and Stiglitz model.

- **Uncertainty, Effort and Disutility**—the probability of being informed increases with the effort, but ... with higher disutility

- **Tradeoff** between the **expected utility gain** from being informed and the **utility loss** from making the effort to be informed:

  - When individuals make more effort to be informed, in aggregate, the price system is expected to become more informative;

  - which then reduces the information advantage and hence the incentive of making the effort.

Research Issues

- What determines the **optimal effort**, hence the **fraction of the informed in aggregate**, individuals are willing to make in equilibrium?
- What are the implications of the effort for **price efficiency** and **social welfare**?
  - Does making more effort always improve price efficiency?
  - Is it possible that public information can be socially valuable?
- **Is it possible to improve both price efficiency and social welfare?**

Model Development

- Build a two-stage optimization scheme and an endogenous information equilibrium
- The optimal effort is the result of a non-cooperative information game—characterized by a **Nash equilibrium**
- The optimal demand is obtained by utility maximization and price is characterized by a **rational expectations equilibrium**
- Characterize a **linear equilibrium price**, uniquely determined by the information structure of the financial market and the sensitivity to the effort

Contributions

- Traders make more effort when **information becomes more valuable**
- **More effort does not always make price more efficient**
- **Improving public information can be socially valuable**

The Model—Information Structure

- A measure-one continuum of traders, indexed by $i \in (0,1)$, risk averse, having a CARA utility $u_i(W) = -e^{-\alpha W}$

- A static one-period model, two assets: one risk-free asset with a payoff of $R\,(>1)$ and a risky asset with payoff

Risky Payoff

$$D = d + \theta + \epsilon, \quad d > R, \quad \theta \sim \mathcal{N}(0, \sigma_\theta^2), \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

- **Information Structure**
  - σ_θ—the **fundamental information component** θ is observable with a probability depending the effort (to be specified)
  - σ_ϵ—the **noise component** of the fundamental ϵ is unobservable.
  - σ_z— **Noise supply**: $z \sim \mathcal{N}(0, \sigma_z^2)$

The Model—Effort

- The more effort an trader makes, the more likely he will become informed, and the higher the disutility for the trader
- With a **probability** $p_i \in (p_0, 1)$, trader i can observe θ, where $p_0 \geq 0$ is a reference probability (to be informed for making no effort)
- The **disutility of the effort,** $c(p_i)$, is an increasing function of the probability p_i.
- Denote type I **as informed** and type U **as uninformed**

The Model—Effort and Choice

- Trader i does not directly decide his type, rather decides the effort he wants to put in place in order to be informed;
- Denote by $\omega_i \in \{0, 1\}$, where $\omega_i = 1$ for type I and $\omega_i = 0$ for type U.

Choice Types

$$\mathbb{P}(\omega_i = 1) = p_i; \quad \mathbb{P}(\omega_i = 0) = 1 - p_i$$

- The trader also chooses his **optimal demand** x_i in the risky asset, conditional on which type (informed or uninformed) he belongs to

The Model—Optimization Problem

The **optimization** of trader i is separated into **two stages** and **solved backward**

- First, trader i decides his **optimal portfolio** choice x_i^* **given his type**
- Second, by averaging on the likelyhood of being informed and forming an expectation about other traders' actions, trader i strategically chooses the **optimal probability**, p_i^*, to become informed.
- The optimal strategy of choosing $p_i^*(i \in [0,1])$ is then determined by a **Nash equilibrium** in which $p^* = p_i^*$ for all the traders
- The **equilibrium price** P^* of the risky asset is determined by the market clearing condition

The Model—Optimization Problem

- The equilibrium price depends on the aggregate market fraction of informed traders
- In turn, traders need to forecast the fraction of informed traders
- In the Nash equilibrium, the **fraction of the informed** $\lambda^* = p^*$ and hence the equilibrium price $P^* = P^*(\lambda^*)$, $\lambda = \int_0^1 \mathbb{I}_{\{\omega_i=+1\}} di$.

The Two-stage Optimization Problem

The two-stage optimization problem

Optimal investment (conditional on the type) based on the expected utility

$$V_I(\lambda) = \max_{x_i} \mathbb{E}\Big[u\big(W(x_i)\big)\Big|\omega_i = I,\, \lambda\Big],$$

$$V_U(\lambda) = \max_{x_i} \mathbb{E}\Big[u\big(W(x_i)\big)\Big|\omega_i = U,\, \lambda\Big].$$

Optimal effort in an information game (information acquisition decision)

$$\max_{p_i} \mathcal{U}(p_i;\lambda) = p_i V_I(\lambda) + (1-p_i)V_U(\lambda) - \mu\, c(p_i),$$

where

$$c(p) = p \ln \frac{p}{p_0} + (1-p)\ln\frac{1-p}{1-p_0}, \quad \mu \geq 0.$$

Disutility Function

The rational cost for probabilistic choice models is of entropic type—Mattsson and Weibull (2002)

Rational Cost

$$c(p) = p \ln \frac{p}{p_0} + (1-p) \ln \frac{1-p}{1-p_0}$$

- Provides a **micro-foundation** for the information game
- In line with classical literature on information theory; Hobson (1969), Hofbauer and Sandholm (2001)
- In principle, we could use any increasing and convex $c(\cdot)$

Optimal Demands

- The optimal demand for the **informed traders**:

$$x_I(P_\lambda, \theta) = \underset{x_i}{\text{argmax}} \left(x_i \mathbb{E}[D - RP_\lambda | P_\lambda, \theta, \lambda] - \frac{\alpha}{2} x_i^2 \mathbb{V}ar[D - RP_\lambda | P_\lambda, \theta, \lambda] \right)$$

leading to

$$x_I(P_\lambda, \theta) = \frac{\mathbb{E}[D - RP_\lambda | P_\lambda, \theta, \lambda]}{\alpha \mathbb{V}ar[D - RP_\lambda | P_\lambda, \theta, \lambda]}.$$

