Trade Disclosure, Signal Jamming, and Strategic Competition

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Abstract

We analyze how public disclosure of informed traders' trades affects competition among informed traders in a dynamic Kyle model. Under disclosure requirement, an informed trader's order flow consists of two components: an information-based component for a profit and a random component for hiding information. The random components from all informed traders collectively equals, in distribution, the random orders from all liquidity traders. Market is more efficient with disclosure.

When signals are substitutes, disclosure makes the market more liquid and reduces informed traders' profits. Surprisingly, when signals are complementary, disclosure can cause the market to be less liquid for some period of time and increase informed traders' expected profits for some parameters.

We further examine how competition affects informed traders' trading strategy under disclosure. With uncorrelated signals, each informed trader behaves exactly like a monopolist. The aggregate profits of informed traders are also maximized when they have uncorrelated signals. When signals are substitues, informed traders trade more aggressively and market is more efficient with competition. The results are the opposite when signals are complementary.

In this paper, we analyze how disclosure affects the trading strategies of competing informed traders with imperfect information, and the implication of such trading strategies on market efficiency, market liquidity, and informed traders' profit in a Kyle (1985) type dynamic model. While informed traders would prefer not to reveal their trades, there are regulations that requires disclosure of trades. For example, corporate insiders are required to disclose their trades to the Securities and Exchange Commission (SEC). Section 16(a) of the SEC Act requires the insiders to report their trades to the Commission within ten days following the end of the month in which the trade occurs.

The disclosure of informed traders' trades creates incentives for informed traders to trade against their private information sometimes so that the market maker for the underlying security cannot perfectly infer information from their trades. As a result, the informed traders randomize to manipulate the market maker's belief until the last moment of trading. The mixed strategy allows the informed traders to maintain an informational advantage over the market for a longer period of time.

Intuitively, one would have thought that disclosure should make the market more efficient and more liquid, and reduce informed traders' expected profits. However, these results may not necessarily go through. Take the effects of disclosure on market efficiency as an example. On the one hand, disclosure improves market efficiency by making more information available to the market. On the other hand, each informed trader recognizes that his trades will be disclosed to other informed traders and the market maker, and he may trade less aggressively as a result. Disclosure will increase market efficiency only if the former effect is dominant. Therefore it is interesting to determine under what conditions disclosure will increase market efficiency, increase market liquidity and reduce informed traders' expected profits.

We consider a Kyle model of multiple informed traders each of whom is required to disclose his trade immediately after the trade is made. In discrete time, we derive a recursive formula for the equilibrium, which can be solved by numerical methods. In continuous time, we derive a closed-form formula for the equilibrium. To determine the impact of disclosure, we compare our closed-form equilibrium formula with that obtained in Back, Cao, and Willard (2000), whose model is the same except no disclosure is required there.

We find that the combined random components in informed traders' trade equals in distribution to that of the liquidity traders. This is intuitively appealing as informed traders and liquidity traders will each contribute to half of the trading volume. Too much randomization will cause informed traders to lose a lot from randomized trade and too little randomization will cause informed traders to lose their informational advantage too early.

We find that the effects of disclosure on market efficiency is unambiguous. Market is more efficient at all times after disclosure. When disclosure is required, informed traders know more about each other's signal than the case without disclosure. This makes informed trader's valuation to converge more quickly and their private information more short-lived. Thus, disclosure causes informed traders to trade more aggressively on their information based trading component, which in turn makes the market more efficient.

The effects on market liquidity is more complicated. Three factors contribute to market liquidity. The first is the randomization effect. As informed traders add noise to their own trades, this will increase market liquidity. The second effect is the trading intensity effect. With disclosure, informed traders trade more intensively with respect to their information. This will decrease market liquidity. The third effect is the residual uncertainty effect. Disclosure makes the market more efficient and this will increase market liquidity. With positively correlated signals, the first and third effects will dominate, resulting in a more liquid market at all times. In addition, informed traders' expected profits are less in any time interval since the inverse of market depth is proportional to informed traders' expected profits.

On the contrary, when informed traders' signals are negatively correlated, the second "trading intensity" effect can dominate and disclosure can cause the market to be illiquid for some period of time. Without disclosure, informed traders enter into a waiting game immediately with negatively correlated signals which cause them to trade very little. Conditional on public information revealed through price, informed traders' signals become more negatively correlated over time and the incentive to wait does not dissipate over time. With disclosure, the conditional correlation of informed traders' signals based on public information revealed through trade disclosure goes to zero and informed traders trade much more aggressively. The aggressiveness in trading dominates the randomization and residual uncertainty effect and causes the market to be illiquid for some period of time. Indeed, since the inverse of market depth is proportional to the expected profits of the informed traders, disclosure can increase informed traders' expected profits for some period of time. Moreover, the total expected profits of informed traders over the whole trading period can increase as a result.

We further examine the effects of competition under disclosure. We find that when informed traders' signals are uncorrelated, each informed trader behaves exactly like a monopolist. The aggregate profits of informed traders are also maximized when they have uncorrelated signals. When informed traders' signals are positively correlated, they trade more aggressively on their information which causes the market to be more efficient. Compared to the monopolistic model, competition makes the market less liquid initially and more liquid later. With negatively correlated signals, the results is the opposite to that of the positively correlated signals case. Moreover, starting out with correlated signals, informed traders' signals become uncorrelated conditional on public information near the end of trading and the informed traders all behave like a monopolist in the end.

The effect of disclosure rules on informed traders' trading has been recently studied by a number of authors including But these articles exclusively focus on the case of a single informed trader. Fishman and Hagerty (1995) study a two period model when an informed trader only possesses inside information with a certain probability. While an *informed* informed trader will never manipulate the market in their model, an *uninformed* informed trader can manipulate the market since the market may mistakenly believe that the *uninformed* informed trader is *informed*. John and Narayanan (1997) extend the Fishman-Hagerty model such that an informed trader receives good or bad signal with different probabilities, and they show that if such difference in probabilities is large enough, even an informed informed trader may manipulate the market. Here, the asymmetry in the likelihood of receiving different signals adds a new factor to induce an informed trader to manipulate: If the prior probability of good news is high, an informed trader with good news will sell initially and then reverse his trades in the next period.[1] While both FH and JN have found that it is possible for disclosure to increase informed trader's expected profits, the intuition is very different from our model. In FH, the result is driven by the assumption that the market does not know if the informed trader indeed has observed a signal or not while in JN the result is driven by the assumption in the asymmetry of the likelihood of receiving different signals. In our model, disclosure increases informed traders' profits because it can reduce the incentive to wait when informed traders are on the opposite side of the market.

The most related paper is by Huddart, Hughes, and Levine (2001) who study disclosure effects in a discrete-time Kyle model with a monopolistic informed trader. They show that the informed trader uses a mixed strategy in which the informed trader attaches a random order flow, for hiding information, to the information-based flow that is exactly the same as in Kyle's model. In addition, mandatory disclosure unambiguously reduces informed trader's profits, increases market liquidity, and improves market efficiency. However, they do not analyze how disclosure will affect informed traders' strategic trading behavior when there are more than one informed trader.

The rest of the paper is organized into sections as follows. The model is described in Section 1. Section 2 describes the equilibrium without public disclosure of informed traders' trades. Section 3 discusses the condition for equilibrium with public disclosure in a discrete-time framework. Section 4 offers a closed-form formula for the equilibrium in a continuous-time framework. Section 5 gives comparative statistics such as the effects of the number of informed traders and the correlation of their signals on the intensity of trading, the rate of information transmission, the depth of the market, and the expected profits of informed traders. Section 6 concludes. All proofs are left to the appendices.

1 The Model

In this section, we describe a model of multiple informed traders who are required to disclose their trades based on the classic model of Kyle (1985). In our model, there are one risk-free asset and one risky asset. An announcement is made at time 1 that reveals the liquidation value of the asset. The risk-free rate is taken to be zero. There are N risk neutral informed traders and many liquidity traders who trade for liquidity reasons. Trading takes place over time interval [0, 1). In the discrete-time version of the model, there are M periods over time [0,1), and the time between any two consecutive trading periods is $\Delta t = 1/M$.

Before any trading starts, each informed trader i ($1 \leq i \leq N$) receives a mean-zero signal s^i at time 0. We assume the signals and the liquidation value of the risky asset has a non-degenerate joint normal distribution that is symmetric in the signals.[2] Let $\tilde{v}$ denote the expectation of the liquidation value conditional on the combined information of the informed traders. By normality, $\tilde{v}$ is an affine

[1] John and Narayanan (1997) contains a brief an extension of their model to allow two informed traders. However, their study on the two-informed trader case is limited to arguing that an informed trader's incentive to manipulate the market decreases when the number of informed traders rises.

[2] Symmetry means that the joint distribution of the asset value and the signals $s_1, \ldots, s_N$ is invariant to a permutation of the indices $1, \ldots, N$.

function of the s^i. By re-scaling the s^i if necessary, we can assume without loss of generality that

$$\tilde{v} = \bar{v} + \sum_{i=1}^{N} s^i, \tag{1.1}$$

for a constant $\bar{v}$. This is a normalization adopted by Foster and Viswanathan (1996, §VI). For simplicity, we also assume $\bar{v} = 0$.

The signals s^i $(1 \leq i \leq N)$ may be correlated. We use ρ to denote the correlation coefficient of s^i with s^j for any (i, j) such that $i \neq j$. In the special case of either $N = 1$ or $\rho = 1$ (i.e., s^i is perfectly correlated with s^j for any i and j), each informed trader has perfect information about $\tilde{v}$. For convenience, we also introduce the following notation

$$\phi = \frac{\mathrm{var}\,(\tilde{v})}{\mathrm{var}\,(N s^i)}. \tag{1.2}$$

This is a measure of the quality of each informed trader information. Specifically, ϕ is the "R-squared" in the linear regression of $\tilde{v}$ on s^i for an arbitrary i, i.e., it is the percent of the variance in $\tilde{v}$ that is explained by a single informed trader's information. It is easy to check that ϕ is related to ρ by the following equation

$$\phi = \frac{1}{N} + \frac{N-1}{N}\rho. \tag{1.3}$$

In each trading period m, a risk-neutral market maker receives the total order from all the informed traders and liquidity traders. Based on such order information, the market maker adjusts the price P_{m-1} to a new price P_m at which he buys or sells the risky security to clear the market in period m. Since the market maker is assumed to be risk neutral, price P_m must be the conditional expectation given all public information. We use x_m^i to denote informed trader i's order, and use z_m^0 to denote the total order by all liquidity traders. We assume that z_m^0 are serially uncorrelated and normally distributed with mean zero and variance

$$E(z_m^0) = 0 \;\text{ and } \mathrm{var}(z_m^0) = \sigma_u^2 \Delta t \quad \text{for all } m.$$

For simplicity, we assume $\sigma_u = 1$. In addition, z_m^0 is independent of all other random variables in the model. Moreover, we assume that informed traders are prevented from any market making activities, and hence when they submit their orders in period m they have no information about the mth-period order flow from any other party.

The only difference between a model with disclosure and a model without disclosure is whether or not each informed trader is required to disclose his mth period trade immediately after all trades are completed in period m. Technically, this implies the following difference in how each of the involved parties behaves in the model. Without disclosure, (1) the market maker sets his price P_m by observing the history of the aggregate order flow $\{y_k : 1 \leq k \leq m\}$, where

$$y_k \equiv z_k^0 + \sum_{1 \leq i \leq N} x_k^i$$

and (2) each informed trader i decides his trade by observing his own past order flow $\{x_k^i : 1 \leq k < m\}$, his own signal s^i, and the past price history $\{P_k : 1 \leq k < m\}$. With disclosure, (1) the market maker sets his price by observing the breakdown of all traders' past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$ together with the current aggregate order flow $\{y_m\}$; and (2) each informed trader i decides his trade by observing all traders' past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$, in addition to his signal s^i and the past price history $\{P_k : 1 \leq k < m\}$. Note that in a model with disclosure, the breakdown of all the past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$ are made public through public disclosure and price history.

The above description has focused on the discrete-time version of the model. A intuitive way to think of the continuous-time version of the model is simply to take the limit of the discrete-time model with $M \to +\infty$. More technical details will be given when it comes to the derivation of our results in the continuous-time version of our model.

2 Informed Trading without Public Disclosure

In this section, we state the main results that have been previously established for the case of no public disclosure. These results will serve as benchmarks for us to compare our new results to be derived for the case with disclosure.

Let $\underline{x}_m^i$ denote the history of trader i's trade in each past period before and including period m (i.e., $\{x_k^i : k = 1, \ldots, m\}$), let $\underline{y}_m$ denote the history of the net trade before and including period m (i.e., $\{z_k^0 + \sum_{1 \leq i \leq N} x_k^i : k = 1, \ldots, m\}$), and let $\underline{P}_m$ denote the price history before and including period m (i.e., $\{P_k : k = 1, \ldots, m\}$). Without disclosure, informed trader i's private information prior to trading in period m includes, his own signal s^i, the history of his own trades $\underline{x}_{m-1}^i$. In addition, all traders know $\underline{P}_{m-1}$. Let

$$x_m^i = x_m^i(s^i, \underline{x}_{m-1}^i, \underline{P}_{m-1})$$

represent the optimal strategy of informed trader i. Let

$$P_m = P_m(\underline{y}_m)$$

represent the optimal strategy of the market maker. Since the prior mean of v is 0, the price before the auction starts is equal to 0 (i.e., $P_0 = 0$).

Let X^i and P represent the strategy functions for informed trader i and the market maker, respectively. The profit of the ith informed trader from trading in period m and on is:

$$\pi_m^i(X^1, \ldots, X^i, \ldots, X^N, P) = \sum_{k \geq m} (\tilde{v} - P_k) x_k.$$

An equilibrium of the trading game exists if there is an $(N+1)$-dimension vector of strategies, $(X^1, \ldots, X^N, P)$ such that:

1. For each $i = 1, \ldots, N$ and for all $m = 1, ..., M$, if $\hat{X}^i \neq X^i$, then

$$\begin{aligned} & E[\pi_m^i(X^1, \ldots, X^i, \ldots, X^N, P) | s^i, \underline{x}_{m-1}^i, \underline{P}_{m-1}] \\ \geq \; & E[\pi_m^i(X^1, \ldots, \hat{X}^i, \ldots, X^N, P) | s^i, \underline{x}_{m-1}^i, \underline{P}_{m-1}] \end{aligned}$$

i.e., the optimal strategy is the best no matter which past strategies informed trader i may have played.

2. For all $m = 1, \ldots, M$,

$$P_m = E[v|\underline{y}_m],$$

i.e., the market maker sets prices equal to the conditional expectation given the history of order flows.

Let F_m and F_m^i denote the information set of the market maker and informed trader i respectively after the mth auction has completed. Define

$$\begin{aligned} \Sigma_m &\equiv \mathrm{Var}[\tilde{v}|F_m], \\ \Omega_m &\equiv \mathrm{Var}[\tilde{v}|F_m^i], \quad \text{and} \\ V_m^i &\equiv E[\tilde{v}|F_m^i]. \end{aligned}$$

For ease of notation, we also define

$$\delta_m \equiv \frac{\Sigma_m - \Omega_m}{\Sigma_m}. \tag{2.1}$$

Foster and Viswanathan (1996) and Cao (1995) have characterized a linear equilibrium in this setting as stated in the following theorem.

