Endogeous Credit Cycles

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June 25, 2011

Introduction

"As we have seen, all financial institutions are at the mercy of our innate inclination to veer from euphoria to despondency." – *The Ascent of Money* by Niall Ferguson

Introduction

Issues:

- We study credit markets with limited commitment.
- Many people these days think credit markets are plagued by problems.
- What is the nature of the equilibrium?
- Why do credit markets fluctuate?
- What are the efficiency properties?

We build a new model that is useful for thinking about these issues.

Introduction

Credit models are like monetary models (Sanches and Williamson 2010):

- Beliefs matter a lot.
  - money: what agents will accept in future and at what terms.
  - credit: whether agents will repay.
- When beliefs matter, interesting equilibrium can emerge. We illustrate how credit markets can display exotic dynamics, including sunspots, cycles, and chaos.

Related literature

- Limited commitment: Kehoe and Levine (1993, 2001), Alvarez and Jermann (2000).
- Credit cycles: Kiyotaki and Moore (1997), etc.
- Mattesini, Monnet and Wright (2010).

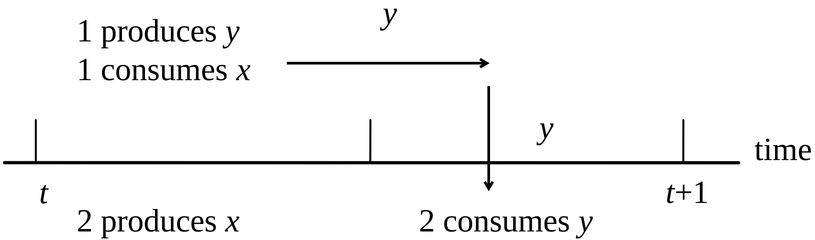
Environment

- Time is discrete and continues forever.
- Each period is divided into two subperiods.
- There are two types of agents: type 1 and type 2, equal measure.
- Two goods every period: good 1 and good 2.
- Type i agents consume good i and produce good $-i$.

Environment

- All production takes place in the first subperiod.
- Type i consumes good i in subperiod i.
- Type 1 agents store good 2 for subperiod 2.
- Type 1: $U^1(x,y)$; Type 2: $U^2 = U^2(y,x)$. Standard curvature assumptions.
- Normal goods.

Environment



Environment

- Discount factor: β
- Partial collateral: type 1 agents receive liquidation value in terms of utility λy. But $U^1(0, y) + \lambda y < 0$ for all $y > 0$.
- Imperfect monitoring: $\pi = \Pr(\text{detect deviation})$
- Markets for date-t goods open only at date t.

Pricing mechanisms

- Nash bargaining
- Walrasian price-taking

Efficient stationary allocations

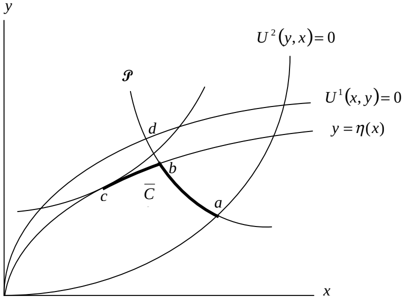
Consider stationary allocations (x, y).

Payoffs: $V^1 = \frac{U^1(x,y)}{1-\beta}, V^2 = \frac{U^2(y,x)}{1-\beta}$

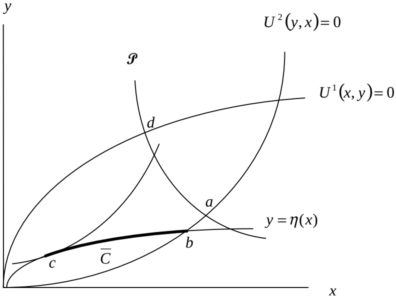
PC: $U^1(x,y) + \beta V^1 \geq 0 \iff U^1(x,y) \geq 0$
$U^2(y,x) + \beta V^2 \geq 0 \iff U^2(y,x) \geq 0$

RC: $\beta V^1 \geq \lambda y + (1-\pi)\beta V^1 \iff \lambda y \leq \frac{\beta\pi}{1-\beta} U^1(x,y)$
$\iff y \leq \eta(x)$

Efficient stationary allocations



$\bar{\mathcal{C}}$ (RC is loose)



$\bar{\mathcal{C}}$ (RC is tight).