- Similarly, for the **uninformed traders**

$$x_U(P_\lambda) = \underset{x_i}{\text{argmax}} \left(x_i \mathbb{E}[D - RP_\lambda | P_\lambda, \lambda] - \frac{\alpha}{2} x_i^2 \mathbb{V}ar[D - RP_\lambda | P_\lambda, \lambda] \right),$$

leading to

$$x_U(P_\lambda) = \frac{\mathbb{E}[D - RP_\lambda | P_\lambda, \lambda]}{\alpha \mathbb{V}ar[D - RP_\lambda | P_\lambda, \lambda]}.$$

Equilibrium

Probabilities $p^* = (p_i^*)_{i\in[0,1]}$, the market fraction of informed traders λ^*, and price P^* form an **equilibrium** if:

(i) $p^* = (p_i^*)_{i\in[0,1]}$ is a **Nash equilibrium**, meaning that for $i \in [0,1]$,

$$\mathcal{U}(p_i^*;\lambda) \geq \mathcal{U}(p_i;\lambda);$$

(ii) the following **consistency equation** is satisfied

$$\lambda = \mathbb{E}\left[\int_0^1 \mathbb{I}_{\{\omega_i^*=I\}}\, di\right] = \int_0^1 p_i^*\, di =: \lambda^*;$$

(iii) optimal demands and price $P^* = P_{\lambda^*}$ satisfy market clearing condition

$$\int_0^1 x_i^*\, di = \lambda^* x_I(P^*,\theta) + (1-\lambda^*)x_U(P^*) = z.$$

Equilibrium Characterization

Under all the aforementioned assumptions, there exists a unique equilibrium where the price is of the form

Equilibrium Price

$$P^* = \frac{1}{R}\left(d + b_\theta \cdot \theta - b_z \cdot z\right)$$

with

$$b_\theta = \frac{\lambda\,\bar{v}}{v_\epsilon}, \quad b_z = \alpha\,\bar{v},$$

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_\epsilon + v_\theta + \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2 v_\epsilon}};$$

and λ satisfies

$$c'(\lambda) = \frac{1}{\mu}\left(V_I(\lambda) - V_U(\lambda)\right).$$

Remarks

- Symmetry. At the equilibrium all agents choose the same $p_i^* = p^*$:

$$\lambda^* = \int_0^1 p_i^* \, di = p^*.$$

- Uniqueness.
  Since $\Delta V(\lambda) = V_I(\lambda) - V_U(\lambda)$ decreases in λ and

$$\mu \, c'(p_o) < \Delta V(p_o) \, ; \quad \mu \, c'(1) > \Delta V(1),$$

  λ^* is unique, hence we have a unique linear equilibrium price.

- Exogenous v.s. endogenous λ.
  The *financial part of the model* (i.e. optimal demands and the linear pricing rule) can be determined also when λ is assumed to be exogenous.

Exogenous Information Equilibrium —Risk Sharing

$$\frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U}.$$

- $\bar{v}$ is the weighted harmonic mean of:

$$v_\epsilon \quad : \text{ risk to the informed traders,}$$

$$v_U := v_\epsilon + v_\theta + \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2 v_\epsilon} : \text{ risk to the uninformed traders.}$$

Indeed, given a linear price, optimal demands read

$$x_I = \frac{d + \theta - RP}{\alpha\, v_\epsilon}; \quad x_U = \frac{d - RP}{\alpha\, v_U}.$$

Exogenous Information Equilibrium —Price Efficiency

Concerning the risk to the uninformed:

$$v_U := v_\epsilon + v_\theta + \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2 v_\epsilon}.$$

- $v_\epsilon + v_\theta = Var(D)$: dividend risk; (recall that $D = d + \theta + \epsilon$)
  $\mapsto$ *The risk for uninformed is larger than the dividend risk.*

- **Price efficiency $Corr(P, \theta)$ increases in λ, making more effort**

Exogenous Information Equilibrium —Price Efficiency

Concerning the risk to the uninformed:

$$v_U := v_\epsilon + v_\theta + \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2 v_\epsilon}.$$

- $v_\epsilon + v_\theta = Var(D)$: dividend risk;

  $\mapsto$ *The risk for uninformed is larger than the dividend risk.*

- $\frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2\, v_\epsilon} \uparrow Corr^2(P,\theta)$: information risk.

  Increasing in the informativeness of price (price efficiency). In fact,

$$Corr^2(P,\theta) = \frac{\mathcal{I}}{1+\mathcal{I}}, \quad \text{where } \mathcal{I} = \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2\, v_\epsilon}\cdot\frac{\lambda}{v_\epsilon}.$$

- **Price efficiency $Corr(P,\theta)$ increases in λ, making more effort**

Exogenous Information Equilibrium —Price Efficiency

Concerning the risk to the uninformed:

$$v_U := v_\epsilon + v_\theta + \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2 v_\epsilon}.$$

- $v_\epsilon + v_\theta = Var(D)$: dividend risk;

  $\mapsto$ *The risk for uninformed is larger than the dividend risk.*

- $\frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2\, v_\epsilon} \uparrow Corr^2(P,\theta)$: information risk.

  Increasing in the informativeness of price (price efficiency). In fact,

$$Corr^2(P,\theta) = \frac{\mathcal{I}}{1+\mathcal{I}}, \quad \text{where } \mathcal{I} = \frac{v_\theta}{v_z}\frac{\lambda}{\alpha^2\, v_\epsilon}\cdot\frac{\lambda}{v_\epsilon}.$$