Theorem 2.1 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are: For all $n = 1, \cdots, M$ and for all informed traders $i = 1, \cdots, N$,*

$$x_m^i = \beta_m \Delta t \left(s^i - \frac{P_{m-1}}{N} \right) = \frac{\beta_m \Delta t}{N \delta_{m-1}} (V_{m-1}^i - P_{m-1}), \tag{2.2}$$

$$P_m = P_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^{N} x_m^i \right) \tag{2.3}$$

$$\lambda_m = \Sigma_m \beta_m \tag{2.4}$$

$$\Sigma_m = (1 - \beta_m \Delta t \lambda_m) \Sigma_{m-1} \tag{2.5}$$

$$\begin{aligned} E[\pi_{(m+1)}^i|F_m^i] &= \alpha_m (N s_i - P_m)^2 + \gamma_m (N s^i - P_m) \left(\frac{V_m^i - \delta_m N s^i}{1 - \delta_m} - P_m \right) \\ &\quad + \eta_m \left(\frac{V_m^i - \delta_m N s^i}{1 - \delta_m} - P_m \right)^2 + \zeta_m \end{aligned} \tag{2.6}$$

where $\alpha, \gamma, \eta, \zeta$ are described below

$$\alpha_{m-1} = \beta_m \Delta t \delta_{m-1} (1 - \lambda_m \beta_m \Delta t)/N + \alpha_m (1 - \lambda_m \beta_m \Delta t \delta_{m-1})^2 \tag{2.7}$$

$$\gamma_{m-1} = \delta_{m-1} [1 - \beta_m \Delta t \lambda_m (1 - 1/N)] (1 - \beta_m \Delta t \lambda_m)/\lambda_m - 2\alpha_{m-1} \tag{2.8}$$

$$\eta_{m-1} = \frac{[1 - (1 - 1/N) \beta_m \Delta t \lambda_m]^2}{4 \lambda_m [1 - (\eta_m + \alpha_m - \gamma_m) \lambda_m]} - \alpha_{m-1} + \gamma_{m-1} \tag{2.9}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \lambda_m^2 \Delta t + \alpha_m \lambda_m^2 N^2 \beta_m^2 (\Delta t)^2 \Sigma_{m-1} \tag{2.10}$$

$$\beta_m \Delta t = \frac{\delta_{m-1} - (\gamma_m + 2\alpha_m)\lambda_m}{\lambda_m [1/N + (1 - (\gamma_m + 2\alpha_m)\lambda_m)\delta_{m-1}]} \tag{2.11}$$

subjecting to the boundary conditions

$$\alpha_M = 0, \;\; \gamma_M = 0, \;\; \eta_M = 0, \;\; \zeta_M = 0, \tag{2.12}$$

and the second order condition,

$$\lambda_m (1 - \eta_m \lambda_m) > 0. \tag{2.13}$$

When the number of trading periods goes to infinity, the model approaches to continuous-time trading. In a continuous-time trading model, Back, Cao, and Willard (2000) show that there exists a unique linear equilibrium described in the next theorem.

Theorem 2.2 *If there is more than one informed trader ($N > 1$) and their signals are perfectly correlated ($\rho = 1$), then there is no symmetric linear equilibrium. Otherwise, there is a unique symmetric linear equilibrium. Set $\Sigma(0) = var(\tilde{v})$, and consider the constant*

$$k = \int_1^\infty x^{\frac{2}{N}(N-2)} e^{-\frac{2}{N}\left(\frac{1-\phi}{\phi}\right)x} \, dx. \tag{2.14}$$

For each $t < 1$, define $\Sigma(t)$ by

$$\int_1^{\Sigma(0)/\Sigma(t)} x^{\frac{2}{N}(N-2)} e^{-\frac{2}{N}\left(\frac{1-\phi}{\phi}\right)x} \, dx = kt. \tag{2.15}$$

The equilibrium is

$$\beta(t) = \left(\frac{k}{\Sigma(0)}\right)^{1/2} \left(\frac{\Sigma(t)}{\Sigma(0)}\right)^{\frac{1}{N}(N-2)} \exp\left\{ \frac{1}{N} \left(\frac{1-\phi}{\phi}\right) \frac{\Sigma(0)}{\Sigma(t)} \right\}, \tag{2.16}$$

$$\lambda(t) = \beta(t)\Sigma(t). \tag{2.17}$$

3 Informed Trading in Discrete Time with Public Disclosure

Under the disclosure requirement, informed traders announce their trades, $\{x_m^i\}, i = 1, \ldots, N$, immediately after the trade is executed. The market maker then adjusts his belief of the asset value from P_m (the market price for the risky asset in period m) to V_m which is defined to be the market maker's estimate of the fair value of the risky asset with all the information up to and including the disclosure made at the end of period m. We can think of V_m as the pseudo-price that market maker would have set for the mth period trading if he had observed the informed traders' order before the execution of trades in the mth period. Although V_m is only a pseudo-price at which no trade ever takes place, it is important since it will be the starting point for the market maker to set P_{m+1} for the $(m+1)$th period of trading. In particular, in a linear equilibrium model that we will focus on, it is $P_{m+1} - V_m$ (as opposed to $P_{m+1} - P_m$) that will be linear to the total order flow submitted in the $(m+1)$th trading period.

With disclosure, informed trader i's private information prior to trading in period m includes his own signal s^i and the history of all past trades and prices $\underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{P}_{m-1}$, where $\underline{x}^i_{m-1}$ and $\underline{P}_{m-1}$ are defined in Section 2. Let

$$x^i_m = x^i_m(s^i, \underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{P}_{m-1})$$

represent the optimal strategy of informed trader i. Let

$$P_m = P_m(\underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{y}_m)$$

represent the optimal strategy of the market maker given the history of all orders and the current aggregate order.

Let X^i and P denote the strategy functions for informed trader i and the market maker, respectively. Given the strategy functions for informed traders and the market maker, the profit of informed trader i from trading in period m and on can be written as:

$$\pi^i_m(X^1, \ldots, X^i, \ldots, X^N, P) = \sum_{k \geq m} (\tilde{v} - P_k) x_k.$$

An equilibrium of the trading game exists if there is an $(N+1)$-dimension vector of strategies, $(X^1, \ldots, X^N, P)$ such that :

1. For any $i = 1, \ldots, N$ and for all $m = 1, ..., M$, if $\hat{X}^i \neq X^i$,

$$\begin{aligned} & E\left[\pi^i_m(X^1, \ldots, X^i, \ldots, X^N, P) | s^i, \underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{P}_{m-1})\right] \\ \geq \quad & E[\pi^i_m(X^1, \ldots, \hat{X}^i, \ldots, X^N, P) | s^i, \underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{P}_{m-1}] \end{aligned}$$

   i.e., the optimal strategy is the best no matter which past strategies informed trader i may have played.

2. For all $m = 1, \ldots, M$, we have

$$P_m = E[v | \underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1}, \underline{y}_m],$$

   i.e., the market maker sets prices equal to the conditional expectation of the asset value given the order-flow history.

In this model, since investor i's trade at period m will be disclosed afterwards, the pricing and trading strategies described earlier for the no-disclosure case cannot be an equilibrium in the new setting. To see this, suppose the informed trader follows a strategy of[3]

$$x^i_m = \beta_m \Delta t s^i_m + L_1(\underline{x}^i_{m-1}) + L_2(\underline{x}^1_{m-1}, \ldots, \underline{x}^N_{m-1})$$

[3] We restrict our attention to symmetric linear equilibria.

where L_i is a linear function of all public information. Then the market maker would infer

$$\tilde{v} = \frac{\sum_{1\le i\le N}[x_m^i - L_1(\underline{x}_{m-1}^i) - L_2(\underline{x}_{m-1}^1, \ldots, \underline{x}_{m-1}^N)]}{\beta_m \Delta t}$$

and choose

$$P_{m+1} = \frac{\sum_{1\le i\le N}[x_m^i - L_1(\underline{x}_{m-1}^i) - L_2(\underline{x}_{m-1}^1, \ldots, \underline{x}_{m-1}^N)]}{\beta_m \Delta t}$$

in the next period. Hence, in the next period, the market depth would be infinity. Understanding this, the informed traders would have incentive to choose $\hat{x}_m^i \neq x_m^i$ which is inconsistent with the proposed equilibrium strategy.

We analyze a symmetric linear equilibrium. In particular, the informed trader's trade can be written as

$$x_m^i = \beta_m \Delta t s^i + L_1(\underline{x}_{m-1}^i) + L_2(\underline{x}_{m-1}^1, \ldots, \underline{x}_{m-1}^N) + z_m^i, \tag{3.1}$$

where (1) $\beta_m \Delta t s^i$ represents a private-information based linear component, (2) $L_1(\underline{x}_{m-1}^i)+L_2(\underline{x}_{m-1}^1, \ldots, \underline{x}_{m-1}^N)$ is a public-information based linear component, and (3) z_m^i is a noise component with z_m^i being normally distributed with mean 0 and variance $\sigma_m^2 \Delta t$. Since informed traders are prevented from market making activities, we further assume that z_m^i are independently distributed across agents. The market maker also uses linear rules for setting prices before disclosure and for updating his value estimate after disclosure. In particular,

$$\begin{aligned} P_m &= V_{m-1} + \lambda_m \left(z_m^0 + \sum_{1\le i\le N} x_m^i \right), \quad \text{and} \\ V_m &= V_{m-1} + \bar{\lambda}_m \left(\sum_{1\le i\le N} x_m^i \right). \end{aligned}$$

The preceding equations imply that the random order from liquidity traders only has a temporary effect on price formation. In particular, liquidity traders' order in period m (i.e., z_m^0) only affects P_m but not P_k for any $k \geq m+1$: Once the mth-period disclosure is made, the market maker immediately abandons z_m^0 and adjusts his belief of asset value to V_m, which is not affected by z_m^0 and will be the base for forming future prices P_k ($k \geq m+1$).

Before stating our result, we first introduce some notation like we did for the case of no disclosure. Let F_m and F_m^i denote the information set of the market maker and informed trader i respectively *after* disclosure has been made in period m. Define

$$\begin{aligned} V_m^i &\equiv E[\tilde{v}|F_m^i], \\ V_m &\equiv E[\tilde{v}|F_m], \\ \Sigma_m &\equiv \text{Var}[\tilde{v}|F_m], \\ \Omega_m &\equiv \text{Var}[\tilde{v}|F_m^i], \quad \text{and} \\ \delta_m &\equiv \frac{\Sigma_m - \Omega_m}{\Sigma_m}. \end{aligned}$$

Theorem 3.1 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all $m = 1, \cdots, M-1$ and for all informed traders $i = 1, \cdots, N$,*

$$x_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}}(V_{m-1}^i - V_{m-1}) + z_m^i \tag{3.2}$$

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^{N} x_m^i \right) \tag{3.3}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^{N} x_m^i \tag{3.4}$$

$$\bar{\lambda}_m = \beta_m \Sigma_m / (N\sigma_m^2) \tag{3.5}$$

$$\lambda_m = \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2) \tag{3.6}$$

$$V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(\tilde{v} - V_{m-1}^i + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{3.7}$$

$$V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left(\tilde{v} - V_{m-1} + \sum_{1 \leq j \leq N} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{3.8}$$

$$\Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t / ((N-1)\sigma_m^2) \tag{3.9}$$

$$\Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t / (N\sigma_m^2) \tag{3.10}$$

$$E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1}(V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \tag{3.11}$$

$$\lambda_m = \alpha_m \bar{\lambda}_m^2 \tag{3.12}$$

$$\lambda_m = \frac{\bar{\lambda}_m}{2 - \bar{\lambda}_m \beta_m \Delta t (1 - 1/(N\delta_{m-1}))} \tag{3.13}$$

$$\alpha_{m-1} = \alpha_m \left(1 - \frac{\beta_m^2 \Delta t \Sigma_m}{N\sigma_m^2} \left(1 - \frac{1}{N\delta_{m-1}} \right) \right)^2 \tag{3.14}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left(\frac{\Omega_m}{(N-1)\sigma_m^2} - \frac{\Sigma_m}{N\sigma_m^2} \right)^2 \left(\Omega_{m-1}\beta_m^2 \Delta t + (N-1)\sigma_m^2 \right) \tag{3.15}$$

subjecting to the boundary conditions

$$\beta_M = \sqrt{\frac{N\delta_{M-1}}{\Sigma_{M-1}\Delta t}}, \tag{3.16}$$

$$\lambda_M = \frac{\sqrt{N\delta_{M-1}\Sigma_{M-1}/\Delta t}}{1 + N\delta_{M-1}}, \tag{3.17}$$

$$\alpha_{M-1} = \frac{1}{\lambda_M (1 + N\delta_{M-1})^2}, \tag{3.18}$$

$$\zeta_{M-1} = 0, \tag{3.19}$$

and the second order condition

$$\lambda_M > 0. \tag{3.20}$$

In general, the system of recursive equations can be solved by conjecturing an initial value of Ω_{M-1} and then solve recursively for $\Omega_{M-2}, ..., \Omega_0$. The initial value of Ω_{M-1} is then adjusted until the derived Ω_0 matches the given Ω_0. Details are given in Appendix A.

In the special case of $\rho = 0$, the model can be solved in closed form:

$$\lambda_m = \frac{\sqrt{\Sigma_0}}{2}$$

$$\bar{\lambda}_m = 2\lambda_m$$

$$\beta_m = \frac{1}{2(M-m+1)\lambda_m},$$

$$\sigma_m^2 = \frac{M-m}{N(M-m+1)}$$

$$\Sigma_m = (1-m/M)\Sigma_0$$

$$\Omega_m = (1-m/M)\Omega_0$$

$$\alpha_m = \frac{1}{4\lambda_m}$$

$$\zeta_m = 0$$

These results are exactly the same as the monopolistic case derived by Huddart, Hughes and Levine (2001). This is in sharp contrast to earlier results on imperfect competition of informed traders without disclosure. Foster and Viswanathan (1996), Cao (1995) and Back, Cao and Willard (2000) have shown that competition causes the market to be very illiquid and inefficient near the end of trade when there is no disclosure. With disclosure, we find that informed traders act as monopolists with respect to their private signals when their signals are uncorrelated. This is because, with disclosure, informed traders will always know as much about others' signals as the market does. If informed traders' signals are uncorrelated to begin with, they remain uncorrelated due to public disclosure of trades after transaction is completed Therefore, each informed trader behaves as if he is a monopolist. On the contrary, without disclosure, each informed trader gradually knows more about others' signals than the market maker since he knows what he traded in the past. Indeed, the correlation coefficient of informed trader's signals goes to -1 in a two-informed trader model without disclosure even when the initial correlation coefficient is zero. The high negative correlation coefficient cause informed traders to trade very aggressively near the end which drives market liquidity to zero.