Equilibrium 1: Nash

The credit limit

$$\begin{aligned} \beta V_{1t+1} &\geq \lambda y_t + \beta(1-\pi) V_{1t+1} \\ \iff & y_t \leq \phi_t \equiv \frac{\beta\pi}{\lambda} V_{1t+1}. \end{aligned}$$

Generalized Nash bargaining problem:

$$\max_{x_t, y_t} U^1(x_t, y_t)^\theta \, U^2(y_t, x_t)^{1-\theta}$$
$$s.t. \; U^1(x_t, y_t) \geq 0, \;\; U^2(y_t, x_t) \geq 0, \; y_t \leq \phi_t.$$

Equilibrium 1: Nash

Solution:

$$\begin{array}{ll} \text{if } \phi_t < y^* \text{ then} & y_t = \phi_t \text{ and } x_t = h^N(\phi_t) \\ \text{if } \phi_t \geq y^* \text{ then} & y_t = y^* \text{ and } x_t = h^N(y^*) \end{array}$$

where y^* solves the bargaining problem without the repayment constraint and $h^N(\cdot)$ solves FOC from Nash Bargaining.

Equilibrium 2: Walrasian

Type 1: $\max U^1(x_t, y_t)$ s.t. $p_t x_t = y_t,\; y_t \leq \phi_t$

Type 2: $\max U^2(y_t, x_t)$ s.t. $p_t x_t = y_t$

Solution:

$$\begin{array}{ll} \text{if } \phi_t < y^* \text{ then} & y_t = \phi_t \text{ and } x_t = h^W(\phi_t) \\ \text{if } \phi_t \geq y^* \text{ then} & y_t = y^* \text{ and } x_t = h^W(y^*) \end{array}$$

where y^* solves the bargaining problem without the repayment constraint and $h^W(\cdot)$ solves agent 2's FOC.

Dynamic credit limits

Type 1's life-time utility

$$V_{1t} = U^1(x_t, y_t) + \beta V_{1t+1}.$$

The dynamic system can be summarized by

$$\phi_{t-1} = \frac{\beta\pi}{\lambda} U^1(x_t, y_t) + \beta\phi_t$$

where (x_t, y_t) is determined as above.

Equilibrium

An equilibrium is given by bounded sequences of credit limits $\{\phi_t\}_{t=1}^{\infty}$ and allocations $\{x_t, y_t\}_{t=1}^{\infty}$ such that for all t:

1. Given ϕ_t, (x_t, y_t) solves

$$\begin{array}{ll} \text{if } \phi_t < y^* \text{ then} & y_t = \phi_t \text{ and } x_t = h(\phi_t) \\ \text{if } \phi_t \geq y^* \text{ then} & y_t = y^* \text{ and } x_t = h(y^*) \end{array}$$

2. Given (x_t, y_t), ϕ_t solves

$$\phi_{t-1} = \frac{\beta\pi}{\lambda} U^1(x_t, y_t) + \beta\phi_t.$$

The dynamical system

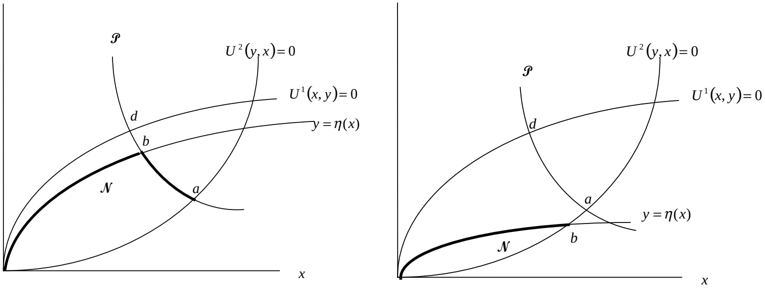
Plug (1) into (2). The equilibrium is characterized by:

$$\phi_{t-1} = f(\phi_t) \equiv \begin{cases} \frac{\beta\pi}{\lambda} U^1 \left[h(\phi_t), \phi_t\right] + \beta\phi_t, & \text{if } \phi_t < y^*, \\ \frac{\beta\pi}{\lambda} U^1 (x^*, y^*) + \beta\phi_t, & \text{otherwise.} \end{cases}$$

Stationary equilibrium – Nash

Define

$$\mathcal{N} = \left\{ (x, y) \, | y = \eta(x) \text{ if } x \leq \hat{x}; \ (x, y) \in \bar{\mathcal{C}} \text{ if } x > \hat{x} \right\}.$$



$\mathcal{N}$ (RC is loose) $\mathcal{N}$ (RC is tight)

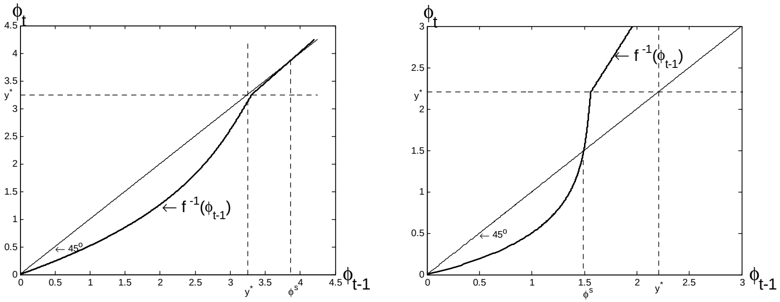
Stationary equilibrium – Nash

- There exists a non-zero stationary equilibrium for all $\theta \in [0, 1]$.
- For all $(x, y) \in \mathcal{N}$, $\exists\ \theta \in [0, 1]$, such that (x, y) is the equilibrium.
- Because $\bar{\mathcal{C}} \neq \mathcal{N}$, the equilibrium (x, y) is in general different from the efficient stationary allocation.
- Because $\bar{\mathcal{C}} \subset \mathcal{N}$, for all allocation $(x, y) \in \bar{\mathcal{C}}$ can be supported by a Nash Bargaining solution with properly chosen θ.

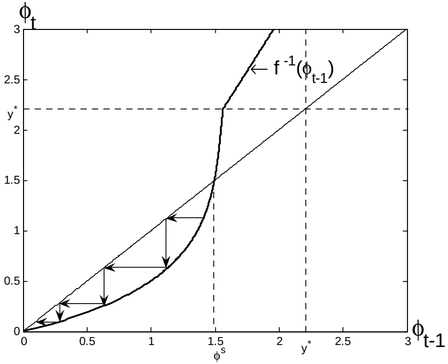
Stationary equilibrium – Walrasian

- There exists a non-zero stationary equilibrium.
- All positive stationary equilibria are in $\bar{\mathcal{C}}$.