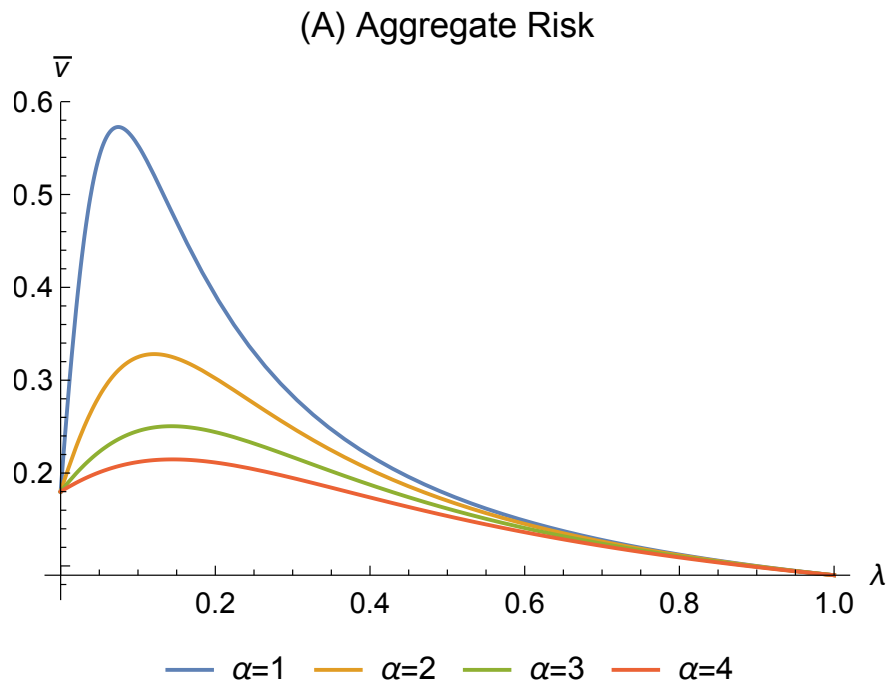
  $\mapsto$ *Uninformed reduce demand when the price is more informative. This effect is stronger when agents are less risk-averse.*

- **Price efficiency $Corr(P,\theta)$ increases in λ, making more effort**

Exogenous Information Equilibrium —Aggregate Risk and Risk Premium

- Aggregate risk $\bar{v}$ as a function of λ.

  $\mapsto$ *$\bar{v}$ is not monotone in λ, in particular when α is small. (Panel A)*



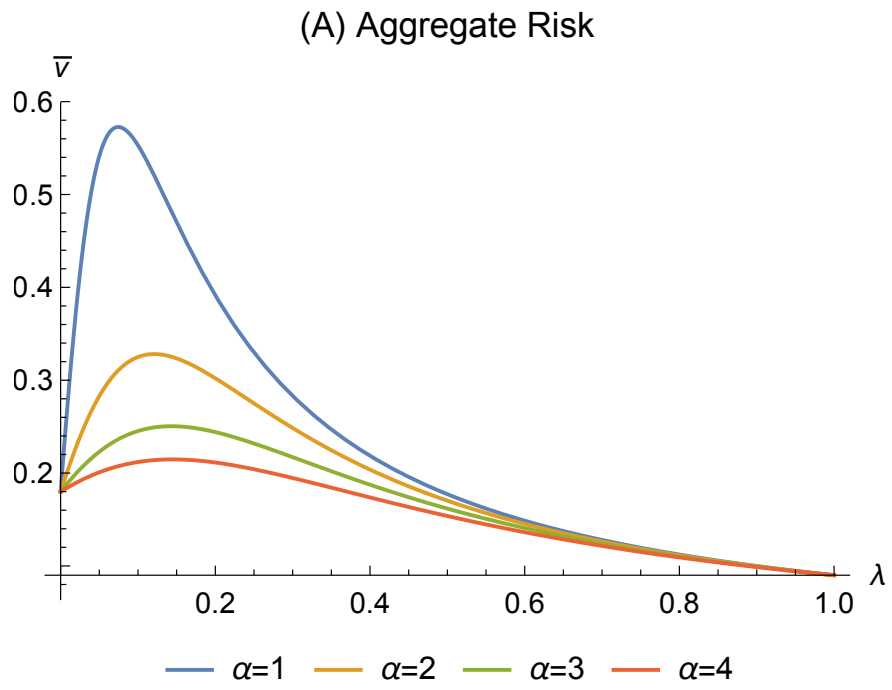
Exogenous Information Equilibrium —Aggregate Risk and Risk Premium

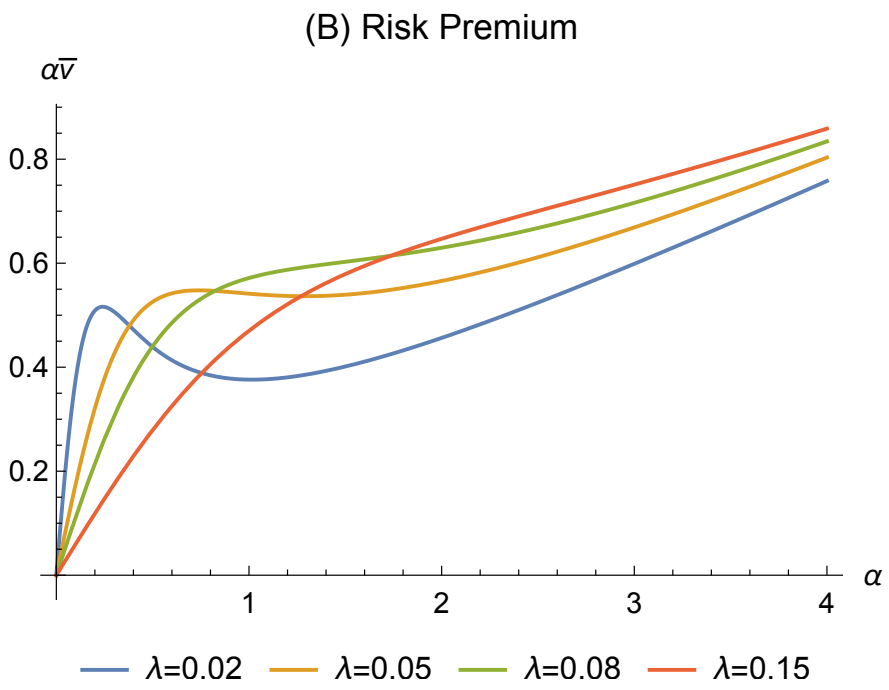
- Aggregate risk $\bar{v}$ as a function of λ.

  $\mapsto$ *$\bar{v}$ is not monotone in λ, in particular when α is small. (Panel A)*

- Risk premium $\alpha\bar{v}$ as a function of α.

  $\mapsto$ *$\alpha\bar{v}$ is not monotone in α when informed trading is low. (Panel B)*





Exogenous Information Equilibrium —Welfare

Recall that, $V_I(\lambda) = \mathbb{E}\Big[u\Big(W\big(x_I(P_\lambda, \theta)\big)\Big)\Big]$, $V_U(\lambda) = \mathbb{E}\Big[u\Big(W\big(x_U(P_\lambda)\big)\Big)\Big]$.

