Bailouts and Bank Runs

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July 8, 2011

Introduction

- Diamond and Dybvig (1983) showed how the bank can improve the welfare by providing the demand-deposit contract. It also indicated that bank runs can occur as a result of coordination failure.

- The recent financial crisis reminded us that bank runs is still a problem today. It also generated a debate about the government bailout.

- This paper tries to study several questions related to bailouts and bank runs by using a Diamond-Dybvig type of model.

Main Results

- When a bank run occurs, a government bailout can improve the efficiency of the resource allocation.
  - Bank runs imply misallocation of the resource in the banking sector.
  - A government bailout is an expost response to this misallocation.
- But government bailouts also increase the ex-ante probability of bank runs.
  - Not because of moral hazard.
  - Given the bailout, depositors have higher probability of getting their money from the bank during the run.
  - Withdrawing money early is more attractive for the depositors.
- The optimal (ex ante) level of bailout is chosen to balance these two effects.

Main Results

- Each bank is small. It won't take into account its effect on the bailouts and public goods provision.
- There is also a time inconsistency problem associated with the government bailouts. Once the bank run occurs, the government is tempted to set the bailout at the expost efficient level (rather than the ex ante efficient level).

Related Papers

- Two types of bank run models:
    - Sunspots Approach: Peck and Shell (2003).
    - Global Game Approach: Goldstein and Pauzner (2005).
- Keister (2010) studied the relation of bailouts and bank runs using the sunspots approach.
- This paper used the global game approach.

Outline:

- ► The model
- ► The post deposit game: given the government policy, what the depositors will do.
- ► The pre deposit choice: knowing the depositors response function, what policy the government will choose.
- ► The analysis of the model taking into the account the time inconsistency problem.

The Model:

- Three periods $t = 0, 1, 2$
- A continuum of agents with measure 1.
- Each agent has 1 unit of endowment at period 0.

The Model: Preferences

- Agents consume both the private good and the public good.

- The public good is provided by the government in period 1.

- As for the consumption of the private good, the agent can be either impatient or patient.

- $$\begin{cases} u(c_1) + v(g) & \text{if impatient} \\ \\ u(c_1 + c_2) + v(g) & \text{if patient} \end{cases}$$

- u and v are continuous, twice differentiable, strictly increasing and concave.

- $u(0) = 0$

- The possibility of being impatient is λ for each agent.

The Model: Asset Return

- ▶ The endowment of each agent can be invested in period 0
- ▶ The return of the asset depends on the fundamental of the economy $\theta \in [0, 1]$
- ▶ For $\theta \in [0, \overline{\theta}]$

	Period 0	Period 1	Period 2	
Asset	-1	r=1		
	-1	0	R	w.p. $p(\theta)$
			0	w.p. $1 - p(\theta)$

Table 1: Asset Return

- ▶
  - ▶ $p(\theta)$ is strictly increasing in θ.

The Model: Asset Return

- For $\theta \in [\overline{\theta}, 1]$

    - $r = R$, $p(\theta) = 1$
    - $\overline{\theta}$ is the point above which the short run liquidation value also improves with the fundamental.

- $\overline{\theta} \to 1$.

- $E_\theta[p(\theta)]u(R) > \overline{\theta}u(1) + (1 - \overline{\theta})u(R)$

- One unit of the proceeds from the asset can be turned into one unit of public good or one unit of private good.

- The public good can only be provided by the government.

The Model: Information

- In period 0:
  - each agent knows that in period 1, he can be "impatient" with probability λ.
  - the fundamental $\theta \sim U[0, 1]$

- In period 1, agent i
  - knows his own "type", which is private information.
  - gets a signal $\theta_i = \theta + \varepsilon_i$. $\varepsilon_i \sim i.i.d\ U[-\varepsilon, \varepsilon]$
  - ε is sufficiently small. Agent i's updated belief about θ is

$$\theta \sim U[\max\{0, \theta_i - \varepsilon\}, \min\{1, \theta_i + \varepsilon\}]$$

The Model: Timeline

Period 0:

- The government collects τ

- The bank announces the demand-deposit contract:
  - The agent need to deposit $1 - \tau$ in period 0
  - Then he can demand withdrawal in either period 1 or 2.
  - If he demands withdrawal in period 1, he gets a fixed payment c_1 until the bank runs out of resource.
  - If he waits until period 2, he shares the leftover resource in the bank with the remaining depositors.