In another special case where the number of trading periods goes to infinity, the model approaches to the continuous-time model. Ignoring higher order terms of Δt, we have the following:

$$\bar{\lambda} = \beta\Sigma$$

$$\lambda = \beta\Sigma/2$$

$$\sigma_m^2 = 1/N$$

$$\frac{\Delta\Omega^{-1}}{\Delta t} = N\beta^2/(N-1)$$

$$\frac{\Delta\Sigma^{-1}}{\Delta t} = \beta^2$$
$$2\alpha\bar{\lambda} = 1$$
$$\frac{\Delta\alpha}{\Delta t} = 2\alpha\beta^2\Sigma\left(1 - \frac{1}{N\delta}\right)$$
$$\frac{\Delta\zeta}{\Delta t} = -\alpha\beta^2[\Omega N - (N-1)\Sigma]^2/[N(N-1)].$$

In the limit, these difference equations converge to the set of differential equations described in Theorem 4.1 and the lemmas in Section 4.

4 Informed Trading in Continuous Time with Public Disclosure

In this section, we derive closed-form formulae for the linear equilibrium of informed trader trading in a continuous-time framework. The section is divided into subsections as follows. Subsection 4.1 introduces necessary notations to state the main theorem. Subsection 4.2 contains the main theorem of the section. Subsections 4.3 and 4.4 outline the proofs of the main theorem by considering the value estimation processes and the informed traders' optimal trading strategy, respectively.

4.1 Model Setup

In this subsection, we introduce the basic notations and concepts for the continuous-time model. Most of these notations (e.g., $\beta(t)$ and $P(t)$) have already been used in the discrete-time model but will be redefined here for an identical or similar quantity in the continuous model.

Like in discrete time, we use s^i to denote the signal of informed trader i and assume $\tilde{v} = \sum_{1\leq i\leq N} s^i$. We use $P(t)$ to denote the price set by the market maker for trading at time t, and we use $V(t)$ to denote the market maker's adjusted belief of the risky-asset value immediately after the disclosure of informed traders' trade at time t. Also, we use $x^i(s^i, t)$ to denote the total order of informed trader i up to time t, and we use $z^0(t)$ to denote the total order from all liquidity traders up to time t.[4]

For the price process, linearity means that there exist functions $\lambda(t)$ and $\bar{\lambda}(t)$ such that the market maker adjusts the risky asset's price and the post-disclosure value estimate by multiplying $\lambda(t)$ and $\bar{\lambda}(t)$ with the new orders from all traders and those from all informed traders, respectively. More precisely, we have

$$dV(t) = \bar{\lambda}(t) \sum_{1\leq i\leq N} dx^i(t), \text{ and} \tag{4.1}$$

$$P(t+dt) - V(t) = \lambda(t)\left(dz^0(t) + \sum_{1\leq i\leq N} dx^i(t)\right). \tag{4.2}$$

[4]In contrast, in the discrete-time model, we have used x^i_m to denote informed trader i's instantaneous order at time m, rather than his cumulative order up to time m.

It should be noted that although at any time t, $P(t)$ and $V(t)$ only differ by an infinitesimal due to the liquidity traders' trades,[5] this infinitesimal will be important in calculating the profit of an informed trader, as we will see in the proof of Lemma 4.5.

We require that the trading strategy $x^i(s^i, t)$ depends only on the trade history up to time t (e.g., it is independent of future value of x^j for any $j = 1, \dots, N$). We also require that the trading strategies to be such that Equation 4.1 with boundary condition $V(0) = 0$ has a unique solution V. Furthermore, we require the solution P to have a finite second moment and to have paths belonging to $\mathcal{C}$, where $\mathcal{C}$ denotes the set of continuous functions f: $[0, 1) \to R$ such that $\lim_{t\to 1} f(t)$ exists and is finite. This is a restriction on the strategy sets of the traders: given that agents $i \neq j$ follow linear strategies to be described in Equation 4.3, we require agent j to follow a strategy such that Equation 4.1 has a solution with the desired properties.[6]

For the trading strategy, linearity means that the rate of purchase for informed trader i can be specified as follows

$$dx^i(s^i, t) = \beta(t)s^i dt + f(t)dt + dz^i(t) \quad \text{for } 1 \leq i \leq N \tag{4.3}$$

where $f(t)$ is a certain function of all public information available up to time t and $z^i(t)$ is a (non-standard) Brownian motion with instantaneous variance

$$\mathrm{var}(dz^i(t)) = \frac{1}{N}dt \quad \text{for } 1 \leq i \leq N. \tag{4.4}$$

In general, the above variance can be assumed to be $\sigma \frac{1}{N} dt$ for a parameter σ to be solved in equilibrium. The fact that σ should be solved endogenously is important in proving the uniqueness of the equilibrium. To be rigorous, Equation 4.4 should not be understood as an exogenous assumption on the variance of the random order term from the informed traders. Rather, is is purely a normalization. We will show that in equilibrium (see Equation B.13), such a normalization leads to normalizing

$$\mathrm{var}(dz^0(t)) = dt. \tag{4.5}$$

Alternatively, we could have chosen to assume Equation 4.5 as a normalization (after all, it is the ratio $\mathrm{var}(dz^i(t))/\mathrm{var}(dz^0(t))$ that is of any interest). Here, we choose the first normalization over the second so that the next few lemmas will appear cleaner.

Since informed traders are assumed not to participate in market making activities, each dz^i is uncorrelated with both noisy trader's trade dz^0 and all other informed traders trade dz^j for all $j \neq i$.

[5] It can be shown that $V(t) - P(t) = \bar{\lambda}(t) \sum_{1\leq i\leq N} dx^i(t) - \lambda(t)\left(dz^0(t) + \sum_{1\leq i\leq N} dx^i(t)\right)$, although we do not need this equation in deriving our equilibrium conditions.

[6] See, e.g., Protter (1990, §V.3) for conditions that guarantee the existence of unique solutions to stochastic differential equations. Our approach has the disadvantage of linking the feasible set for each trader to the strategies assumed to be chosen by the other traders and the market maker. In this respect, we are modeling a generalized game rather than a game. It would be better to define a feasible set for each trader and a set of $\bar{\lambda}$ functions for the market maker such that, given any vector of choices from these sets, the stochastic differential equation defining the price has a unique solution with the desired limits existing. However, this approach would lead us into a thicket of technicalities that we prefer to avoid.

While we have only included t in our notation $f(t)$, it should be emphasized that $f(t)$ can be an arbitrarily complex function of all public information available before and including time t, such as the history of all the orders submitted by all the informed traders and the liquidity traders, which are revealed to the public through disclosures. We leave $f(t)$ in this very general form for now and will make it more explicit later.

In most previous studies in the informed trader trading literature, $f(t)$ is simply the asset price at time t multiplied by a certain function $\alpha(t)$, which solely depends on time t but no other information (see Kyle (1995), Back, Cao, and Willard (2000), and Huddart, Hughes, and Levine (2001)). In our current model, however, $f(t)$ has to depend on more public information other than price. Indeed, it can be shown in the discrete-time model that informed traders' order flows of the form $\beta(t)s^i+\alpha(t)P(t)$ (where $\alpha(t)$ is a function of time t only) does not constitute an equilibrium.

Like discussed in Back, Cao, and Willard (2000), it may be natural to consider trading that is linear in a trader's updated estimate of the asset value rather than linear in a trader's initial signal. One difficulty with such an approach arises in calculating each informed informed trader's dynamic estimates of the asset value, because each trader's estimate would depend on other agents' trades, which depend on their estimates of the asset value, which depend on other agents' trades, etc. This is what is called the "forecasting the forecasts of others" problem (see Foster and Viswanathan (1996)). By specifying the trading strategy as linear in a trader's initial signal, we can avoid this problem. In the end, Strategy 4.3 can be shown to be a linear functions of value estimates in equilibrium. In particular, although the trading strategy assumed in Equation 4.3 is quite different from that in Back, Cao, and Willard (2000) in that our strategy cannot even be expressed as a function of the *initial signals* and price, when expressed as a function of a trader's *estimate of the asset value* and the price, the deterministic component of our trading strategy follows the same formula as theirs (compare our Equation 4.14 and their Equations 1.6 and 3.11).

4.2 Equilibrium

We define a symmetric linear equilibrium to be functions $\beta(t)$ and $\bar{\lambda}(t)$ such that (1) they are positive and continuous on $[0,1)$ and continuously differentiable on $(0,1)$, (2) $P(t)$ and $V(t)$ calculated from Equations 4.1 and 4.2 are both rational expectations of the asset value at all time t, and (3) the trading strategy in Equation 4.3 for each informed trader i is feasible and maximizes his expected profit over the set of feasible strategies. The following theorem is our main result.

Theorem 4.1 *If there is more than one informed trader ($N > 1$) and their signals are perfectly correlated ($\rho = 1$), then there is no symmetric linear equilibrium. Otherwise, there is a unique symmetric linear equilibrium specified as follows*

$$\beta = \frac{1}{\Sigma}\sqrt{-\Sigma'}, \lambda = \frac{\sqrt{-\Sigma'}}{2}, \bar{\lambda} = \sqrt{-\Sigma'},$$

where $\Sigma(t)$ is specified as

$$\Sigma(t) \quad = \quad \Sigma(0)(1-t) \;\; \textit{for } \rho = 0 \textit{ and}$$

$$\Sigma(t) = \frac{(1-\rho)\Sigma(0)}{\rho N}\left[((1-B)\,t+B)^{-\frac{1}{3-\frac{4}{N}}}-1\right] \quad \textit{for } \rho \neq 0$$

$$\textit{where } B = \left(\frac{1-\rho}{1-\rho+\rho N}\right)^{3-\frac{4}{N}} \quad \textit{and}$$

$$\Sigma(0) = var(\tilde{v}), \ \textit{as estimated by market marker at time 0.}$$

In equilibrium, the expected profit of each informed trader is

$$\frac{1}{N}\int_0^1 \lambda(t)dt = \frac{1}{2}\sqrt{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{3N-4}{1-B}\right)}\,\frac{\left|1-B^{\frac{N-2}{3N-4}}\right|}{N|N-2|}. \tag{4.6}$$

Note that in the case of $N = 1$, the second formula for $\Sigma(t)$ is independent of ρ and is equal to the first formula for $\Sigma(t)$. In such a case, the theorem implies that the market depth λ is a constant, as shown by Huddart, Hughes, and Levine (2001) in the discrete model. Surprisingly, λ remains constant as long as $\rho = 0$, even in the case $N > 1$. This is in sharp contrast to the result in Back, Cao, and Willard (2000).

4.3 Value Estimates and Variances

In this subsection, we consider the filtering problems of the traders and market maker. Throughout this section, we assume $\beta(t)$ used in Strategy 4.3 is a continuous and non-negative function.

Let $\mathbf{F} \equiv \{\mathcal{F}(t)|0 \le t < 1\}$ denote the filtration generated by the aggregate informed traders' order process

$$\sum_{i=1}^{N} x^i(t).$$

We interpret $\mathbf{F}$ as the market maker's information structure. Under the new notation, $V(t) = E[\tilde{v}|\mathcal{F}(t)]$ where the conditional expectation is taken after the disclosure at time t. We define $\Sigma(t)$ as[7]

$$\frac{1}{\Sigma(t)} \equiv \int_0^t \beta(u)^2\,du + \frac{1}{\Sigma(0)}. \tag{4.7}$$

Lemma 4.1 *Assume each trader i follows a linear strategy as in Equation 4.3. Then* $\Sigma(t) = var[\tilde{v}|\mathcal{F}(t)]$, *where the variance is calculated after disclosure at time t. Define*

$$W(t) \equiv \sum_{1\le i\le N} z^i(t) + \int_0^t \beta(u)\left\{\tilde{v} - V(u)\right\}\,du. \tag{4.8}$$

[7] In the discrete-time model, we define $\Sigma(t)$ as the variance of the asset value conditional on the market maker's information. Here, we choose to define $\Sigma(t)$ by a mathematical equation and then prove that it is equal to the same conditional variance under certain conditions (see Lemma 4.1). Alternatively, we could define $\Sigma(t)$ as the desired conditional variance and then prove Equality 4.7 in Lemma 4.1. But such an alternative approach does not offer us a easy-to-use mathematical formula for $\Sigma(t)$ when conditions in Lemma 4.1 do not hold. Finally, we remark that it can be verified (see the proof of Theorem 4.1) that the function $\Sigma(t)$ defined here is the same as that used in the statement of Theorem 4.1.

The process W is a Wiener process on the market maker's information structure $\mathbf{F}$. *Furthermore,*

$$V(t) = \int_0^t \beta(u)\Sigma(u)dW(u). \tag{4.9}$$

The process W is called the "innovation" process for the market maker's estimation problem. The differential

$$dW = \sum_{1\leq i\leq N} dz^i + \beta\left\{\tilde{v} - V\right\} dt$$

is the unpredictable part of the order flow from informed traders (recall that from the market maker's viewpoint, the expected order from informed traders is 0). The lemma shows that the market's estimate of $\tilde{v}$ is revised according to $dV = \beta\Sigma\, dW$. Moreover, having the changes of both value estimates and prices proportional to orders as in Equations 4.1 and 4.2 implies that these changes are unpredictable, as they must be when the market maker is risk neutral and competitive.

Consider an arbitrary informed trader j $(1 \leq j \leq N)$. Assume that each trader i $(i \neq j)$ follows a linear strategy as in Equation 4.3, and assume that j follows an arbitrary strategy, which may or may not follow Eqaution 4.3. Let $\mathbf{F}^j \equiv \{\mathcal{F}^j(t)|0 \leq t < 1\}$ denote the filtration generated by s^j and the order flow of all traders i $(i \neq j)$. This is informed trader i's information structure. We want to describe the conditional expectation and conditional variance of $\tilde{v}$, given his information. In particular, we define

$$\begin{aligned} U^j &\equiv E[\tilde{v} - s^j|\mathcal{F}^j(t)], \quad \text{and} \\ V^j &\equiv s^j + U^j \end{aligned}$$

where the expectation is taken at time t after the informed traders' disclosure. We also define[8]

$$\frac{1}{\Omega(t)} \equiv \begin{cases} \frac{N}{N-1}\int_0^t \beta(u)^2\, du + \frac{N}{(1-\rho)(N-1)\Sigma(0)} & \text{if } \rho \neq 1 \\ +\infty & \text{if } \rho = 1. \end{cases} \tag{4.10}$$

Lemma 4.2 *Consider an arbitrary informed trader j $(1 \leq j \leq N)$. Assume each trader $i \neq j$ follows a linear strategy as in Equation 4.3. Then, $\Omega(t) = var[\tilde{v}|\mathcal{F}^j(t)]$, where the variance is calculated after disclosure at time t. Define*

$$W^j(t) \equiv \sqrt{\frac{N}{N-1}}\left[\sum_{i\neq j} z^i(t) + \int_0^t \beta(u)\{\tilde{v} - V^j(u)\}\, du\right]. \tag{4.11}$$

The process W^j is a Weiner process on the information structure $\mathbf{F}^j$, *and*

$$U^j(t) = (N-1)\rho s^j + \int_0^t \sqrt{\frac{N}{N-1}}\beta(u)\Omega(u)dW^j(u). \tag{4.12}$$

[8]Here we choose to define $\Omega(t)$ by a mathematical equation, rather than by defining it to be the conditional variance of the asset value as in the discrete-time model. The reason is the same as that given in Footnote 7 for $\Sigma(t)$.