Concerning welfare analysis

- Marginal welfare: $V_I(\lambda) > V_U(\lambda)$ for all λ.

  $\mapsto$ *Single trader is always better-off being informed rather uninformed.*

- Social welfare (extreme cases): $V_I(1) < V_U(0)$.

  $\mapsto$ *A society where nobody is informed is socially preferable to a society where everybody is informed.*

  This fact results in a sort of **Prisoner's Dilemma**

Prisoner's Dilemma—Example

Suppose there are just two agents of type U or I. $v_\epsilon = v_z = v_\theta = 1$, $\alpha = 1$. Then, payoffs are such that

$$V_U(0.5) < V_I(1) < V_U(0) < V_I(0.5)$$

(where 0.5 accounts for one agent of type I and one agent of type U).

The normal-form game is as follows:

	I	U
I	-0.70 ; -0.70	-0.56 ; -0.76
U	-0.76 ; -0.56	- 0.57 ; -0.57

Exogenous Information Equilibrium —Social Welfare

- For $\lambda \in (0,1)$, the welfare of both informed and uninformed traders is decreasing in λ

- The difference between their welfare is also decreasing in λ,

$$\frac{d}{d\lambda} V_k(\lambda) < 0, \quad k \in \{I, U\} \qquad \text{and} \qquad \frac{d}{d\lambda}\left(V_I(\lambda) - V_U(\lambda)\right) < 0.$$

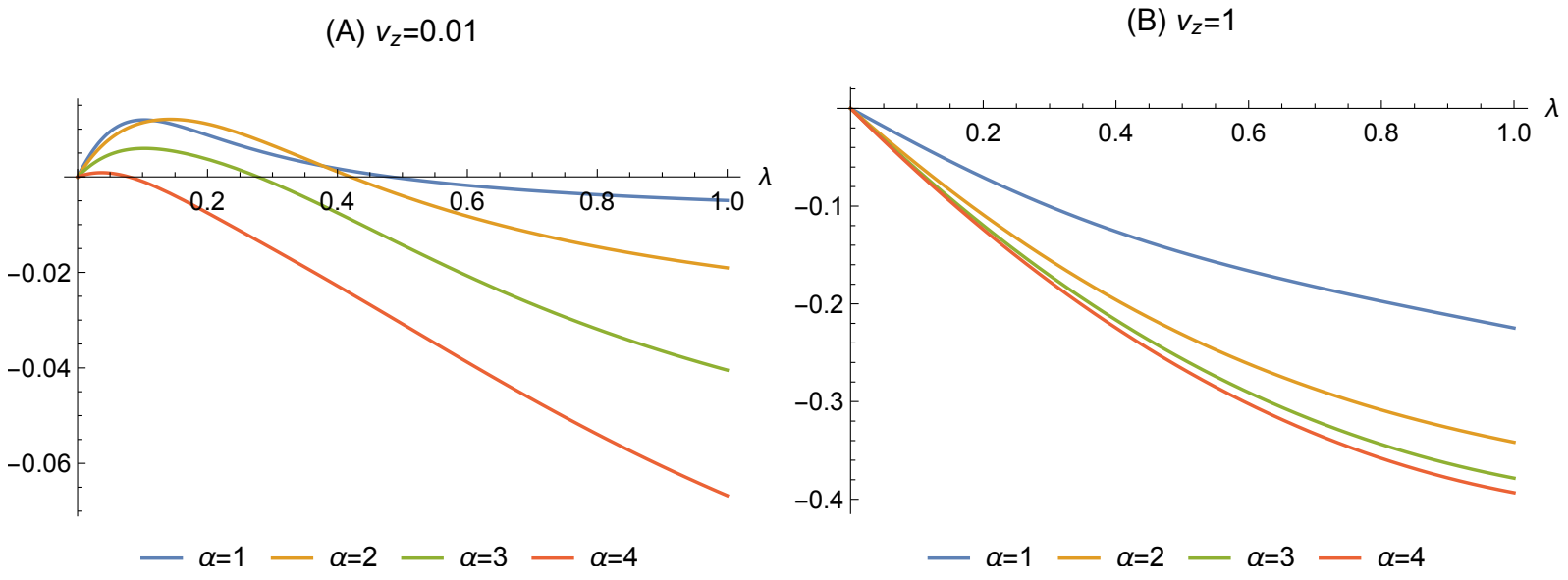
- Social welfare:

$$\mathcal{U}(\lambda) = \lambda\, V_I(\lambda) + (1-\lambda)\, V_U(\lambda),$$

$$\mathcal{U}'(\lambda) = \underbrace{\left(V_I(\lambda) - V_U(\lambda)\right)}_{>0} + \underbrace{\frac{d}{d\lambda} V_U(\lambda)}_{<0} + \underbrace{\lambda \frac{d}{d\lambda}\left(V_I(\lambda) - V_U(\lambda)\right)}_{<0}. \tag{1}$$

