

A Theory of Demand Driven Liquidity Commonality

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Abstract

Many empirical studies suggest that correlated demand (such as those from mutual funds) is important in driving liquidity commonality among stocks. However, theoretical studies on how demand-side factors cause and affect liquidity and return commonality are still very limited, if any. We propose a tractable equilibrium model with asymmetric information and imperfect competition among market makers to study the effect of correlated liquidity-based demand on the commonality in liquidity, in liquidity risks and in returns. We solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels all in closed-form. We find that price and liquidity correlations can be non-monotonic in the quality of stock payoff information, the stock return volatility and the volatility of liquidity demands. In the absence of asymmetric information, price correlations have the same sign as the demand correlation and liquidity correlation is always positive, even when demands are negatively correlated. In the presence of asymmetric information, however, price correlation can be negative even when demand correlation is positive and liquidity correlation can become negative. In addition, liquidity correlation may decrease with information asymmetry and information about one asset can affect the price and the liquidity of another asset even though they have independent payoffs. Moreover, as the quality of the private information about a stock improves, the uninformed's estimation precision of another stock's payoff worsens and thus both the liquidity risk and the price impact measures for one stock may change in opposite directions to those for the other stock.

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I. Introduction

It has been widely documented that liquidity strongly covaries across stocks even under normal economic conditions (e.g., Chordia, Roll and Subrahmanyam (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Eckbo and Norli (2002), Coughenour and Saar(2004), Korajczyk and Sadka (2008), and Corwin and Lipson (2011)).¹ The recent financial crisis further highlighted the importance of understanding the mechanism through which assets exhibit illiquidity commonality and how this commonality is impacted by various factors such as information asymmetry and trading correlation. Correlated illiquidity has important implications for asset pricing and financial market regulations because liquidity risk can become undiversifiable and systemic.²

However, the cause of correlated liquidity is still not well understood. Broadly speaking, liquidity commonality can arise from both the supply side and the demand side. The existing theories have focused on the liquidity supply side factors. For example, Kyle and Xiong (2001) show that trading losses in one market of financial intermediaries who supply liquidity in two risky asset markets can cause them to supply less liquidity in both markets, resulting in reduced market liquidity and increased correlation, due to the wealth effect. Brunneimeier and Pedersen (2009) illustrate that financial constraints of liquidity suppliers may lead to co-variations in liquidity because they can restrict liquidity suppliers in different securities simultaneously. Cespa and Foucault (2011) show that cross-security learning by dealers may cause liquidity spillovers and thereby co-movements in liquidity.³ However, although many empirical studies suggest that correlated trading by liquidity demanders such as mutual funds can be the main source of illiquidity commonality (e.g.,

¹For example, Chordia, Roll and Subrahmanyam (2000) find that quoted spreads, quoted depth, and effective spreads comove with market- and industry-wide liquidity. Coughenour and Saar(2004) show that stocks with the same specialist exhibit strong commonality in liquidity.

²Pastor and Stambaugh (2003), Acharya and Pedersen (2005) argue that returns of financial assets should depend on commonality in liquidity.

³Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010) provide evidence that the aggregate inventory of all NYSE specialists is an important determinant of aggregate market liquidity.

Koch, Ruenzi and Starks (2010), Corwin and Lipson (2011), Karolyi, Lee, and van Dijk (2011)), theories on how demand-side factors cause and affect liquidity commonality are still lacking.⁴

In this paper, we develop an equilibrium model to study the impact of correlated demands on the commonality of liquidity across different markets in the presence of asymmetric information, imperfect competition among market makers and risk aversion. We model the correlated liquidity demands for different assets as the need to trade these assets to hedge against certain non-traded assets.⁵ To focus on the correlated demands as the single source of liquidity commonality, we assume all stocks have independent payoffs and in contrast to Kyle and Xiong (2001), there is no wealth effect for any trader. We show that asset prices and their illiquidity (measured by the bid-ask spread) can become highly correlated. In the absence of asymmetric information, liquidity is always positively correlated whether hedging demands are positively or negatively correlated and price correlation always has the same sign as the liquidity demand correlation. In the presence of asymmetric information, however, price correlation can be negative even when hedging demand correlation is positive and liquidity correlation can also become negative. Information about one asset can affect the price and the liquidity of another asset even though they have independent payoffs and liquidity correlation may decrease with information asymmetry. In addition, we also study the price impact of an informed trader's trade on stock prices as another measure of illiquidity. We find that the price impact on one stock is dependent on the characteristics of the other stock and can be non-monotonic in the private information quality. Moreover, more precise information about one asset *reduces*