The differential of the innovation process W^j is again the difference between the actual order and the expected order, but now we are computing the expected order using trader j's information. The lemma shows that his estimate of the asset value $\tilde{v}$ is revised as $dV^j = \sqrt{N/(N-1)}\beta\Omega\, dW^j$.

For ease of notation, we define

$$\delta(t) \equiv \frac{\Sigma(t) - \Omega(t)}{\Sigma(t)}.$$

Lemma 4.3 *Assume (1) each informed trader believes that all other informed traders follow Strategy 4.3, and (2) the market maker believes that all informed traders follow Strategy 4.3. Then,*

$$\sum_{1 \le i \le N} \left(V^i(t) - V(t)\right) = N\delta(t)(\tilde{v} - V(t)). \tag{4.13}$$

The next lemma gives explicitly formula for each informed trader's trading strategy in equilibrium. It may be worth noting that, somewhat surprisingly, the deterministic part of an informed trader's order flow is identical to that in the no-disclosure case (see Equations 1.6 and 3.11 in Back, Cao, and Willard (2000)).

Lemma 4.4 *Assume that each informed trader believes that all other informed traders follow Strategy 4.3. The following is the only trading strategy*[9] *such that (1) it satisfies Equation 4.3 and (2) Equation 4.1 is a rational pricing rules for the market maker:*

$$dx^i(t) = \frac{\beta(t)}{N\delta(t)}\left(V^i(t) - V(t)\right)dt + dz^i(t), \quad 1 \le i \le N. \tag{4.14}$$

Moreover,

$$\bar{\lambda}(t) \;=\; \beta(t)\Sigma(t), \tag{4.15}$$

and the trading strategy supports pricing rule given in Equation 4.2 with

$$\lambda(t) \;=\; \beta(t)\Sigma(t)\frac{\sum_{1\le i\le N} var(dz^i(t))}{var(dz^0(t)) + \sum_{1\le i\le N} var(dz^i(t))}. \tag{4.16}$$

Given Equation 4.15, the entire equilibrium is determined by $\bar{\lambda}(t)$. To see this, note that

$$\bar{\lambda}(t)^2 = \beta(t)^2\Sigma(t)^2 = -\Sigma'(t),$$

where the second equation follows from Equation 4.7. Therefore, the function $\Sigma(t)$ is determined by $\bar{\lambda}(t)$. The condition $\bar{\lambda}(t) = \beta(t)\Sigma(t)$ then determines $\beta(t)$.

To determine $\bar{\lambda}(t)$ or, equivalently $1/\bar{\lambda}(t)$, which Kyle (1985) calls "the depth of the market," we turn to the equilibrium condition that has not yet been exploited, namely, the requirement that each informed trader's trading strategy be optimal.

[9] Unlike in Back, Cao, and Willard (2000), the deterministic part of $dx^{i(t)}$ cannot be expressed as linear functional of initial signal s^i and $V^i(t)$.

4.4 Optimal Trading and Market Depth

In this subsection, we derive the optimality condition for an informed trader's trading rules. Such a condition turns out to be a restriction on market depth.

Throughout the subsection, we focus on an arbitrarily chosen trader, say trader j. Assume that each trader $i \neq j$ follows Strategy 4.14. By Lemma 4.4, trader j's trading strategy can be written as $x^j(s^j, t, P^{x^j})$, where we use P^{x^j} to emphasize that trader j's strategy affects the price process. We define a trading strategy x^j to be feasible for trader j if there exists a unique solution P^{x^j} to Equation 4.1 (with boundary condition $P^{x^j}(0) = 0$) for the given $\bar{\lambda}$ and for the given β that characterizes the other traders' strategies and if

$$\lim_{t \to 1} P(t) \text{ exists and is finite a.s.,} \tag{4.17}$$

$$\int_0^1 dx^j\left(s^j, u, P^{x^j}\right) \text{ exists and is finite a.s., and} \tag{4.18}$$

$$E \int_0^1 P^{x^j}(t)^2\, dt < \infty. \tag{4.19}$$

Note that the integral in Expression 4.19 is the limit of the integral over $[0, t]$ as $t \to 1$. The limits in Expressions 4.17 and 4.18 define, respectively, the price and number of shares held by trader j just before the announcement. Condition 4.19 is the "no doubling strategies" condition introduced in Back (1992). Given the existence of the limits, the integral

$$\int_0^1 \left(\tilde{v} - P^{x^j}(t + dt)\right) dx\left(s^j, t, P^{x^j}\right), \tag{4.20}$$

exists and equals to the profit of trader j. The formula is is derived from the Merton-type wealth equation, and the existence of the integral can be verified by integrating by parts as in Back (1992).

Lemma 4.5 *Assume each trader $i \neq j$ plays a linear strategy as in Equation 4.14. The conditions*

$$\frac{d}{dt}\left(\frac{1}{\bar{\lambda}(t)}\right) = \frac{2\beta(t)}{N}\left(N - \frac{\Sigma(t)}{\Sigma(t) - \Omega(t)}\right), \tag{4.21}$$

$$\lambda = \frac{\bar{\lambda}}{2}, \tag{4.22}$$

and

$$\lim_{t \to 1} \Sigma(t) = 0 \;\; or \;\; \lim_{t \to 1} \bar{\lambda}(t) = +\infty \tag{4.23}$$

are necessary and sufficient for Strategy 4.14 to create an optimal expected profit for trader j, which is equal to

$$\frac{((1 + (N-1)\rho)s^j)^2}{2\bar{\lambda}(0)} + \frac{N-1}{2N} \int_0^1 \frac{1}{\bar{\lambda}(u)} \left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1}\right)^2 du. \tag{4.24}$$

If $N = 1$ or $\rho = 0$, then the right-hand side of Equation 4.21 is zero. Therefore, market depth (which is $1/\lambda = 2/\bar{\lambda}$) must be constant. (The special case of $N = 1$ has been derived by Huddart, Hughes, and Levine (2001).) If $N \neq 1$ and $\rho \neq 0$, then the right-hand side of Equation 4.21 is always positive. This implies that in such a case market depth $1/\lambda$ must be rising over time, in contrast to the result of no-disclosure case obtained by Back, Cao, and Willard (2000), in which market depth first rises to its maximum and then fall to 0.

Condition 4.21 is a local condition for optimality at each $t < 1$, which we will discuss below. Condition 4.23 means there is no money "left on the table" an instant before the announcement. If the second condition of 4.23 holds, then the market's information about $\tilde{v}$ is precise by the announcement date, and the asset will be correctly priced. If the first condition of 4.23 holds, then the market is completely illiquid just before the announcement, so, even if the asset were mis-priced, there would be no profitable trades available. These conditions are not mutually exclusive. In fact, only the first condition holds in our case, which is contrasting with both conditions hold in Back, Cao, and Willard (2000).

5 Comparative Dynamics

In this section, we use the closed-form equilibrium formula derived in the previous section to study the comparative dynamics of the equilibrium and compare the equilibrium against that obtained by Back, Cao and Willard (2000) in the case of no disclosure. For comparison, we use $\hat{\Sigma}$ to denote the conditional variance in the BCW model and the same holds for other variables.

The parameters of the model are the variance of $\tilde{v}$, the number N of informed traders and the correlation ρ. We will fix $\text{var}(\tilde{v}) = 1$ and examine the effects of varying N and ρ.

In each period, the informativeness of informed traders' trade is measured by β because the total information based trade in period t to $t + dt$ is proportional to $\beta(v - V)dt$. The increase in market maker's precision is $\beta^2 dt$. The derivative of market maker's conditional precision is $(1/\Sigma(t))' = \beta^2(t)$. The following describes how disclosure affects β, Σ.

Corollary 5.1 *When $N = 2$, then $\beta > \hat{\beta}, \Sigma < \hat{\Sigma}$. More generally, we have for any $N > 1$,*

$$\lim_{t\to 1} \frac{\beta}{\hat{\beta}} = \infty, \quad \lim_{t\to 1} \frac{\Sigma}{\hat{\Sigma}} = 0,$$

Disclosure makes the market more efficient. Since informed traders' information based trade is mixed with more noise trades, they trade more aggressively with respect to their signal. This effect is most profound near the end of trade as the ratio of Σ with and without disclosure goes to zero. Figure 1A shows intensity of informed traders' trading in relation to that of trading without disclosure. The intensity is greater when disclosure is required. Figure 1B shows the ratio of trading intensity with and without disclosure. It is always larger than 1 and goes to infinity near the end of trade.

As a result of more aggressive informed traders trading and the fact that the random order from all informed traders collectively equals, in distribution, to that of the liquidity traders, market becomes more efficient under the disclosure rules. This is clearly demonstrated in Figure 2.

Next we examine market depth, $1/\lambda(t)$ defined by Kyle (1985). The expected profits $\pi(0)$ is related to market depth as described below:

$$\pi(0) = \int_0^1 \lambda(t)dt.$$

Notice that $\lambda(t)$ represents the expected loss per unit of trade for liquidity traders arriving at time t. The previous relationship holds because the expected profits of informed traders equal to the expected loss of liquidity traders. The following describes the effects of disclosure on these variables.

Corollary 5.2 *When $N = 2$ and $\rho \geq 0$, then $1/\lambda > 1/\hat{\lambda}$. In addition, $\pi(0) < \hat{\pi}(0)$. More generally, for any $N > 1$, any ρ, in the limit, we have:*

$$\lim_{t \to 1} \frac{1/\lambda}{1/\hat{\lambda}} = \infty,$$

In brief, with positively correlated signals, informed traders trade more aggressively on their perceived difference from market expectation under disclosure. Market depth is higher with disclosure due to the randomization component in informed trader's trade which makes the proportion of informed trade less significant. Figure 3 plots the market depth with positively correlated signals. Clearly the results in corollary 5.2 also hold for $N > 2$. It is interesting to observe that market depth changes over time in a pattern that is different from no-disclosure case. In the case of multiple informed traders of positively correlated signals, market first rises and then declines to 0 when this is no disclosure requirement; but market depth always rises in when there is disclosure requirement.

Corollary 5.3 *When $N = 2$, then there exists $\rho < 0, t$ such that $1/\lambda(t) < 1/\hat{\lambda}(t)$, $\pi(0) > \hat{\pi}(0)$.*

This is a rather surprising result. Intuitively, one would have expected that disclosure should decrease market liquidity. Notice the ratio of market liquidity can be decomposed into three components:

$$\frac{1/\lambda}{1/\hat{\lambda}} = \frac{2}{1} \times \frac{\hat{\beta}}{\beta} \times \frac{\hat{\Sigma}}{\Sigma}$$

There are three factors that affect market liquidity. The first effect is the randimization effect which will increase market liquidity under disclosure. Other things being equal, this effect will double market liquidity which is true with a single informed trader. The second effect is the trading intensity effect which decrease market liquidity under disclosure. The third effect is the residual uncertainty effect which increase market liquidity under disclosure because of a more efficient market. With positively correlated signals, the last two effects roughly offset each other except near the end of trade. Therefore, the first effect is dominant in early part of the trading period while the third effect is dominant with the later part of the trading period. Therefore, market is always more liquid with disclosure. However, with negatively correlated signals, informed traders does not trade very much. This cause the third effect insignificant for much of the early trading period. Therefore, market liquidity can decrease as the trading intensity effect can dominate the randomization effect. The reduction in market liquidity can be strong enough such that informed traders' expected profits increases with disclosure.

The effect of disclosure on informed traders' profits is ambiguous. Other things being equal, disclosure causes the informed traders to lose half of their information based trading profits due to randomization. This results in a reduction of informed traders' profits when informed traders' signals are positively correlated. With negatively correlated signals, the results can be reversed. Notice that informed traders' profits would be maximized if they trade like a monopolist. Their profits reduces when they trade more intensively or less intensively than a monopolist. Without disclosure and with negatively correlated signals, informed traders trade less than a monopolist. Disclosure increases informed traders' information based trading and causes their profits from information based trading to increase. This effect can dominate the randomization effect when informed traders' correlation is sufficiently low and the number of informed traders is small.

Figure 4 plots market depth for negatively correlated signals. Notice that market depth are reduced by disclosure for some period of time. However, disclosure always increases market depth near the end of trade.

For three dimensional plots, we introduce a new variable $\bar{\rho}$ which measures the correlation coefficient of informed trader i's signal with that of the sum of other informed traders signals, that is

$$\bar{\rho} = \frac{\mathrm{cov}(\tilde{v} - s_i, s_i)}{\sqrt{\mathrm{var}(\tilde{v} - s_i)\mathrm{var}(s_i)}} = \frac{\rho}{\sqrt{(1 + (N-2)\rho)/(N-1)}}$$

The variable $\bar{\rho}$ has the same sign as ρ and has a range of [-1,1] while ρ has a range of [-1/(N-1],1]. Using $\bar{\rho}$ allows us to plot the profit ratio of different N and ρ on the same graph. Figure 5 plots the ratio of informed traders' profits as a function of N and $\bar{\rho}$. Notice that the profit ratio could be larger than 1 for $N = 2$ but is strictly less than 1 for $N \geq 3$ or $\rho \geq 0$.

Finally, we examine how competition affects market efficiency, market liquidity and trading intensity under disclosure.

Corollary 5.4 *With disclosure and uncorrelated signals, market efficiency, market liquidity, and trading intensity remain the same as the monopolistic case. With correlated signals, β, Σ^{-1} all increase with ρ. $\lambda(0)$ increases with ρ and $\lambda(1)$ decreases with ρ. The variable $\lambda(t)$ decreases over time when $\rho > 0$ while $\lambda(t)$ increases over time when $\rho < 0$. In the end of the trading period as $t \to 1$, $N\delta \to 1$ and informed traders' signals become uncorrelated and they all behave like monopolist. We have*

$$\lim_{t \to 1} \frac{\beta}{1/(\sqrt{S_0}(1-t))} = 1, \quad \lim_{t \to 1} \frac{\Sigma}{S_0(1-t)} = 1, \quad \lim_{t \to 1} \frac{\lambda}{\sqrt{S_0}/2} = 1.$$

Here, $S_0 = \frac{(1-\rho)(1-B)\Sigma 0}{\rho(3N-4)}$, B is defined in theorem 4.1. Finally, informed traders' profits are maximized when $\rho = 0$.

Competition has no effect when informed traders' signals are uncorrelated. The effects of competition on $\beta, \Sigma, \lambda, \pi_0$ are plotted in Figures 6-9. When $\rho = 0$, the results are the same as those in a monopolistic model. As a result, we can examine competition effects by simply looking at how $\beta, \Sigma, 1/\lambda, \pi_0$ changes with ρ, the correlation coefficient of informed traders' signals.

In Figure 6, it is clear that β increases with ρ. High correlation makes informed traders compete more intensively to trade before others. Thus, with positively correlated signals market is more efficient with competition and less efficient with negatively correlated signals as show in Figure 7. Near the end of trading, informed traders correlation coefficient goes to zero and each behaves like a monopolist.