Exogenous Information Equilibrium —Social Welfare

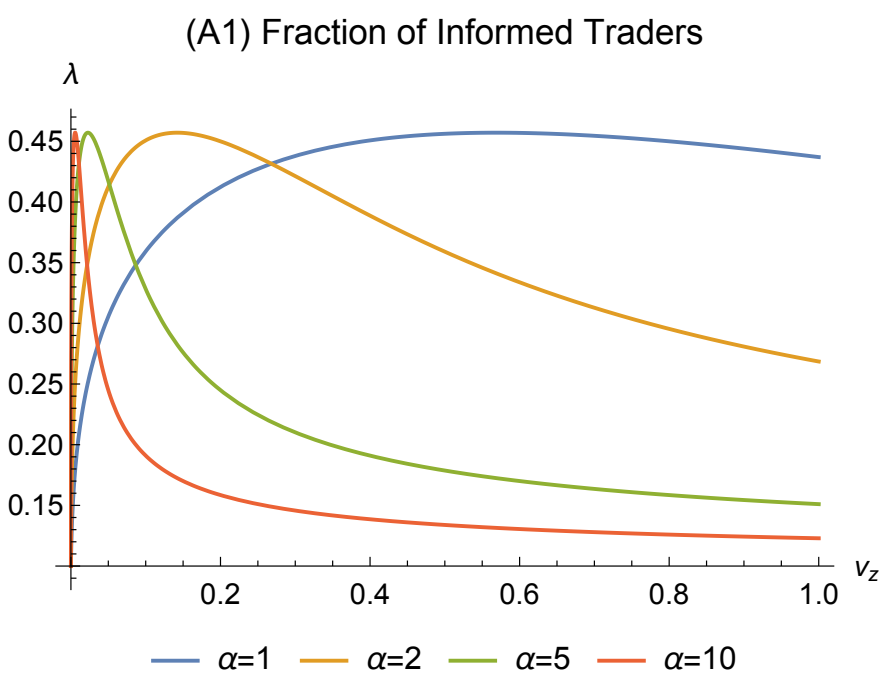
By looking at the percentage of change in welfare $1 - \mathcal{U}(\lambda)/\mathcal{U}(0)$, we see that informed trading (price efficiency) improves aggregate welfare only when: (i) λ is small (i.e., the price is less informative), (ii) supply risk v_z is small and (iii) agents are less risk-averse.

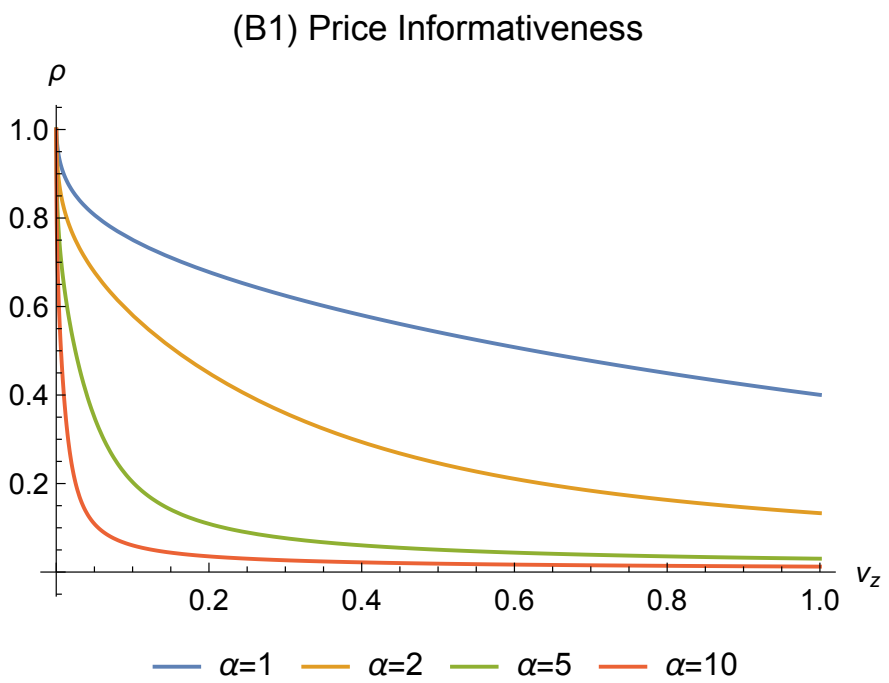


Endogenous Information Equilibrium—Informed Trading and Price Efficiency

The effect of noise supply

- In contrary to the GS model, the equilibrium fraction of informed traders does not monotonically increase with the noise supply;
- Informed trading improves price efficiency in general, but not necessarily when traders are either less risk averse or the supply is less noisy.