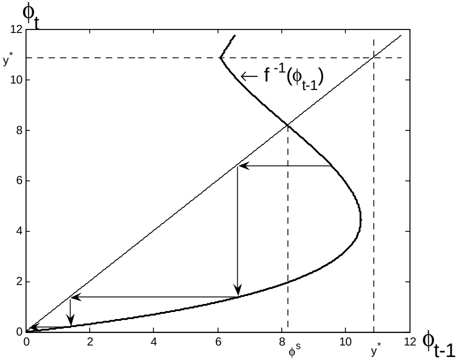
The dynamical system



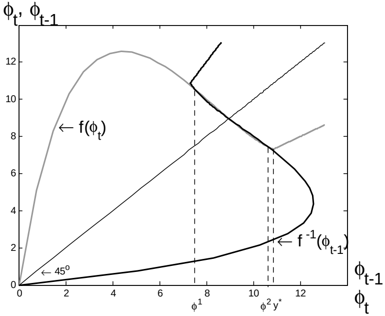
The dynamical system



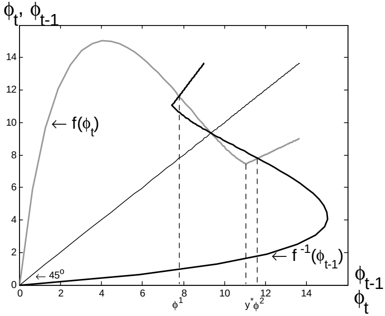
The dynamical system



Deterministic cycles (NB)

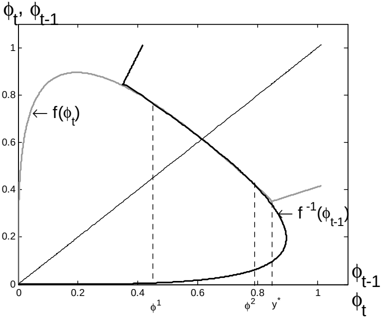


Nash cycle, $\phi^1, \phi^2 < y^*$

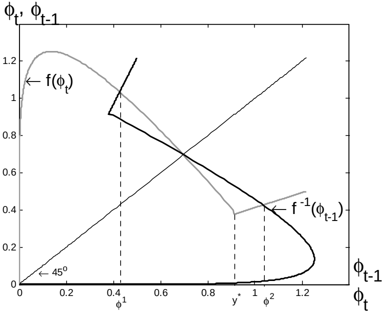


Nash cycle, $\phi^1 < y^* < \phi^2$

Deterministic cycles (WP)



Walras cycle, $\phi^1, \phi^2 < y^*$



Walras cycle, $\phi^1 < y^* < \phi^2$

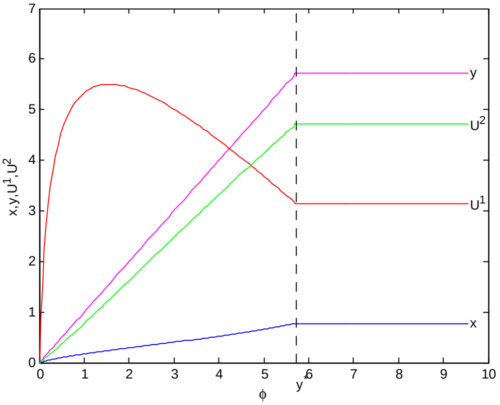
Intuition

The existence of cycles relies on the downward sloping $f\left(\phi_t\right)$ curve $\Rightarrow$ downward sloping $U^1\left(h\left(\phi_t\right), \phi_t\right)$.

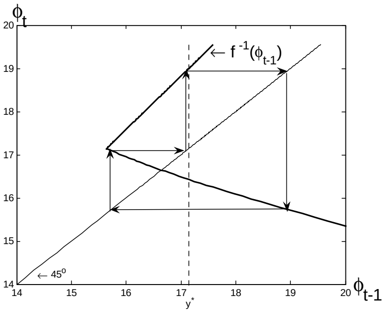
Prop: If good y is normal for type 1 and type 2, equilibrium implies $\partial U^1 / \partial \phi < 0$ for $\phi \approx y^*$ in either Nash or Walrasian equilibrium.

Remark: Consider the labor market...