This is in contrast to the BCW model without disclosure in which informed traders' signals become perfectly uncorrelated and the ratio of informational intensity, trading intensity, market depth over that of a monopolist goes to zero while the ratio of conditional variance goes to infinity. Figure 8 plots market depth with disclosure. With positive correlation, market depth increases over time as informed traders' signals become less correlated. With negative correlation, market depth decreases over time as the correlation of informed traders' signals increases. In addition, informed traders' profits is maximized with when $\rho = 0$ as shown in Figure 9 while in BCW, informed traders'profits are maximized when signals are slightly positive.

6 Conclusion

We have analyzed the effects of trade disclosure when there are multiple informed traders. We show that informed traders will employ a randomization strategy to hide their information. The instantaneous variance of informed traders' trade is the same as that of the liquidity traders. Because of the randomization of the informed traders' trade, the sensitivity of informed traders' trade to their private information is higher with disclosure and this cause the market to be more efficient. Market is more liquid and informed traders make less profits with positively correlated signals. This result is consistent with the initial motivation of mandatory disclosure to reduce informed traders' profits and level the playing field for small investors.

However the market can be less liquid and informed traders can make more profits with negatively correlated signals. This arises because disclosure discourages informed traders' incentive to wait as with disclosure informed traders' signals eventually become uncorrelated. The stimulation of active trading more than offset the loss from randomization for low correlation and small N. We also examine the effects of competition when disclosure is required. We find that competition does not affect market liquidity, market efficiency, or informed traders' profits when informed traders' signals are uncorrelated. With correlation, market is more liquid and more efficient with positive correlation and the opposite holds for negative correlation. In addition, market depth increases over time with positive correlation and decreases over time with negative correlation. Competition strictly decreases informed traders' profits, with the exception of zero correlation. Informed Traders's signals become uncorrelated near the end of trade and they eventually behave like a monopolist. This is in contrast to the BCW model in which informed traders' signals become almost perfectly negatively correlated near the end of trade which cause the market to be very illiquid and inefficient compared to the monopolistic case.

A Proofs for Section 3

Proof of Theorem 3.1 We focus on proving the necessity of the claimed equations. The sufficiency of these equations can be established by reversing the necessity arguments (see the end of this proof for more details). So in the rest of this proof except in the last paragraph, we assume that a symmetric linear equilibrium exists, and we prove the claimed equations.

We first prove Equations 3.7, 3.8, 3.9, and 3.10 simply by assuming that each informed trader follows Strategy 3.1. These equations will be used in the inductive proofs for other equations.

First, we can easily check the correctness of Equations 3.7 and 3.8 by the fact that the expectation of a normal variable is precision-weighted average of all received signals. Moreover, the updating rules of normally distributed variables states that posterior precision equals prior precision plus the precision of the noise of the signals. Hence, we immediately establish the correctness of Equations 3.9 and 3.10.

Before proving the rest of the desired equations, we first establish the following useful lemma.

Lemma A.1 *Assume (1) each informed trader believes that all other informed traders follow Strategy 3.1, and (2) the market maker believes that all informed traders follow Strategy 3.1. Then,*

$$\sum_{1\le i\le N}(V_m^i - V_m) = N\delta_m(\tilde{v} - V_m).$$

Proof First, it is easy to check the correctness of the following mathematical identidy by properties of normal variables

$$\Omega_0 = \frac{N-1}{N}(1-\rho)\Sigma_0. \tag{A.1}$$

Using this relation and Equations 3.9 and 3.10, we can easily check

$$\frac{\Omega_m}{\Omega_0}(N-1)\rho + 1 = N\delta_m. \tag{A.2}$$

In what follows, define

$$U_m^i \equiv E[\tilde{v} - s^i | F_m^i]$$

where the expectation is computed after trade disclosures in period m. Equivaently, we could have defined $U_m^i \equiv V_m^i - s_m^i$.

Since the expected value of a normal variable is equal to the precision weighted average of all received signals, we have

$$\begin{aligned} U_m^j &= \frac{\Omega_m}{\Omega_0}U_0^j + \Omega_m \sum_{1\le k\le m}\left[\left(\frac{1}{\Omega_k} - \frac{1}{\Omega_{k-1}}\right)\sum_{i\ne j}\left(s_k^i + \frac{z_k^i}{\beta_k \Delta t}\right)\right] \\ &= \frac{\Omega_m}{\Omega_0}U_0^j + \Omega_m \sum_{1\le k\le m}\left[\frac{\beta_k^2 \Delta t}{(N-1)\sigma_m^2}\sum_{i\ne j}\left(s_k^i + \frac{z_k^i}{\beta_k \Delta t}\right)\right], \end{aligned} \tag{A.3}$$

where the second equation follows from Equation 3.9. (It is easy to verify that Equation 3.9 holds when each informed trader merely *belives* all other informed traders follow Strategy 3.1.) Similarly,

$$V_m = 0 + (\Sigma_m)\cdot \sum_{1\le k\le m}\left[\frac{\beta_k^2 \Delta t}{N\sigma_m^2}\sum_{1\le i\le N}\left(s_k^i + \frac{z_k^i}{\beta_k \Delta t}\right)\right]. \tag{A.4}$$

Summing up Equation A.3 over $j = 1, 2, \ldots, N$, we have

$$\begin{aligned}
\sum_{1 \le j \le N} U_m^j &= \frac{\Omega_m}{\Omega_0}(N-1)\rho \sum_{1 \le j \le N} s^j + \Omega_m \sum_{1 \le k \le m} \left[\frac{\beta_k^2 \Delta t}{\sigma_m^2} \sum_{1 \le i \le N} \left(s_k^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right] \\
&= \frac{\Omega_m}{\Omega_0}(N-1)\rho\tilde{v} + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by Equation A.4)} \\
&= (N\delta_m - 1)\tilde{v} + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by Equation A.2).}
\end{aligned}$$

The last equation is only a slight variation of the equatlity claimed in the lemma.

We have thus completed the proof of Lemma A.1. Using the results established in proving the lemma, we next prove that Strategy 3.2 satisfies Equation 3.1. In Equation 3.2, x_m^i consists of a random component (z_m^i), a component based on public information ($\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}$), and a private-information-related component ($\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i$). By Equation A.3, the only private component in $\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i$ is equal to

$$\frac{\beta_m \Delta t}{N\delta_{m-1}} \left(s^i + \frac{\Omega_{m-1}}{\Omega_0}(N-1)\rho s^i \right) = \beta_m \Delta t s^i \quad \text{(by Equation A.2).}$$

This proves that Strategy 3.2 satisfies Equation 3.1. Moverover, our arguments also imply that to support a symmetric linear equilibrium, x_m^i must have the following form:

$$x_m^i - z_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i + \text{ a public-information-based component.} \tag{A.5}$$

Using Lemma A.1 and Equation 3.2, we have

$$\sum_{1 \le i \le N} x_m^i = \beta_m \Delta t \left(\tilde{v} - V_{m-1} + \sum_{1 \le i \le N} \frac{z_m^i}{\beta_m \Delta t} \right). \tag{A.6}$$

Therefore, using Equation 3.8 we immediately obtain Equation 3.4 and Equation 3.5 (the derivation of Equation 3.5 also needs Equation 3.10). Note that in a symmetric linear equilibrium, the value updating rules must be of the form specified in Equation 3.4. Our arguments in this paragraph together with Equation A.5 also show that to support a symmetric linear equilibrium, Equation 3.2 must hold.

Using Equation A.6 and the rules of conditional expectation of normally distributed variables, we immediately obtain Equation 3.3 with

$$\begin{aligned}
\lambda_m &= \frac{\operatorname{cov}\left(\tilde{v}, \sum_{1 \le j \le N} x_m^j + z_m^0\right)}{\operatorname{var}(z_m^0 + \sum_{1 \le j \le N} x_m^j)} \\
&= \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2).
\end{aligned}$$

The last equation is exactly Equation 3.6.

We next proceed to prove Equations 3.11 to 3.20 by backward induction on m, starting with the last period $m = M$. As there is no more trading opportunities after the last period, the maximization problem for each informed trader i is the same as the case without disclosure which has been derived in Foster and Viswanathan (1996) and Cao (1995). In particular, applying Thereom 2.1, we know that the expected profit function of informed trader i has the form described in Equation 3.11 with the boundary conditions specified in Equations 3.16 to 3.20.

Thus, we have completed the base step. Next, we assume Equations 3.11 to 3.15 are correct for period $m+1$ and prove them for period m. By the induction hypothesis, immediately after the mth period disclosure, the expected profits for future trades (i.e., trades from period $m+1$ onwards) can be written as,

$$E^i_m[\pi^i_{(m+1)}] \equiv E[\pi^i_{(m+1)}|F^i_m] = \alpha_m(V^i_m - V_m)^2 + \zeta_m. \tag{A.7}$$

Hence, the maximization problem of informed trader i immediately after the $(m-1)$th period trade disclosure is:

$$\max_{x^i_m} E^i_{m-1}\left[x^i_m\left(\tilde{v} - V_{m-1} - \lambda_m\left(z^0_m + \sum_{1\le j\le N} x^j_m\right)\right) + \alpha_m(V^i_m - V_m)^2\right] + \zeta_m, \tag{A.8}$$

where the two terms inside the squared brackets represent the profit of the mth trade and the total profit of all future trades.

For informed trader i to follow a random strategy, he must be indifferent between different values of x^i_m. Thus, the coefficients of $(x^i_m)^2$ and x^i_m in Expression A.8 must be zero. These two restrictions respectively imply

$$\lambda_m = \alpha_m\bar{\lambda}^2_m, \quad \text{and} \tag{A.9}$$

$$E^i_{m-1}\left[\tilde{v} - V_{m-1} - \lambda_m\sum_{j\ne i} x^j_m\right] = 2\alpha_m\bar{\lambda}_m E^i_{m-1}\left[V^i_m - V_{m-1} - \bar{\lambda}_m\sum_{j\ne i} x^j_m\right]. \tag{A.10}$$

Note that Equation A.9 is the same as Equation 3.12. In what follows, we show that Equations A.9 and A.10 together imply Equation 3.13. On the other hand, by Lemma A.1,

$$\sum_{j\ne i} x^j_m \;=\; \beta_m\Delta t\left(\tilde{v} - V_{m-1} - \frac{1}{N\delta_{m-1}}(V^i_{m-1} - V_{m-1})\right) + \sum_{j\ne i} z^j_m. \tag{A.11}$$

Hence,

$$E^i_{m-1}\left[\sum_{j\ne i} x^j_m\right] = \beta_m\Delta t\left(1 - \frac{1}{N\delta_{m-1}}\right)\left(V^i_{m-1} - V_{m-1}\right)$$

Applying this relation to Equation A.10, we obtain

$$\frac{1 - \lambda_m\beta_m\Delta t + \lambda_m\beta_m\Delta t/(N\delta_{m-1})}{1 - \bar{\lambda}_m\beta_m\Delta t + \bar{\lambda}_m\beta_m\Delta t/(N\delta_{m-1})} = 2\alpha_m\bar{\lambda}_m.$$

Now we multiply both sides of the preceding equation with the denominator of the left-hand side of the equation, and then we use Equation A.9 to substitute all the $\alpha_m\bar{\lambda}^2$ terms by λ_m. This leads to

$$2\alpha_m\bar{\lambda}_m = 1 + \lambda_m\left(\beta_m\Delta t - \frac{\beta_m\Delta t}{N\delta_{m-1}}\right).$$

Next, multiplyuing both sides of the above equation with $\bar{\lambda}_m$ and using Equation A.9 to substitute $\alpha_m\bar{\lambda}^2$ by λ_m, we immediately obtain Equation 3.13.

Since we have established that informed trader i is indifferent to x_m^i, Expression A.8 can be simplified by setting $x_m^i = 0$. Thus,

$$\begin{aligned} E_{m-1}^i[\pi_m] &= \alpha_m E_{m-1}^i\left[\left(V_m^i - V_m\right)^2\right] + \zeta_m \\ &= \alpha_m \left(E_{m-1}^i\left[V_m^i - V_m\right]\right)^2 + \alpha_m \mathrm{var}_{m-1}^i(V_m^i - V_m) + \zeta_m \end{aligned} \tag{A.12}$$

On the other hand, since we have assumed $x_m^i = 0$ in the profit calculation, using the updating rule for normal variables we have

$$\begin{aligned} V_m^i &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(\tilde{v} + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t}\right) \\ &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} \left(\tilde{v} + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t}\right), \end{aligned} \tag{A.13}$$

where the second equation follows from Equation 3.9. Moreover, using the pricing rules by market maker and applying Equation A.11, we have

$$\begin{aligned} & V_m \\ = \;& V_{m-1} + \bar{\lambda}_m \beta_m \Delta t \left(\tilde{v} - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N\delta_{m-1}}\right) + \bar{\lambda}_m \sum_{j \neq i} z_m^j \\ = \;& V_{m-1} + \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2} \left(\tilde{v} - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N\delta_{m-1}} + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t}\right), \end{aligned} \tag{A.14}$$

where the second equation follows from Equation 3.5.

Now, using Equations A.14 and A.13 and the fact that $\tilde{v}$ is independent of $\sum_{j \neq i} z_m^j$, we have

$$\mathrm{var}_{m-1}^i(V_m^i - V_m) = \left(\frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} - \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2}\right)^2 \left(\Omega_{m-1} + \frac{(N-1)\sigma_m^2 \Delta t}{(\beta_m \Delta t)^2}\right) \tag{A.15}$$

Moreover,

$$\begin{aligned} E_{m-1}^i[V_m^i - V_m] &= V_{m-1}^i - E_{m-1}^i[V_m] \\ &= \left(1 - \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2}\left(1 - \frac{1}{N\delta_{m-1}}\right)\right)(V_{m-1}^i - V_{m-1}), \end{aligned} \tag{A.16}$$

where the last equation follows from Equation A.14. Substituting Equations A.15 and A.16 into Equation A.12, we immediately see that Equation 3.11 is correct for m with α and ζ satisfying Equations 3.14 and 3.15. This completes our inductive step.