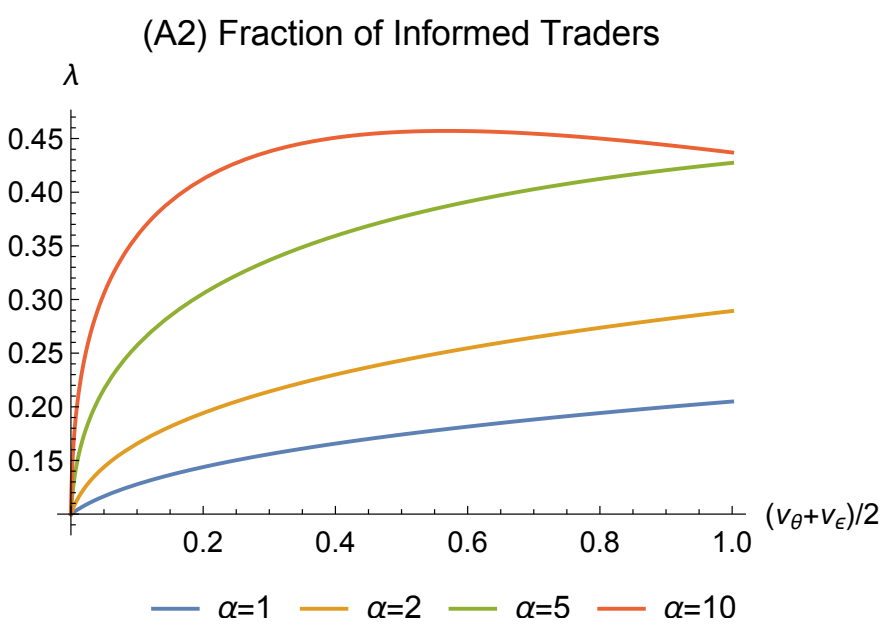


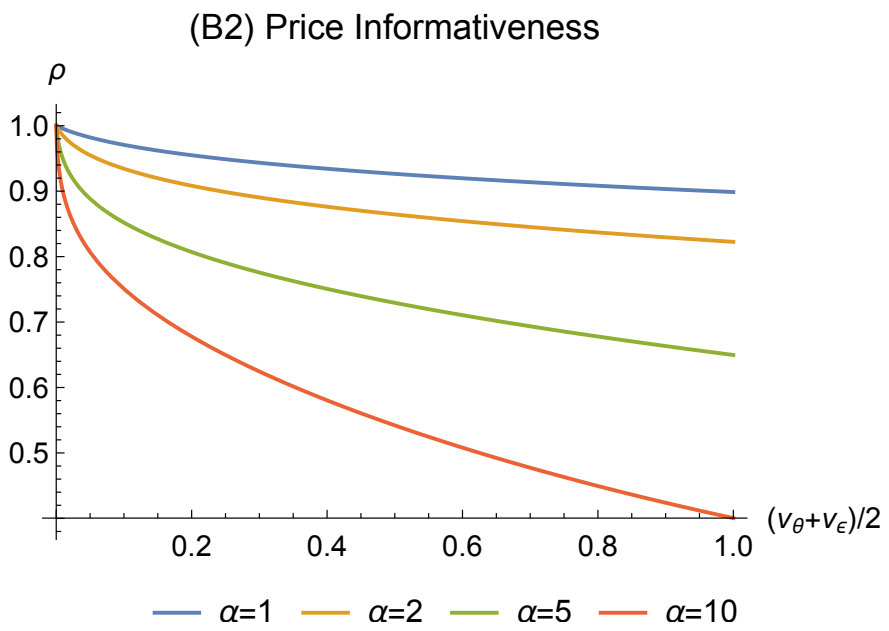


Endogenous Information Equilibrium—Informed Trading and Price Efficiency

The effect of dividend risk

- With an increase in the dividend risk, traders are making more effort to be informed but the price becomes less efficient
- The effect becomes more significant when traders become more risk averse.

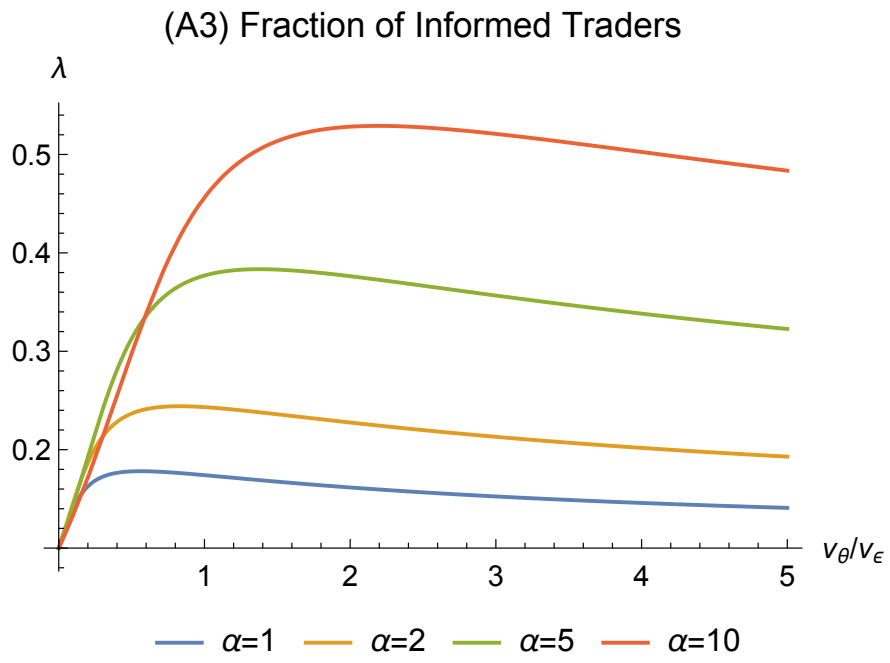


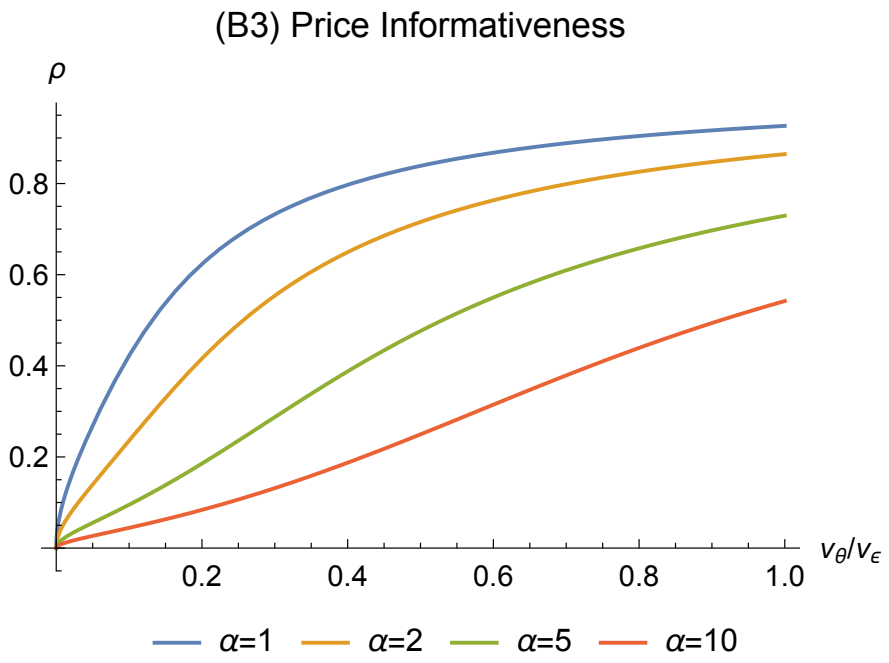


Endogenous Information Equilibrium—Informed Trading and Price Efficiency

The effect of informativeness of the signal

- Different from the GS model, with an increase in the informativeness, traders are making more effort to be informed so that price becomes more informative.





Endogenous Information Equilibrium—Informed Trading and Price Efficiency

- The information becomes **more valuable when the noise supply and the unobserved noise are neither too small nor too large** relative to the noise in the information,
- **Making more effort to becoming informed not necessarily makes the price more efficient**; in particulary
  - when more risk averse traders face increasing dividend risk;
  - when less risk averse traders face increasing supply risk

Endogenous Social Welfare

- Consider two different cost functions:

$$c_1(p) = p \ln \frac{p}{p_0} + (1-p) \ln \frac{1-p}{1-p_0}, p_0 \in (0, p);$$

$$c_2(p) = p \tanh^{-1}(p) + \frac{1}{2} \ln(1-p^2), \ p_0 = 0.$$

  - c_1 is of *entropic type* and is related to information theory (Hofbauer & Sandholm (2001), Mattsson & Weibull (2002)).
  - c_2 is of *hyperbolic type* and is in line with discrete choice models (Brock & Durlauf (2001)).