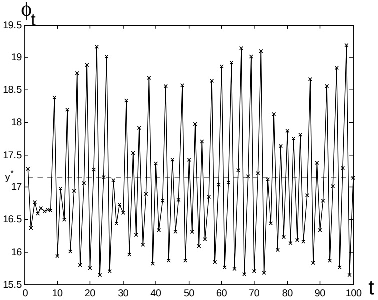
Intuition



Deterministic cycles and chaotic dynamics (NB)



A three-period cycle



Chaotic dynamics

Stochastic cycles: sunspot equilibrium

- A sunspot variable $s_t \in \{1, 2\}$ in each period t.
- The distribution of s_t follows a Markov process.
  $\Pr(s_{t+1} = s | s_t = s) = \sigma^s$, $s = 1, 2$.
- The repayment constraint for a type 1 agent in state s is

$$y_t^s \leq \frac{\beta\pi}{\lambda}\left[\sigma^s V_{1t+1}^s + (1 - \sigma^s) V_{1t+1}^{-s}\right] \equiv \phi_t^s.$$

Stochastic cycles: sunspot equilibrium

An equilibrium is given by bounded sequences of credit limits $\{\phi_t^s\}_{t=1}^{\infty}$ and allocations $\{x_t^s, y_t^s\}_{t=1}^{\infty}$ such that for all t and $s = 1, 2$:

1. Given ϕ_t^s, (x_t^s, y_t^s) solves

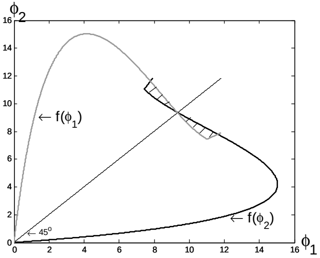
$$\begin{array}{lll} \text{if } \phi_t^s < y^* \text{ then} & y_t = \phi_t^s \text{ and } x_t^s = h\left(\phi_t^s\right) \\ \text{if } \phi_t^s \geq y^* \text{ then} & y_t = y^* \text{ and } x_t^s = h\left(y^*\right) \end{array}$$

2. Given (x_t^s, y_t^s), ϕ_t^s solves

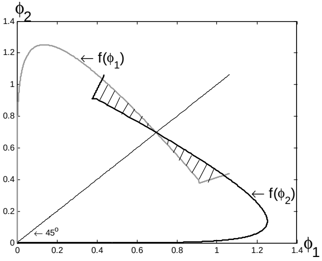
$$\begin{aligned} \phi_{t-1}^s &= \sigma^s \left(\frac{\beta\pi}{\lambda} U^1 \left(x_t^s, y_t^s\right) + \beta\phi_t^s \right) + \\ &\quad (1 - \sigma^s) \left(\frac{\beta\pi}{\lambda} U^1 \left(x_t^{-s}, y_t^{-s}\right) + \beta\phi_t^{-s} \right) \end{aligned}$$

Stochastic cycles

If $f'(\phi^s) < -1$ and $\phi^s < y^*$, there exist (σ^1, σ^2), $\sigma^1 + \sigma^2 < 1$, such that given the sunspot distribution the economy has a sunspot equilibrium in the neighborhood of ϕ^s.



Nash sunspot equilibria



Walras sunspot equilibria

A few remarks on efficiency

A planner's problem with limited commitment:

$$V^2\left(V^1\right) = \max_{x,y,V^{1\prime}} U^2\left(y,x\right) + \beta V^2\left(V^{1\prime}\right)$$

$$s.t.\ U^1\left(x,y\right) + \beta V^{1\prime} = V^1$$

$$\beta\pi V^{1\prime} \geq \lambda y$$

$$V^2\left(V^1\right) \geq 0$$

$$V_0^1 \geq 0$$

A few remarks on efficiency

- Planner's allocation exhibits back-loading property and converges to stationary allocation.
- Cycles are not efficient.
- Stationary equilibria are not efficient if it is credit constrained.
- Equilibria cannot be Pareto ranked in general.

Conclusion

- We studied dynamic credit markets with
  - limited commitment
  - imperfect collateral
  - random monitoring
- We consider Nash bargaining and Walrasian pricing
- Easy to get equilibria with exotic dynamics.
- Exotics dynamics are not efficient.