- Agents decide whether to make a deposit or not.

The Model: Timeline

Period 1:

- ▶ Agent i learns his type, receives his signal θ_i. he also decides whether to demand withdrawal.

- ▶ The bank pays c_1 to depositors demanding withdrawal until it runs out of resources.

- ▶ Once the bank runs out of resources, it will seek government's help.

- ▶ If the government helps, it will use the tax revenue τ to satisfy the withdrawal demand. And the government also provides the public good.

The Model: Risk Sharing through the Bank

- In autarky,
    - the patient consumer: $E_\theta[p(\theta)]u[(1 - \tau)R] + v(\tau)$.
    - the impatient consumer:
      $\overline{\theta}u(1 - \tau) + (1 - \overline{\theta})u[(1 - \tau)R] + v(\tau)$
- As Diamon-Dybvig (1983), the expected utility can be improved by making agents share the risk of being impatient.

The Model: Risk Sharing through the Bank

- ▶ If "type" is observable, then only the impatient depositors are allowed to withdraw in period 1.
- ▶ Free entry in the banking sector. The bank will maximize the depositors expected utility from the private good consumption.
- ▶ $w(\tau) = \max\limits_{c_1} \lambda u(c_1) + (1-\lambda) E_\theta[p(\theta)] u(\frac{1-\tau-\lambda c_1}{1-\lambda} R)$
    - ▶ FOC : $u'(c_1^*) = E[p(\theta)] R u'[\frac{1-\tau-\lambda c_1^*}{1-\lambda} R]$
    - ▶ $1 - \tau < c_1^*(\tau) < (1-\tau)R$

The Model: Resource Allocation Between Public and Private Good

- The government is benevolent. It will maximize the expected utility of the agents by choosing a tax rate τ

- $\max\limits_{\tau} w(\tau) + v(\tau)$

- $FOC : v'(\tau) = -w'(\tau)$

- Resource Allocation:
$$\begin{cases} v'(\tau^{FB}) = E[p(\theta)]Ru'[\frac{1-\tau^{FB}-\lambda c_1^{FB}}{1-\lambda}R] \\ \\ u'(c_1^{FB}) = E[p(\theta)]Ru'[\frac{1-\tau^{FB}-\lambda c_1^{FB}}{1-\lambda}R] \end{cases}$$

The Model: "Types" are not observable

- When "types" are not observable, we may have bank runs.

- Bank runs: a positive measure of patient depositors demand withdrawal in the first period.

- The patient depositors may demand withdrawal in the first period because they expect other patient depositors will do so or they expect the fundamental will be very bad or both.

The Model: Government Bailout

- After the government starts the bailout, it pays c_1 to each depositor who wants to withdraw until the government has paid out $\overline{b}$

- So the actual resource the government spends in bailout, b, depends on the number of depositors who want to withdraw early (n) and $\overline{b}$.

- $b = \begin{cases} 0 & \text{if } nc_1 \leq (1-\tau) \\ \\ nc_1 - (1-\tau) & \text{if } \overline{b} + (1-\tau) \geq nc_1 > (1-\tau) \\ \\ \overline{b} & \text{if } nc_1 > \overline{b} + (1-\tau) \end{cases}$

- So the government will choose $\overline{b}$.

- $g = \tau - b$

Post Deposit Game: Period 1.

- To study the model, we need to analyze it backward.
- For given τ and $\overline{b}$ chosen by the government and c_1 chosen by the bank, if the agent has deposited his resources, will he demand withdrawal in period 1?
- Impatient depositors will.
- The withdrawal decision of a patient depositor depends.
- The game among the patient depositors is called post deposit game.