- The unique equilibrium $\lambda \in (p_0, 1)$ is then determined by

$$\lambda = g^{-1}(h(\lambda)) = \begin{cases} \frac{p_0}{p_0 + (1-p_0)e^{-h(\lambda)}}, & p_0 \in (0, p); \\ \tanh(h(\lambda)), & p_0 = 0. \end{cases}$$

Endogenous Social Welfare

- The ex-ante welfare in equilibrium can be written as a function of λ,

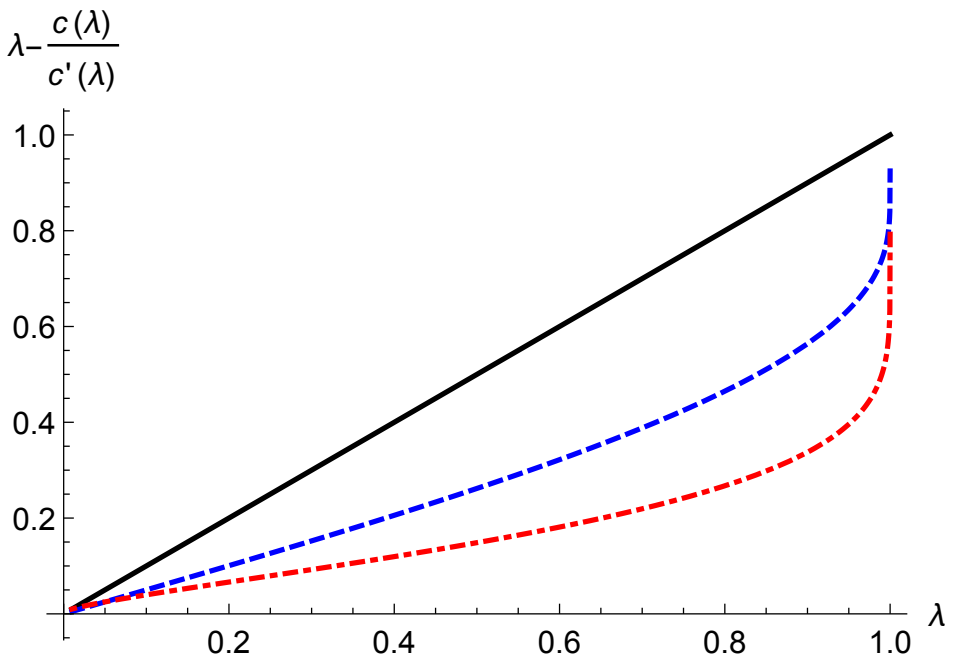
$$\mathcal{U}_i(\lambda) = V_U(\lambda) + \left(\lambda - \frac{c(\lambda)}{c'(\lambda)}\right)(V_I(\lambda) - V_U(\lambda)).$$

- **Marginal benefit of becoming informed**

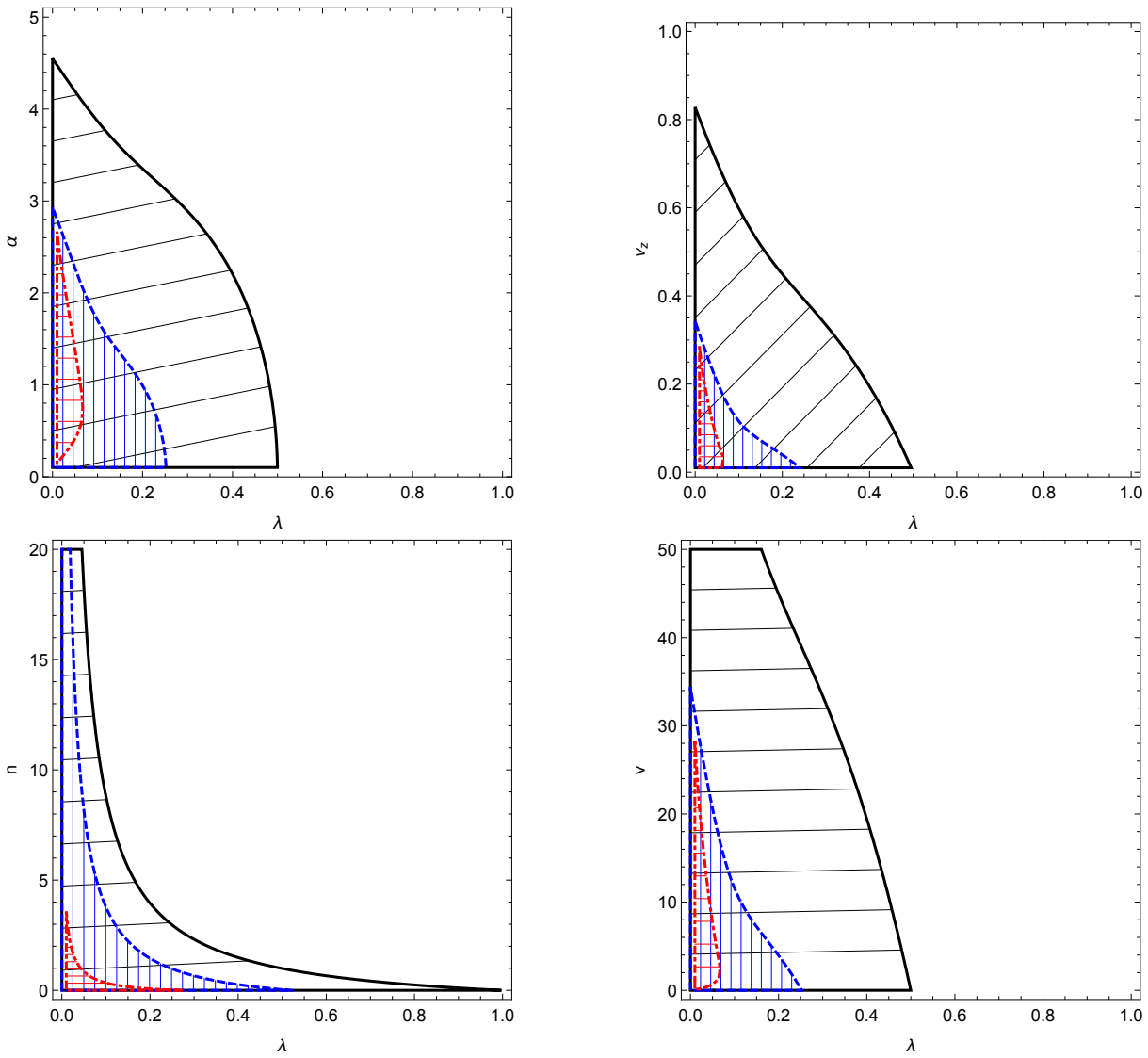
$$m(\lambda) \equiv \lambda - \frac{c(\lambda)}{c'(\lambda)} = \begin{cases} \frac{\ln\left(\frac{1-\lambda}{1-p_0}\right)}{\ln\left(\frac{1-\lambda}{1-p_0}\right) - \ln\left(\frac{\lambda}{p_0}\right)}, & p_0 \in (0, \lambda); \\ -2\ln(1-\lambda), & p_0 = 0, \end{cases}$$