So far, we have proved all the desired equations as necessary conditions to support a symmetric linear equilibrium. In proving these equations, we have used (1) the rationality of the market maker's pricing rules and value updating rules, and (2) the optimality of all informed traders' trading strategies. Moreover, by reversing these arguments, we can easily check that when these equations indeed hold, (1) the pricing rules and value updating rules are indeed rational for the market maker, and (2) the trading strategies of all informed traders are indeed optimal. Therefore, all these equations collectively form a set of sufficient conditions to support a symmetric linear equilibrium. □

Discussion on Solving the System of Equations in Theorem 3.1

The whole recursive system of $\alpha_m, \beta_m, \lambda_m, \bar{\lambda}_m, \Sigma_m, \Omega_m, \zeta_m$ can be numerically solved by first conjecturing a value of Ω_{M-1} and then solving recursively for $\Omega_{M-2}, \ldots, \Omega_0$. Given the conjectured Ω_{M-1}, we can compute δ_{M-1}, since the definition of δ_M and Equations 3.9 and 3.10 imply

$$N\delta_{M-1} = 1 + \frac{\Omega_{M-1}}{\Omega_0}(N\delta_0 - 1).$$

From Ω_{M-1} and δ_{M-1}, we can now derive Σ_{M-1}. From the boundary condition in Equation 3.18, we can determine α_{M-1}. Now again we conjecture a value for Ω_{M-2}, which allows us to derive δ_{M-2} and Σ_{M-2} as before. From Equations 3.10 and 3.5,

$$\Sigma_{M-1}^{-1} = \Sigma_{M-2}^{-1} + \bar{\lambda}_{M-1}\beta_{M-1}\Delta t/\Sigma_{M-1}.$$

Consequently, we obtain $\beta_{M-1}\bar{\lambda}_{M-1}$. Comparing Equations 3.6 and 3.12, we arrive at

$$\beta_{M-1}\Sigma_{M-2}/(\beta_{M-1}^2\Delta t\Sigma_{M-2} + 1 + N\sigma_{M-1}^2) = \bar{\lambda}_{M-1}^2\alpha_{M-1}.$$

In the preceding equation, we can use the derived expression for $\beta_{M-1}\bar{\lambda}_{M-1}$ to substitute $\bar{\lambda}_{M-1}$ for β_{M-1}, and we can use Equation 3.5 to substitute $\bar{\lambda}_{M-1}$ for σ_{M-1}^2. Doing so results in an equation with $\bar{\lambda}$ being the only unknown. Solving the resulting eqaution gives a formula for $\bar{\lambda}_{M-1}$. Next we can derive β_{M-1} from $(\beta_{M-1}\bar{\lambda}_{M-1})/\bar{\lambda}_{M-1}$, λ_{M-1} from Equation 3.12, and σ_{M-1}^2 from Equation 3.5. Given the expressions for $\lambda_{M-1}, \bar{\lambda}_{M-1}, \beta_{M-1}$ and σ_{M-1}^2, we can now check whether Equation 3.13 holds or not. If it doesn't, we modify our initial value of Ω_{M-2} until it holds. We repeat the procedure to derive $\Omega_{M-3}, ..., \Omega_0$. If the derived Ω_0 is different from the initial given value, we adjust Ω_{M-1} and repeat the whole procedure until the derived Ω_0 equals to the initial given value.

B Proofs for Section 4

Proof of Lemma 4.1 Recall that each dz^i $(1 \le i \le N)$ is a non-standard Brownian motion and that dz^i and dz^j are independent for $i \neq j$. Hence, by Equation 4.4, $\sum_{1\le i\le N} dz^i$ is a standard Brownian motion with instantaneous variance dt. The correctness of the lemma then follows from the Kalman-Bucy filter (see, e.g., Kallianpur (1980)). □

Proof of Lemma 4.2 Note that $U^j(0) = (N-1)\rho s^j$ and that $\sum_{i\neq j} z^i(t)$ is a Brownian motion with instantaneous variance equal to $\frac{N-1}{N}dt$. On the other hand, Equation A.1 implies $\Omega(0) = \text{var}(\tilde{v}|\mathcal{F}^j(0))$ (since there is no trade at time 0, it makes no difference whether the variance is taken before or after disclosure at time 0). Now, the correctness of the lemma follows from the Kalman-Bucy filter (see, e.g., Kallianpur (1980)). □

Proof of Lemma 4.3 Consider an arbitrary informed trader j ($1 \le j \le N$). If indeed all other traders follow Strategy 4.3, then from Equations 4.11 and 4.12,

$$dU^j + \left(\frac{N}{N-1}\beta^2\Omega\right)U^j dt = \frac{N}{N-1}\beta\Omega\left(\sum_{i\neq j} dz^i + \beta\sum_{i\neq j} s^i dt\right).$$

Together with Equation 4.10, this immediately implies

$$\frac{d}{dt}\left(\frac{1}{\Omega}U^j\right) = \frac{N}{N-1}\beta\left(\sum_{i\neq j} dz^i + \beta\sum_{i\neq j} s^i dt\right).$$

Since trader j believes all other traders follow Strategy 4.3, he expects $dz^i + \beta s^i dt = dx^i - f(t)dt$. Hence, he uses the following rule to update his U^j,

$$\frac{d}{dt}\left(\frac{1}{\Omega}U^j\right) = \frac{N}{N-1}\beta\sum_{i\neq j}(dx^i - f(t)dt).$$

Therefore,

$$\begin{aligned}
& U^j(t) \\
= \; & \frac{\Omega(t)}{\Omega(0)}U^j(0) + \Omega(t)\frac{N}{N-1}\int_0^t \beta(t)\sum_{i\neq j}(dx^i - f(t)dt) && \text{(B.1)} \\
= \; & \frac{\Omega(t)}{\Omega(0)}(N-1)\rho s^j + \Omega(t)\frac{N}{N-1}\int_0^t \beta(t)\sum_{i\neq j}(dx^i - f(t)dt). && \text{(B.2)}
\end{aligned}$$

Note that Equation B.1 can also be directly derived by the fact that (under our normality assumption) the conditional expectation of the asset value is the precision-weighted average of all observable signals.

By exactly the same reasoning, the market maker, who believes that all informed traders follow Strategy 4.3, has his estimate of asset value as

$$V(t) = 0 + \Sigma(t)\int_0^t \beta(t)\sum_{1\le i\le N}(dx^i - f(t)dt). \quad \text{(B.3)}$$

Summing up Equation B.2 over all j, we obtain

$$\begin{aligned}
\sum_{1\le j\le N} U^j(t) &= \frac{\Omega(t)}{\Omega(0)}(N-1)\rho\tilde{v} + \Omega(t)N\int_0^t \beta(t)\sum_{1\le i\le N}(dx^i - f(t)dt) \\
&= (N-1)\frac{\Omega(t)}{\Omega(0)}\rho\tilde{v} + N\frac{\Omega(t)}{\Sigma(t)}V(t) \quad \text{(by Equation B.3).} && \text{(B.4)}
\end{aligned}$$

Now the correctness of Equation 4.13 reduces to the following

$$\frac{\Omega(t)}{\Omega(0)}(N-1)\rho + 1 = N\frac{\Sigma(t)-\Omega(t)}{\Sigma(t)}. \quad \text{(B.5)}$$

This equality can be directly verified by Equations 4.7 and 4.10.

Proof of Lemma 4.4 By Eqaution B.2, V^j consists of two components.[10] The first component is based on private-information (i.e., it depends on s^j) and is equal to

$$\left(1 + \frac{\Omega(t)}{\Omega(0)}(N-1)\rho\right) s^j.$$

The second component is purely based on public information. Hence, the private-information component in dx^j (i.e., the component dependent on s^j) is equal to

$$\frac{\beta}{N\delta}\left(1 + \frac{\Omega(t)}{\Omega(0)}(N-1)\rho\right) s^j = \beta s^j,$$

where the equality follows from Equation B.5 (which is purely a mathematical identity). This proves that Stratergy 4.14 satisfies Equation 4.3. The above arguments also show that if a strategy satisfies Equation 4.3 and its deterministic part can be decomposed into a public-information component and another component involving V^j, then the latter component must be of the form specified in Equation 4.14.

On the other hand, if all informed traders indeed follow Strategy 4.14, then by Lemma 4.1,

$$dV(t) = \beta(t)\Sigma(t)\left[\sum_{1\le i\le N} dz^i + \beta(\tilde{v} - V(t))dt\right]. \tag{B.6}$$

However, the market maker does not observe $\tilde{v}$ directly. Hence, believing that all informed traders follow Strategy 4.14, he can use Equation 4.13 to substitute the $\tilde{v} - V(t)$ term in the above equation and obtain

$$dV(t) = \beta(t)\Sigma(t)\left[\sum_{1\le i\le N} dz^i + \frac{\beta}{N\delta}\sum_{1\le i\le N}\Big(V^i(t) - V(t)\Big)\, dt\right]. \tag{B.7}$$

Since we have already proved that each informed trader i's information-based component has the form of $\frac{\beta(t)}{N\delta(t)}V^i(t)$, the above equation is consistent with Equation 4.1 if and only if the public-information component of each informed trader i's deterministic trade is equal to $\frac{\beta(t)}{N\delta(t)}V(t)$. Hence, we conclude that Strategy 4.14 is the unique trading strategy with the claimed properties.

Now, comparing Equations 4.1 and B.7, we immediately obtain Equation 4.15. Finally, given that Strategy 4.14 supports the pricing rule in Equation 4.1 with $\bar{\lambda}$ specified in Equation 4.15, a direct application of the Kalman-Bucy filter (see, e.g., Kallianpur (1980)) proves that Strategy 4.14 also supports the pricing rule in Equation 4.2 with λ specified in Equation 4.16. □

Proof of Lemma 4.5 Since we will focus on an arbitrary informed trader j ($1 \le j \le N$) throughout this proof, we use $dx(t)$ as a shorthand for $dx^j(s^j, V^j, V^{x^j})$. Also, we rewrite $V(t)$ and $P(t)$ as $V^x(t)$ and $P^x(t)$, respectively, to emphasize that trading strategy x affects the processes V and P. Using Expression 4.20 and the law of iterated expectations, we know that the objective of trader j is to maximize

$$E\int_0^1 \Big(V^j(t) - P^x(t+dt)\Big)\, dx(t)$$

[10] We can use this equation here since it is derived from merely assuming that each informed trader believes all other informed traders follow Strategy 4.3.

under the dynamics of the state variables V^j, V^x, and P^x.

From Equation 4.12, V^j follows the following dynamics

$$dV^j(t) = \sqrt{\frac{N}{N-1}}\beta(t)\Omega(t)dW^j(t). \tag{B.8}$$

On the other hand, the instantaneous order submitted by all traders $i \neq j$ sum to

$$\begin{aligned} & \sum_{i\neq j}\frac{\beta}{N\delta}\left(V^i - V^x\right)dt + \sum_{i\neq j}dz^i \\ = & \left[\beta(\tilde{v} - V^x) - \frac{\beta}{N\delta}\left(V^j - V^x\right)\right]dt + \sum_{i\neq j}dz^i \quad \text{(by Equation 4.13)} \\ = & \sqrt{\frac{N-1}{N}}dW^j + \beta\left(1 - \frac{1}{N\delta}\right)(V^j - V^x)dt \quad \text{(by Lemma 4.2)} \end{aligned}$$

Hence, by the pricing rule in Equation 4.1,

$$\begin{aligned} dV^x(t) &= \bar{\lambda}(t)\sqrt{\frac{N-1}{N}}dW^j(t) + \bar{\lambda}(t)\beta(t)\left(1 - \frac{1}{N\delta(t)}\right)\cdot \\ &\quad \cdot\left(V^j(t) - V^x(t)\right)dt + \bar{\lambda}(t)dx(t). \end{aligned} \tag{B.9}$$

The optimization problem is a Markovian stochastic control problem with state variables $(V^j(t), V^x(t), P^x(t))$. Let $J(t, s, V^j, V^x)$ denote a candidate for the following value function

$$\begin{aligned} & \sup_x E\int_t^1 \left(V^j(u) - P^x(u+du)\right)dx(u) \\ = & \sup_x E\int_t^1 \left(V^j(u) - V^x(u) - \lambda(u)dx(u)\right)dx(u), \end{aligned}$$

where the expectation is conditioned on $F^j(t)$ and the equality follows from Equation 4.2. Note that we have dropped "$-\lambda(dz^0(u) + \sum_{i\neq j}dx^i)$" in the parentheses on the right-hand side of the above equation. All of the droped terms there are either a random variable uncorrelated with dx^i or a deterministic term of an order at least dt, and therefore they do not contribute to the expectation.

The Bellman equation for this control problem is

$$\begin{aligned} 0 &= \max_x E_t^j\Big[\left(V^j - V^x - \lambda dx\right)dx + J_t dt + J_{V^x}dV^x + J_{V^j}dV^j + \\ &\quad + \frac{1}{2}J_{V^xV^x}(dV^x)^2 + J_{V^xV^j}dVdV^j + \frac{1}{2}J_{V^jV^j}(dV^j)^2.\Big] \end{aligned} \tag{B.10}$$

Here, J_x, J_{xy} are the first- and second-order partial derivatives of J with respect to x and x, y. Intuitively, the Bellman equation states that the over x of the drift of J plus the instantaneous profit $(V^j - P^x)x$ equals zero; i.e., the expected decline in future profit should be exactly offset by the realized current profit.

Note that the right-hand side of the above Bellman equation depends on x through a quadratic function of dx. In particular, since dx only appears in the dynamics of dV^x but not in the dynamics of dV^j, the only terms involving dx (except those of higher orders) on the right-hand side of the Bellman equation are: $(-\lambda + \frac{1}{2} J_{V^x V^x} \bar{\lambda})(dx)^2$ and $(\bar{\lambda} J_{V^x} + V^j - V^x)dx$. For trader j to follow a random strategy, he must be indifferent across the various possible orders induced by the random strategy; i.e., the coefficient of dx and $(dx)^2$ must be zero. Reasoning from the linear term, we have

$$J_{V^x} = \frac{V^x - V^j}{\bar{\lambda}}. \tag{B.11}$$

Reasoning from the quadratic term, we have $\frac{1}{2} J_{V^x V^x} \bar{\lambda}^2 = \lambda$. Then, applying Equation B.11, we obtain

$$\lambda = \frac{\bar{\lambda}}{2} \tag{B.12}$$

which establishes Equation 4.22. Recall that our postulated equilibrium strategy in Equation 4.14 includes a stochastic term in trader j's order flow. For trader j to follow such a random strategy, he must be indifferent across the various possible orders induced by the random strategy. The above two equations serves to ensure that trader j will be indeed indifferent.

From Equations B.12, 4.16, and 4.15, we immediately know that $\text{var}(dz^0(t)) + \sum_{1 \le i \le N} \text{var}(dz^i(t)) = 2 \sum_{1 \le i \le N} \text{var}(dz^i(t))$. This confirms our earlier claim (see Equation 4.5) that Equation 4.4 leads to

$$\text{var}(dz^0(t)) = dt. \tag{B.13}$$

Using Equations B.8, B.9, B.11, and B.12, we can simplify Equation B.10 to

$$\begin{aligned} 0 &= J_t + J_{V^x} \bar{\lambda} \beta \left(1 - \frac{\Sigma}{N(\Sigma - \Omega)} \right) (V^j - V^x) + \\ &+ \frac{1}{2} J_{V^x V^x} \bar{\lambda}^2 \frac{N-1}{N} + J_{V^x V^j} \bar{\lambda} \beta \Omega + \frac{1}{2} J_{V^j V^j} \beta^2 \Omega^2 \frac{N}{N-1}. \end{aligned} \tag{B.14}$$

By taking the derivatives of Equation B.14 with respect to V^x and using Equations B.11 and B.12 to simplify terms, we arrive at

$$0 = \frac{d}{dt} \left(\frac{V^x - V^j}{\bar{\lambda}} \right) + \frac{d}{dV^x} \left(\beta \left(1 - \frac{\Sigma}{N(\Sigma - \Omega)} \right) (-1)(V^j - V^x)^2 \right).$$

It is straightforward to show that this is equivalent to Equation 4.21. Since Bellman equation is a necessary condition for the optimality of the trading strategy for trader i, the above arguments prove the necessity of Equation 4.21.