⁴For example, Corwin and Lipson (2011) suggest that commonality in returns and liquidity is driven by the correlated trading decisions of institutional traders. In studying cross-country liquidity commonality, Karolyi, Lee, and van Dijk (2011) conclude that the commonality is more reliably consistent with demand-side explanations than supply-side ones like the funding liquidity hypothesis.

⁵One example of correlated liquidity trading is portfolio rebalancing of a mutual fund upon a liquidity shock such as fund-flow shock and index reconstitution. To keep the correct stock composition, the fund needs to trade (buy or sell) multiple stocks simultaneously even though they are independent and there is no new information about their payoff. Because many funds own similar stocks, their aggregate rebalancing needs can cause commonality in liquidity and returns.

the estimation precision of the uninformed about another stock. Consistent with our model, Koch, Ruenzi and Starks (2010) show that the correlated trading of mutual funds has an important role in explaining commonality in liquidity and Karolyi, Lee, and van Dijk (2011) conclude that the commonality is more reliably consistent with demand-side explanations than supply-side ones. Although this model incorporates many important features such as asymmetric information, imperfect competition, and risk aversion and allows market makers to choose bid and ask prices and depths, it is still tractable. Indeed, we solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels all in closed-form. These explicit solutions make it possible to verify and clarify the economic intuitions behind our main results and to conduct reliable comparative statics with respect to a wide range of parameters.

Specifically, we consider an economy with three types of risk averse investors: informed investors, uninformed investors, and designated market makers who are also uninformed. There are one risk-free asset and two risky assets (“stocks”) with independent payoffs. Both informed and uninformed investors optimally choose how to trade these assets to maximize their expected CARA utility and all are endowed with some shares of the stocks but none of the risk-free asset. Type 1 market makers make market in Stock 1 and Type 2 market makers make market in Stock 2. Informed investors can observe a private signal about the payoff of each stock before the terminal date and thus they have trading demand motivated by the private information. They are also subject to a liquidity shock modeled as a random endowment of a nontradable asset (e.g., highly illiquid asset) whose payoff is correlated with both stocks. Accordingly, informed investors also have trading demand motivated by the needs for hedging. Because both stocks, although independent of each other, are correlated with the non-traded asset, the hedging demands across the two markets are correlated. It is this correlated hedging demands that drive the correlated stock returns and liquidity.⁶ The population of the informed and the uninformed is large

⁶Alternatively, one can directly model the extra trading needs from a liquidity shock.

and thus neither the informed nor the uninformed trade strategically. Informed and uninformed investors trade through market makers. Following Kyle (1989), we assume that informed and uninformed investors submit their demand schedules simultaneously before trading. Different from Kyle (1989), however, the demand schedules are dependent on bid and ask prices and are submitted not to an auctioneer but to the designated market makers who then determine how to trade at the bid and ask. This assumption is consistent with a market microstructure where designated market makers observe the order flow before determining bid and ask prices and bid and ask depths (e.g., NYSE and Euronext, the Stockholm Stock Exchange). We allow the competition among market makers to be imperfect. In contrast to the standard literature that implicitly assumes Bertrand competition among market makers, we model the competition among market makers as a Cournot competition: they choose simultaneously how much to buy at the bid and how much to sell at the ask, taking into account the price impact of their trades. The equilibrium bid and ask prices are then determined by the market clearing conditions at the bid and at the ask, i.e., the total amount market makers buy at the bid is equal to the total amount other investors sell, and the total amount market makers sell at the ask is equal to the total amount other investors buy. In equilibrium, both the stock markets and the risk-free asset market clear.