Post Deposit Game: Period 1.

- **Definition:** in the post deposit game, a strategy for patient depositor i is a mapping $s_i : [0 - \varepsilon, 1 + \varepsilon] \to \{\text{withdraw,wait}\}$.
- **Definition:** An equilibrium of the post deposit game is a profile of strategies-one for each patient depositor-such that for each patient depositor i, s_i maximizes his expected utility given s_{-i}
- We restrict our attention to the monotone and symmetric strategies: that is $s_i(\theta_i) = \text{withdraw}$ if and only if $\theta_i < k$.

Post Deposit Game: Period 1.

- Given the strategy of the patient depositors, the number of depositors who demand withdrawal is a function of θ and k:

  - $$n(\theta; k) = \begin{cases} \lambda \text{ if } \theta > k + \varepsilon \\[2ex] \lambda + (1 - \lambda)[\frac{k - (\theta - \varepsilon)}{2\varepsilon}] \text{ if } k + \varepsilon \leq \theta \leq k + \varepsilon \\[2ex] 1 \text{ if } \theta < k + \varepsilon \end{cases}$$

- $A_1(k) = \{ \theta \mid n(\theta; k)c_1 \leq (1 - \tau)r \}$
- $A_2(k) = \{ \theta \mid (1 - \tau)r < n(\theta; k)c_1 \leq \overline{b} + (1 - \tau)r \}$

Post Deposit Game: Period 1.

- the difference between "wait" and "run" is $\delta(\theta, k) =$

$$
\begin{cases}
p(\theta)u(\frac{1-\tau-nc_1}{1-n}R) - u(c_1), & \text{if } \theta \in A_1(k) \\[2ex]
-u(c_1) & \text{if } \theta \in A_2(k) \\[2ex]
-\frac{\overline{b}+(1-\tau)}{nc_1}u(c_1) & \text{if } \theta \notin A_1(k) \cup A_2(k)
\end{cases}
$$

Post Deposit Game: Period 1

$$\Delta(k, k) = \int_{\lambda}^{(1-\tau)/c_1} [p(\theta(k, n))u(\frac{1-\tau-nc_1}{1-n}R) - u(c_1)]dn$$

▶

$$- \int_{(1-\tau)/c_1}^{[(1-\tau)+\overline{b}]/c_1} u(c_1)dn$$

$$- \int_{[(1-\tau)+\overline{b}]/c_1}^{1} \frac{1-\tau+\overline{b}}{nc_1}u(c_1)dn$$

Post Deposit Game: Period 1.

- **Theorem:** *In the post deposit game, there is a unique threshold equilibrium. To be more specific, there is a unique k^* such that a patient depositor demands withdrawal in the first period if and only if he receives a signal below k^*.*

Post Deposit Game: Period 1

- **Theorem**: *The probability of bank run (k^*) is an increasing function of bailout limit ($\overline{b}$), if the bailout can't rule out the bank runs ($\overline{b} + (1 - \tau) < c_1$)*

- **Theorem:** *The probability of bank run (k^*) is an increasing function of tax level (τ) and first-period promised return by the bank (c_1)*

Deposit Contract Announced in Period 0

- Given τ and $\bar{b}$, the bank will choose c_1 to maximize the depositors' expected utility from the private good consumption.

-
$$EU_p(c_1) = \int_0^{k^*} \left[\frac{1-\tau+\bar{b}}{c_1} u(c_1)\right] d\theta$$

$$+ \int_{k^*}^1 \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left[\frac{1-\tau-\lambda c_1}{1-\lambda}R\right]\right] d\theta$$

Deposit Contract Announced in Period 0

- $p(\underline{\theta}) = u(1)/u(R)$
- **Theorem:** If $\underline{\theta}$ is sufficiently small, the demand-deposit contract chosen by the bank enables risk sharing among the depositors. To be more specific $c_1 > 1 - \tau$
- **Corollary:** Agents will always choose to deposit their resource in period 0.