To prove the necessary and sufficient conditions for the optimality of the trading strategy, we can assume in the rest of the proof that Equations 4.21 and 4.22 hold, and we only need to show that Equation 4.23 is necessary and sufficient for the optimality of trader j's strategy.

First, straightforward calculations show that the following function J does satisfy the Bellman equation as specified in Equations B.11, B.12, and B.14.

$$J(t, s, V^j, V^x) = \frac{1}{2\bar{\lambda}(t)} (V^x - V^j)^2 + \frac{N-1}{2N} \int_t^1 \frac{1}{\bar{\lambda}(u)} \left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1} \right)^2 du. \tag{B.15}$$

Reasoning with the above J as in Back (1992), we can show that an optimal strategy should not include discrete orders (this is due to the convexity of J as a function of V^j and V^x). Given any trading strategy x with continuous orders, we can apply Ito's lemma to obtain

$$\begin{aligned}
& J(1,s,V^j(1-),V^x(1-)) - J(0,s,V^j(0),V^x(0)) \\
= & \int_0^1 \left(J_t dt + J_{V^x} dV^x + J_{V^j} dV^j + \frac{1}{2} J_{V^x V^x} (dV^x)^2 + J_{V^x V^j} dV^x dV^j + \frac{1}{2} J_{V^j V^j} (dV^j)^2 \right) \\
= & J(0,s,V^j(0),V^x(0)) + \int_0^1 g(t) dW^j(t) - \int_0^1 (V^j - V^x - \lambda dx) dx
\end{aligned}$$

for some function g that depends on time t only and it's easy to verify $\mathrm{E}\Big[\int_0^1 g(t)^2 dt\Big] < \infty$,

where the last equality holds since J satisfies Equations B.11, B.12, and B.14. Thus,

$$E\left(\int_0^1 (V^j - V^x - \lambda dx) dx \right) = J(0,s,V^j(0),V^x(0)) - E(J(1,s,V^j(1-),V^x(1-))).$$

By the definition of J, $-E(J(1,s,V^j(1-),V^x(1-))) \le 0$. Thus from the preceding equality, we see that the proposed trading strategy is optimal if and only if $J(1,s,V^j(1-),V^x(1-)) = 0$, *a.s.*, which is equivalent to

$$\lim_{t\to 1} V^x(t) - V^j(t) = 0 \ a.s. \ \text{ or } \ \lim_{t\to 1} \bar{\lambda}(t) = +\infty. \tag{B.16}$$

To complete the correctness proof that Equations 4.21 and 4.23 are indeed necessary and sufficient. We are left to prove that Equations B.16 and 4.23 are equivalent. First, if $\lim_{t\to 1} \Sigma(t) = 0$, then $\lim_{t\to 1} V^x(t)$ is a precise estimate of $\tilde{v}$ and so $V^j(t)$ should also approach to $\tilde{v}$. On the other hand, by Lemma 4.3, we know that $\lim_{t\to 1} V^x(t) - V^j(t) = 0$ *a.s.* imply:

$$\begin{aligned}
\lim_{t\to 1} \sum_j [V^x(t) - V^j(t)] &= \lim_{t\to 1} N\delta(t)(\tilde{v} - V^x) \\
&= 0
\end{aligned}$$

This must imply $\lim_{t\to 1} \Sigma(t) = 0$, otherwise we have $\lim_{t\to 1} N\delta(t) \neq 0$ (by Equation (B.5)), $\lim_{t\to 1} \tilde{v} - V^x \neq 0$, a contradiction.

Finally, to complete the proof, note that the above argument implies that the expected trading profit for Strategy 4.14

$$\begin{aligned}
& E(J(0,V^j(0),V^x(0))) \\
= & \frac{1}{2\bar{\lambda}(0)} (V^j(0) - 0)^2 + \frac{N-1}{2N} \int_0^1 \frac{1}{\bar{\lambda}(u)} \left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1} \right)^2 du.
\end{aligned}$$

as claimed. □

Proof of Theorem 4.1 This proof consists of two parts: (1) assuming Σ as given, we first prove the formulae for all other quantities; (2) then we prove that Σ exists if and only if $\rho < 1$ or $N = 1$ and that when σ exists it is uniquely determined by the formula given in the lemma.

First, we prove the formulae for all the other formulae assuming the correctness of the formula for Σ. The formula for $\beta(t)$ as a function of $\Sigma(t)$ follows directly from Equation 4.7. The formula for $\bar{\lambda}(t)$ follows from the fact $\bar{\lambda}(t) = \beta(t)\Sigma(t)$ (see Lemma 4.4), and hence the formula for $\lambda(t)$ follows from the fact that $\lambda(t) = \frac{1}{2}\bar{\lambda}(t)$.

Since the market maker makes no profit, the expected profit of all the informed traders is equal to the loss of liquidity traders, which is equal to

$$\int_0^1 \lambda(t) dz^0(t) dz^0(t) = \int_0^1 \lambda(t) dt.$$

By symmetry, each informed trader's profit is $\frac{1}{N}$ of the total expected profits of all informed traders, and hence it is equal to $\frac{1}{N}\int_0^1 \lambda(t)dt$, as claimed. To prove the correctness of Expression 4.6, note that

$$\begin{aligned} \lambda &= \frac{1}{2}\bar{\lambda} \\ &= \frac{1}{2}\beta\Sigma \\ &= \frac{1}{2}\sqrt{\left(\frac{1}{\Sigma}\right)'}\Sigma \\ &= \frac{1}{2}\sqrt{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{1-B}{3N-4}\right)}((1-B)t+B)^{-\frac{2N-2}{3N-4}}, \end{aligned}$$

where the last equality follows from the formula for Σ in Theorem 4.1. Algebraic calculations then show that the integral of the last expression with respect to t from 0 to 1 is equal to Expression 4.6 times N, as desired.[11]

We have thus proved the correctness of all the formulae except the one for Σ. Moreover, this means that the existence, non-existence, or uniqueness of the equilibrium is equivalent to the existence, non-existence, or uniqueness of $\Sigma(t)$, respectively. So in what follows, we only need to derive the formulae for $\Sigma(t)$ or prove its non-existence. We will do so by solving the differential Equation 4.21 with boundary condition 4.23.

From Equation 4.21, we have

$$\frac{d}{dt}\left(\frac{1}{\bar{\lambda}}\right) = \frac{d}{dt}\left(\frac{1}{\beta\Sigma}\right) = -\frac{\beta'}{\beta^2\Sigma} + \beta,$$

where we have used Equation 4.7 to derive the last equality. Thus, Equation 4.21 implies

$$-\frac{\beta'}{\beta^3\Sigma} = 1 - \frac{2\Sigma}{N(\Sigma-\Omega)}. \tag{B.17}$$

On the other hand, the definitions of $\Sigma(t)$ and $\Omega(t)$ (Equations 4.7 and 4.10) implies

$$\frac{N-1}{N}\frac{1}{\Omega(t)} - \frac{1}{\Sigma(t)} = \frac{1}{(1-\rho)\Sigma(0)} - \frac{1}{\Sigma(0)} = \frac{A}{N},$$

[11] The absolute-value operator is needed for the case $\rho < 0$, which implies that the term inside the absolute value operator is negative. Also, we remark that the expected-profit formula can be alternatively derived by taking expectation (at time 0) of Expression 4.24.

where

$$A = \frac{\rho N}{(1-\rho)\Sigma(0)}$$

is a constant. Substituting $\Sigma(t)$ for $\Omega(t)$ in Equation B.17, we get

$$-\frac{\beta'}{\beta^3} = \left(1 - \frac{2}{N}\right)\Sigma + \frac{2(N-1)}{(-A - \frac{1}{\Sigma})N}$$

In what follows, we let $\Gamma = \frac{1}{\Sigma}$. Using the fact $\frac{d}{dt}(\frac{1}{\Sigma}) = \beta^2$, we can rewrite the preceding differential equation as

$$0 = \frac{\Gamma''}{\Gamma'} + \left(2 - \frac{4}{N}\right)\frac{\Gamma'}{\Gamma} + \frac{4(N-1)\Gamma'}{(-A-\Gamma)N} \tag{B.18}$$

In the case $N > 1$ and $\rho = 1$, $A = \infty$, and hence the above equation implies[12]

$$0 = \frac{d}{dt}\left[\log\left(\Gamma'\Gamma^{2-\frac{4}{N}}\right)\right].$$

Thus,

$$\Gamma'\,\Gamma^{2-\frac{4}{N}} = C_0 \text{ for some constant } C_0 > 0,$$

which in turns implies

$$\Sigma(t) = \frac{1}{\Gamma(t)} = (C_1 t + C_2)^{\frac{-1}{3-\frac{4}{N}}} \text{ for some constants } C_1 \text{ and } C_2. \tag{B.19}$$

But when $N > 1$, there is no constants $C_1 = C_0(3 - 4/N) > 0$ and C_2 which can make the above $\Sigma(t)$ satisfy either $\Sigma(1) = 0$ or $\lim_{t\to 1}\bar{\lambda}(t) = \sqrt{-\Sigma'(1)} = +\infty$, as required by Equation 4.23. This completes the proof that a linear equilibrium does not exist for $N > 1$ and $\rho = 1$.

In the rest of the proof, we assume either $\rho \neq 1$ or $N = 1$. Under these assumptions, we prove that Equation B.18 has a unique solution of $\Sigma(t)$ as described in the theorem. Now, the only possible case with $\rho = 1$ happens is when $N = 1$. But when $N = 1$, there is no competing informed traders, and ρ is irrelevant. Without loss of generality, we make the additional assumption $\rho \neq 1$. This will ensure a finite A in the rest of the proof.

By Equation B.18,

$$0 = \frac{d}{dt}\left[\log\left(\Gamma'\Gamma^{2-\frac{4}{N}}(\Gamma + A)^{-\frac{4(N-1)}{N}}\right)\right].$$

Hence,

$$\Gamma'\,\Gamma^{2-\frac{4}{N}}\,(\Gamma + A)^{\frac{-4(N-1)}{N}} = C_3 \text{ for some constant } C_3,$$

which in turns implies

$$(\Gamma)^{2-\frac{4}{N}}\,(\Gamma + A)^{\frac{4(1-N)}{N}}\,\Gamma' = C_4 \text{ for some constant } C_4. \tag{B.20}$$

In the case of $\rho = 0$, we have $A = 0$. Hence, the above equation is equivalent to $\Gamma^{-2}\Gamma' = C_4$, which implies that $\Sigma = 1/\Gamma$ is linear in t. Hence, the desired formula for Σ follows immediately from the boundary condition $\Sigma(1) = 0$.

[12] To be completely formal and to avoid dividing by 0, we should have directly derived the desired equation below. But this is a straightforward exercise by using the argument for obtaining Equation B.18.

For the case of $\rho \neq 0$, we can make a change of variable as $\Gamma = A\frac{r}{1-r}$, the above equation becomes

$$r^{2-\frac{4}{N}} r' = C_4.$$

From this and the boundary condition on $r(0)$ and $r(1)$, we obtain

$$\frac{1}{A\Sigma(t)+1} = \left(\left[\left(\frac{1}{A\Sigma(1)+1} \right)^{3-\frac{4}{N}} - B \right] t + B \right)^{\frac{1}{3-\frac{4}{N}}}. \tag{B.21}$$

Taking derivatives with respect to t in the above equation, we know that $\Sigma'(1)$ is bounded. Hence, from the proved formula $\bar{\lambda}(t) = \sqrt{-\Sigma'(t)}$, we know that $\lim_{t\to 1} \bar{\lambda}$ is finite. Hence, from Equation 4.23, we must have $\Sigma(1) = 0$. Plugging $\Sigma(1) = 0$ into Equation B.21, we immediately arrive at the claimed formula for $\Sigma(t)$. □

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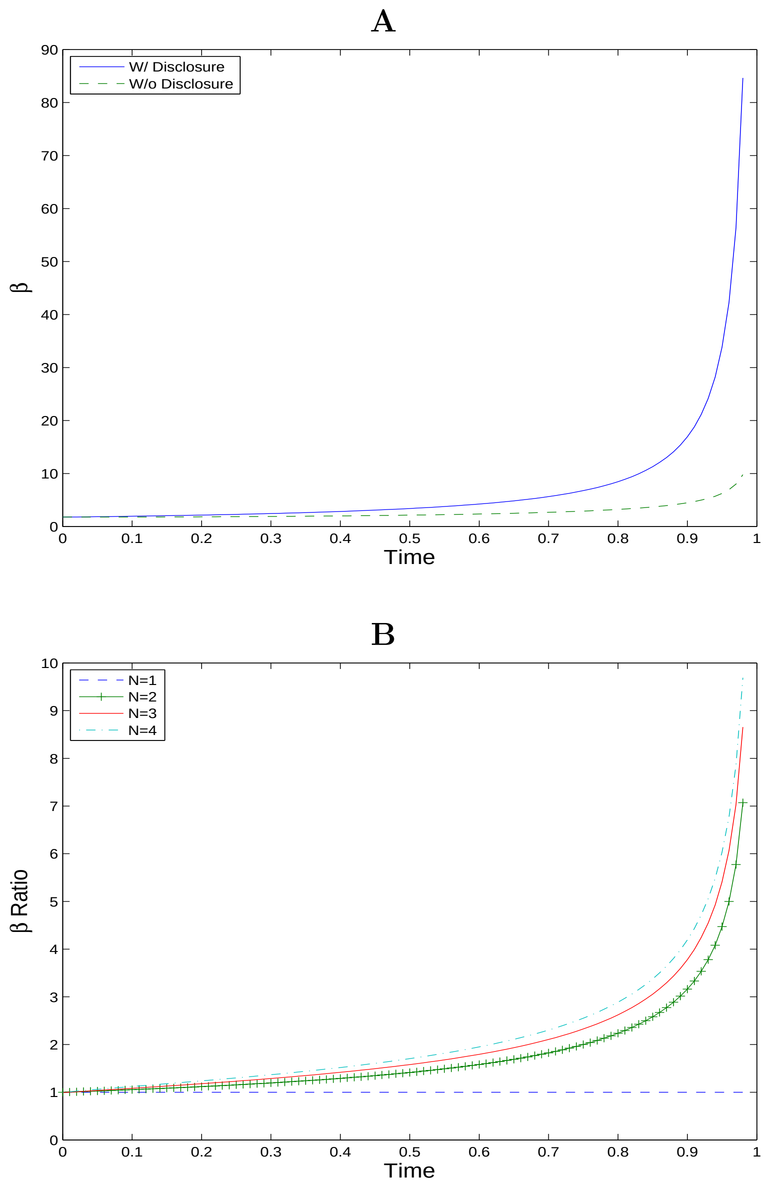


Figure 1: Figure 1.A: Trading intensity β as a function of time for $\rho = 0.3$ and $N = 3$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 1. B: Ratio of β with and without disclosure as a function of t for $\rho = 0.3$ and $N = 1, 2, 3, 4$.