As in Liu and Wang (2011), investors buy (sell) a stock if and only if the market price is lower (higher) than their reservation price, which increases with their conditional expectation of the stock payoff and decreases with the conditional variance. Relative to the informed, the uninformed always have greater conditional variances and may underestimate or overestimate the expected stock payoff. The bid ask spread is equal to the absolute value of the reservation price difference between the informed and the uninformed, divided by the number of market makers plus one. In addition, bid-ask spread can be lower with asymmetric information.

We find that even though asset payoffs are independent, their prices and liquidity

can become highly correlated when hedging demands for the assets are correlated. With perfect competition and information asymmetry, the price correlation is non-monotonic in the quality of the information about stock payoffs, the volatility of stock payoff, the risk aversion and the volatility of the liquidity shock. In addition, the price correlation can become negative even when the hedging demands are positively correlated. Intuitively, in the presence of asymmetric information, the uninformed investors use the equilibrium prices in both markets to infer about the conditional distribution of a stock's payoff. When the equilibrium price of a stock increases, it may be due to good information about the stock or due to a large liquidity shock to the informed. Although the information about the stock payoff is irrelevant for the other stock because their payoffs are independent, the information about the liquidity shock embedded in the equilibrium price of one stock is important for the uninformed to better estimate the payoff of the other stock. This is why information about one asset can affect the price and the liquidity of another asset even though they have independent payoffs. As the equilibrium price of one stock increases, it becomes more likely that the informed have a larger liquidity shock. If the hedging demands for the two stocks are positively correlated, then it is also more likely that the informed's trades in the second stock are from a larger hedging demand rather than from good information about the second stock and therefore the conditional expected payoff of the second stock estimated by the uninformed decreases, which drives down the equilibrium price of the second stock. Thus, when this information filtering effect dominates the effect of the positively correlated hedging demands, the equilibrium prices become negatively correlated. All the non-monotonicity mentioned above comes from affecting this balance between the information filtering effect and the correlated hedging effect. In addition, the informed can trade in the opposite directions in the two stocks even when their hedging demands are positively correlated due to the information filtering effect. Furthermore, we show that as the precision of the private information about a stock improves, the uninformed's estimation precision (measured by the inverse

of the uninformed's conditional variance) of another stock's payoff worsens. Intuitively, as the precision of the private information about one stock improves, the stock price becomes less informative about the hedging demand because the informed put a greater weight on the private information (rather than hedging demands) in their trades. Therefore, the stock price is less useful for the uninformed to separate the effect of hedging demand and the effect of private information about the other stock on the other stock's price and thus their estimation of the other stock's payoff becomes less precise.

In the absence of asymmetric information, illiquidity (measured by stock spreads) is always positively correlated whether hedging demands are positively or negatively correlated. This is because the reservation price difference between the informed and the uninformed is a constant multiple of the hedging demand and spreads are proportional to the *absolute value* of the reservation price difference. In the presence of asymmetric information, however, liquidity can become negatively correlated if hedging demands are negatively correlated, because of the impact of information filtering on the reservation prices. For the main intuition, suppose relative to the informed, the uninformed underestimate the expected payoff of stock 1 and therefore have a lower reservation price. In addition, although the uninformed overestimate the expected payoff of stock 2, the higher estimation risk faced by the uninformed makes the uninformed's reservation price for stock 2 still lower than that of the informed. Thus the informed buy both stock 1 and stock 2. When a better signal about stock 1 is observed by the informed, the uninformed underestimate even more the expected payoff of asset 1 relative to the informed due to information asymmetry, which implies that reservation price difference between the informed and the uninformed increases and so does the spread of asset 1. With a better signal for stock 1, the uninformed attribute less of the informed's trade in stock 1 to hedging demand and therefore attribute more of the informed's trade in stock 2 to the information about stock 2's payoff. Thus, the uninformed overestimate more the expected payoff of stock 2 which in turn offsets more of the estimation risk premium and drives