Government Policy in Period 0

- The government will choose τ and $\overline{b}$ to maximize the expected utility of the agents from both the private good consumption and the public good consumption.

$$EU(\tau, \overline{b}) = \int_0^{k^*} \left[\frac{1-\tau+\overline{b}}{c_1} u(c_1) + v(\tau - \overline{b})\right] d\theta$$

$$+ \int_{k^*}^1 \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left[\frac{1-\tau-\lambda c_1}{1-\lambda} R\right] + v(\tau)\right] d\theta$$

Government Policy in Period 0

- We have seen that the bailout can increase the probability of bank runs.

- On the other hand, the bailout can improve the resource allocation if a bank run does occur.

  - The benefit of providing one unit of resource to the banking sector and satisfy more withdrawal demand is $u(c_1)/c_1$
  - This will cost one unit of public good $v'(\tau)$
  - The net benefit is $u(c_1)/c_1 - v'(\tau) > 0$

Government Policy in Period 0

▶ The government will find the optimal ex ante level of $\overline{b}$ by balancing these two effects.

$$k^*[\tfrac{1}{c_1}u(c_1) - v'(\tau - \overline{b})]$$

$$= \tfrac{\partial k^*}{\partial \overline{b}}[\lambda u(c_1) + (1 - \lambda)p(k^*)u[\tfrac{1-\tau-\lambda c_1}{1-\lambda}R]$$

$$+ v(\tau) - \tfrac{1-\tau+\overline{b}}{c_1}u(c_1) - v(\tau - \overline{b})]$$

Regulation is necessary in Period 0

- From the government planner's point of view of a social planner, the bank's provides a too high c_1.

- $EU = EU_p + k^* v(\tau - \overline{b}) + (1 - k^*) v(\tau)$

- $\partial EU / \partial c_1 = EU_p / \partial c_1 - \underbrace{\partial k^* / \partial c_1 [v(\tau) - v(\tau - \overline{b})]}_{\textit{Externality}}$

Comparison Among Different Regimes:

- If the bank can't make an commitment, once the bank run occurs. The government will choose the ex post efficient bailout level: $v'(\tau - \overline{b}^{NC}) = \frac{1}{c_1}u(c_1)$

- If the government can't make a commitment on the bailout level, then we have:

  - (1)In the post deposit game, there exists a unique threshold equilibrium k^{*NC}.
  - (2) k^{*NC} is an increasing function of τ and c_1.
  - (3) If $\underline{\theta}$ is sufficiently small, the demand-deposit contract chosen by the bank enables risk sharing among the depositors. To be more specific $c_1 > 1 - \tau$

Comparison Among Different Regimes:

- $\overline{b}^{NC}(\tau, c_1) > \overline{b}^{C}(\tau, c_1)$

-
$$\max_{\tau, \overline{b}} EU^C > \max_{\tau} EU^{NC}$$

- It is interesting to compare no commitment and no bailout.

Comparison Among Different Regimes:

- $k^{*NC}(\tau, c_1) > k^{*NB}(\tau, c_1)$.
- Stability VS Discretion.
- **Theorem:** If $p'(k^{*NB})$ is sufficiently large,
  $$\max_{\tau,\, c_1} EU^{NC} > \max_{\tau,\, c_1} EU^{NB}$$

Conclusion:

- ▶ When a bank run occurs, a government bailout can improve the efficiency of the resource allocation.
- ▶ But government bailouts also increase the ex-ante probability of bank runs.
- ▶ The optimal level of bailout is chosen to balance these two effects.
- ▶ The bank will underestimate the cost of bank runs because of the government bailouts. So regulation in the banking sector is necessary given the bailout.
- ▶ There is also a time inconsistency problem associated with the government bailouts. Once the bank run occurs, government is tempted to give more bailout than the ex ante optimal level. If the government can't make a commitment, bailout may not be desirable.