Endogenous Social Welfare

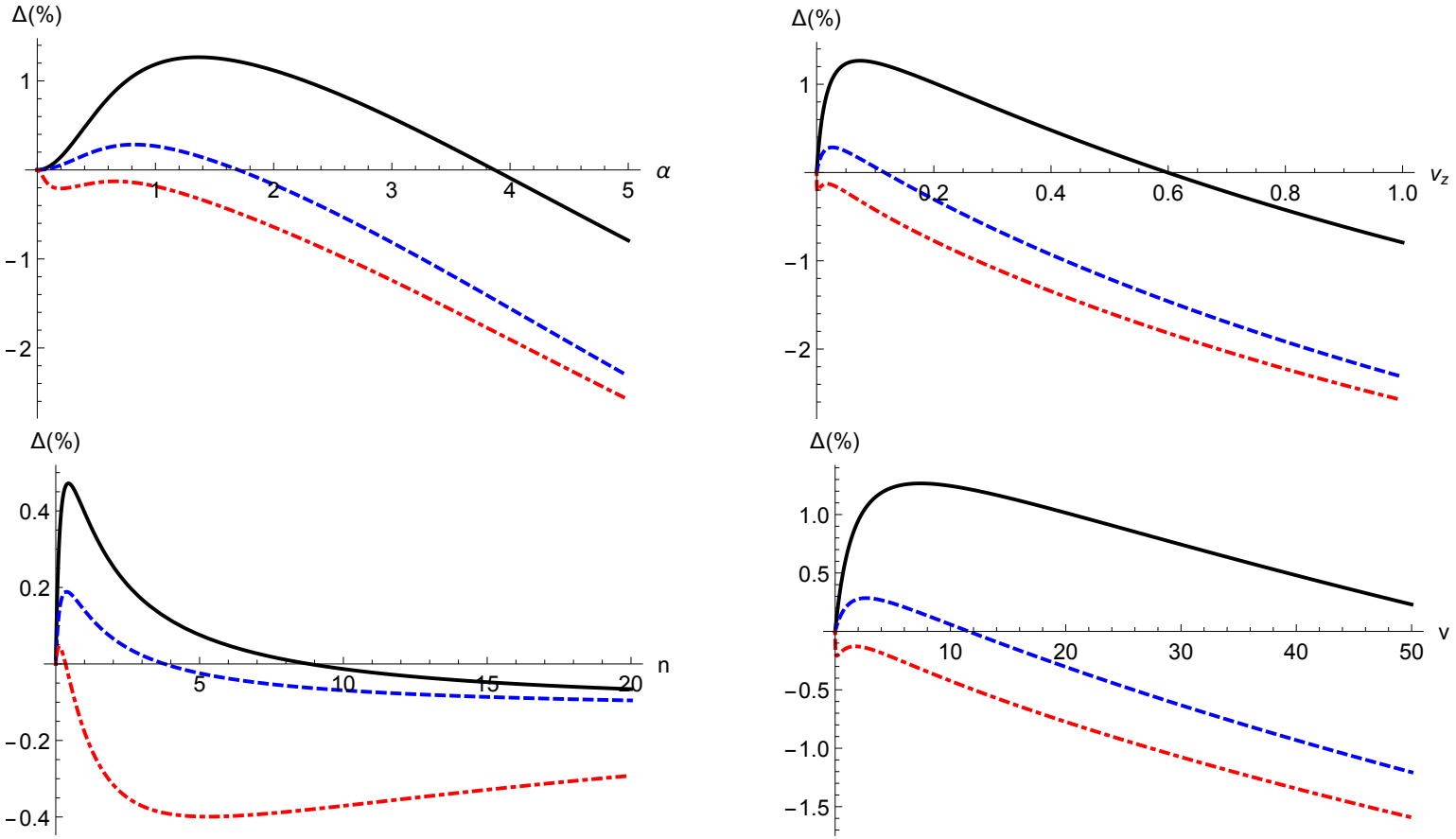
The marginal benefit is strict larger for $c_2(\cdot)$ than for $c_1(\cdot)$, so informed trading is more likely to improve investors' welfare under the cost structure $c_2(\cdot)$ than $c_1(\cdot)$.



Endogenous Social Welfare—Improvement



Endogenous Social Welfare—Net Gain



Endowment Shocks

- Rather assuming noise in supply, we explicitly to motivate trading using *endowment shocks*
- Each trader $i \in \{I, U\}$ receives an endowment, $e_i \tilde{D}$, at the end of the trading period, is known to trader i, but not known to other traders
- Moreover $e_i = \tilde{z} + \tilde{u}_i$, where $\tilde{z} \sim \mathcal{N}(0, v_z)$ is an aggregate endowment shock and $\tilde{u}_i \sim \mathcal{N}(0, v_u)$ is an idiosyncratic shock
- The main results still hold

Conclusion

- Considering uncertainty to be informed
- Making effort improves expected utility but also increases the disutility of such effort
- Characterize the optimal effort in a Nash equilibrium and the REE price
- Traders make more effort when the information becomes more valuable
- More effort does not always make price more efficient
- Improving public information can be socially valuable