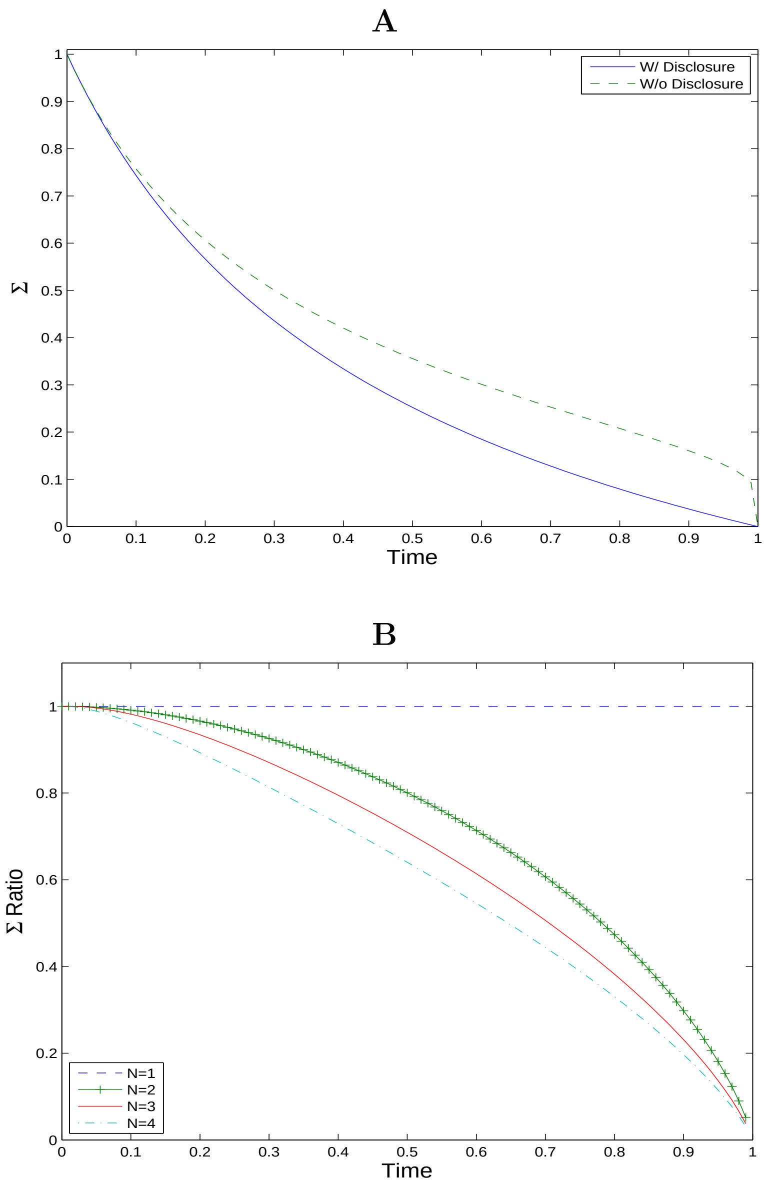
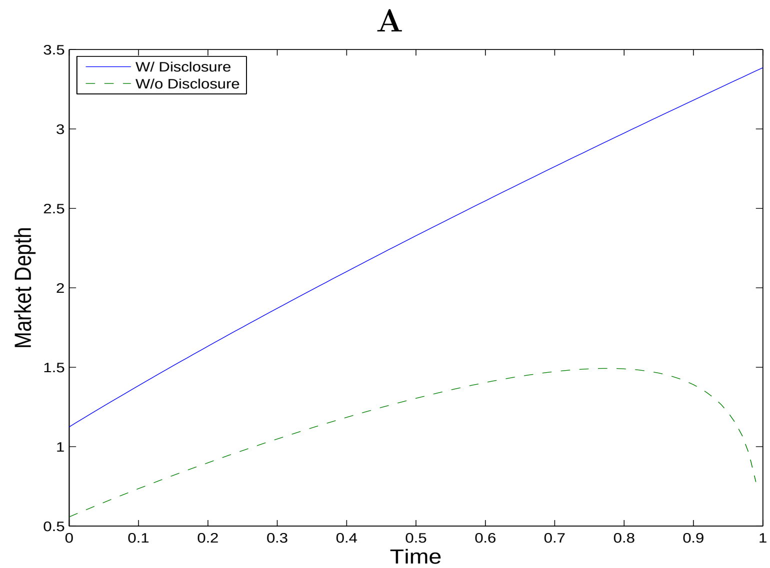


Figure 2: Figure 2A: Residual uncertainty Σ as a function of time for $\rho = 0.3$ and $N = 3$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 2B: Ratio of Σ with and without disclosure as a function of t for $\rho = 0.3$ and $N = 1, 2, 3, 4$.



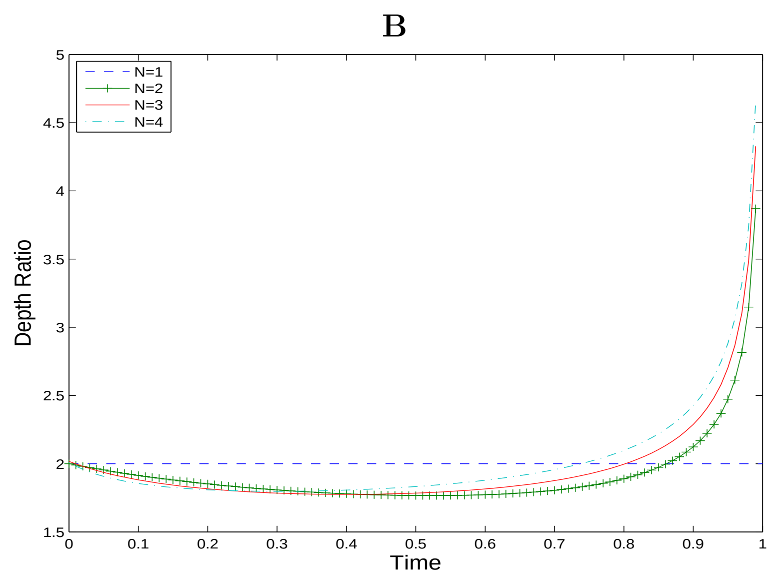


Figure 3: Figure 3A:Market depth $1/\lambda$ as a function of time for $\rho = 0.3$ and $N = 3$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 3B: Market depth ratio with and without disclosure as a function of t for $\rho = 0.3$ and $N = 1, 2, 3, 4$.

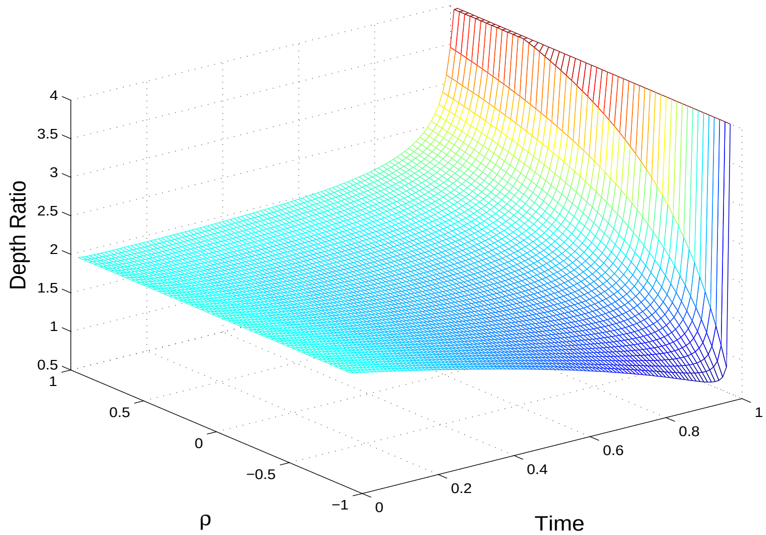


Figure 4: The ratio of market depth $1/\lambda$ with disclosure and without disclosure as a function of ρ, t for $N = 2$.

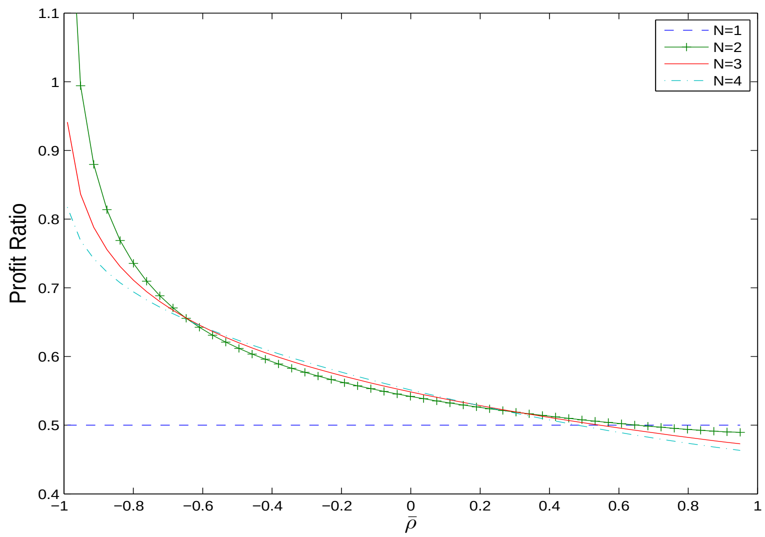


Figure 5: The ratio of informed traders' total profits $\pi(0)$ with disclosure and without disclosure as a function of $\bar{\rho}$ for $N = 1, 2, 3, 4$.

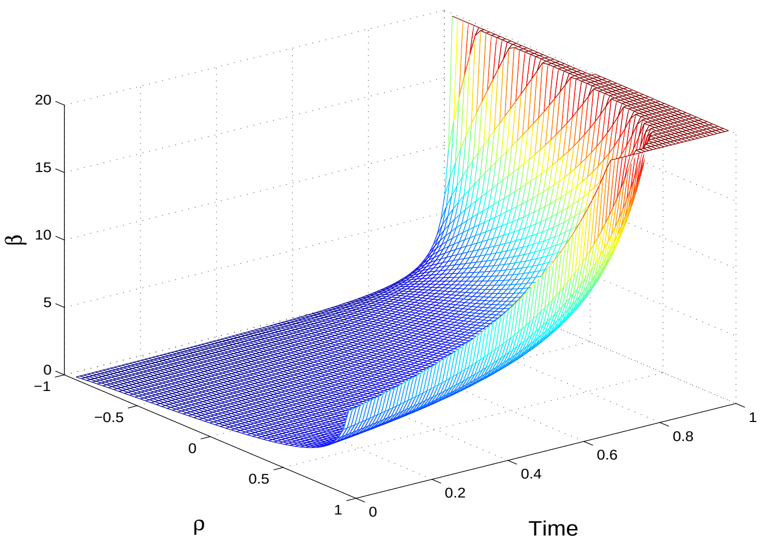


Figure 6: Trading intensity β as a function of ρ, t for $N = 2$ with disclosure

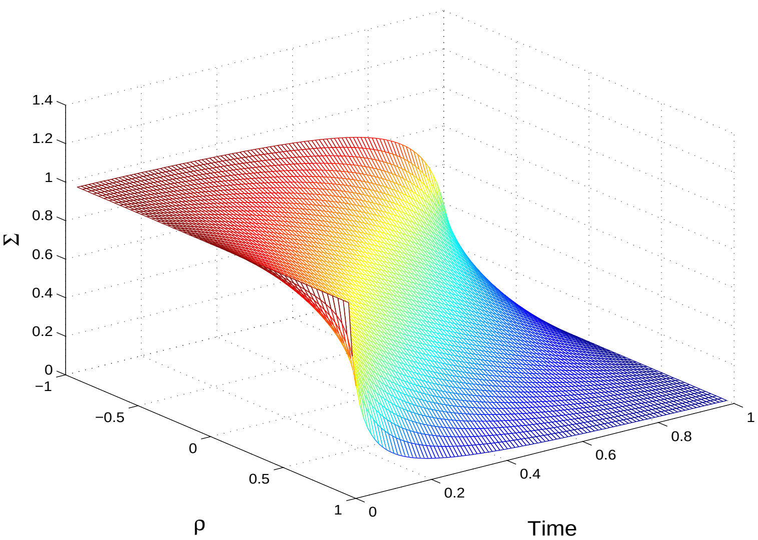


Figure 7: Residual uncertainty Σ as a function of ρ, t for $N = 2$ with disclosure

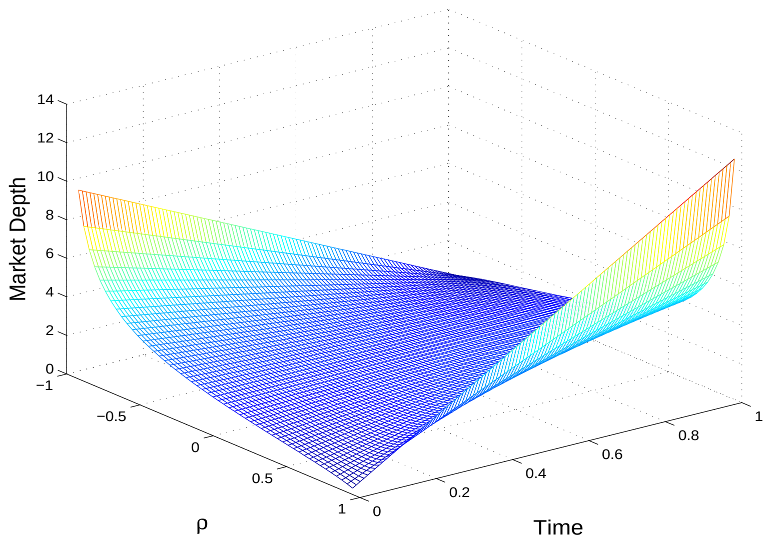


Figure 8: Market depth $1/\lambda$ as a function of ρ, t for $N = 2$ with disclosure.

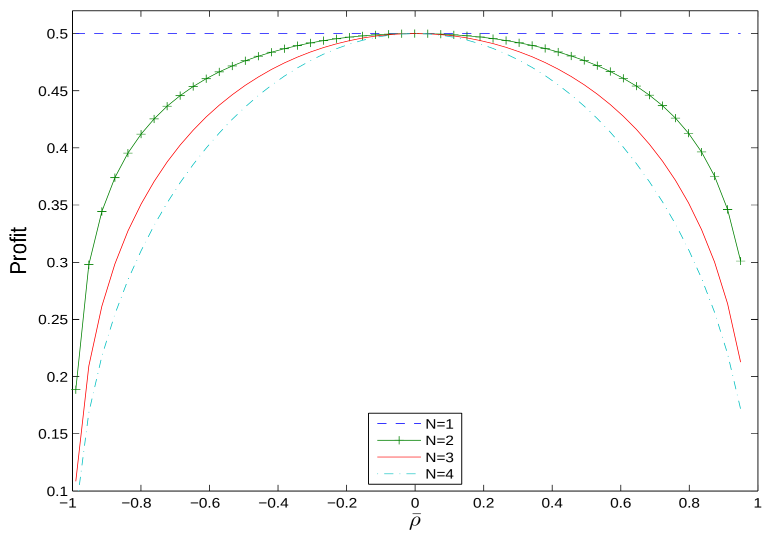


Figure 9: Informed Traders' expected profit $\pi(0)$ as a function of $\bar{\rho}$ for $N = 1, 2, 3, 4$ with disclosure.