down the spread of stock 2. Therefore, the spreads of the two stocks move in the opposite directions. As the quality of the private information about a stock's payoff increases or the liquidity shock volatility increases, the spread correlation can change from negative to positive. On the other hand, as the payoff of a stock becomes more volatile, the spreads can become more negatively correlated or less positively correlated. These patterns arise because the reservation price difference variations are affected by these changes in the parameter values. In addition, the magnitude of the correlation between the spreads is higher with symmetric information than that with asymmetric information and the magnitude of the correlation between the spreads can decrease with information asymmetry for either stock 1 or stock 2. We also find that the liquidity risk of one stock is affected by the quality of the private information about the other stock. When the correlation between hedging demands for the two stocks is negative, the liquidity risk of stock 1 may decrease when the liquidity risk of stock 2 increases.

We also show that the price impact of an informed investor's trade on a stock is nonmonotonic in the private signal quality about this stock. The non-monotonicity is driven by two effects of a private signal: an information quality effect and an information asymmetry effect. As the signal becomes more precise, both the information quality and the information asymmetry increase. The price impact for a stock decreases in the information quality and increases in the information asymmetry. When the private signal is less noisy, the information quality effect dominates and thus the price impact for a stock decreases in the precision of the private signal about this stock. When the private signal is noisy, the reverse is true. Interestingly, the price impact of stock 1 decreases in the quality of the private signal about stock 2's payoff. This is because uninformed investors can estimate the hedging demand better from the equilibrium price of stock 2 when informed investors' private signal about stock 2's payoff is less precise and thus uninformed investors' uncertainty about stock 1's payoff decreases. This implies that the price impact of stock 1 may increase when the price impact of stock 2 decreases.

The remainder of the paper proceeds as follows. In Section II we present the model. In Section III we solve the case with symmetric information, and in Section IV we derive the equilibrium under asymmetric information and provide some comparative statics on asset prices and illiquidity. We conclude in Section V. All proofs are in the Appendix.

II. The model

We consider a one period setting with trading dates 0 and 1. There are N investors: N_I informed investors (I), N_U uninformed investors (U), and $N_M \equiv N - (N_I + N_U)$ market makers who are also uninformed. Both I and U investors can trade one risk-free asset and two risky assets on date 0 and date 1 to maximize their expected constant absolute risk aversion (CARA) utility from the terminal wealth on date 1. Among the N_M market makers, N_{Mk} ($k = 1, 2$) of them (M_k) only trade and make the market of risky asset k .⁷ There is a zero net supply for the risk-free asset, which also serves as the numeraire and thus the risk-free interest rate is normalized to 0. The total supply of the asset k is $N_k \bar{\theta}_k$ ($N_k = N_I + N_U + N_{Mk}$) shares and the date 1 payoff of each share of asset k is $\tilde{V}_k \sim \mathbf{N}(\bar{V}_k, \sigma_{V_k}^2)$, where \bar{V}_k is a constant, $\sigma_{V_k} > 0$, and $\mathbf{N}(\cdot)$ denotes the normal distribution. The payoffs of asset 1 and asset 2 are independent.⁸ Each I or U investor is endowed with $\bar{\theta}_1$ shares of the asset 1 and $\bar{\theta}_2$ shares of the asset 2. Each market maker of asset k is only endowed with $\bar{\theta}_k$ shares of the asset k . No one is endowed with the risk-free asset.

On date 0, informed investors observe a private signal $\hat{s}_k = \tilde{V}_k - \bar{V}_k + \tilde{\varepsilon}_k$ about the payoff \tilde{V}_k , where $\tilde{\varepsilon}_k$ is independently normally distributed with mean zero and variance

⁷Our results do not change if we assume all market makers trade and make the market of both assets. We divide the market makers into two groups to rule out commonality in liquidity arises simply because the same market makers are making the market of both assets.

⁸We assume the payoffs of the two assets are independent to rule out the commonality in illiquidity arises simply because their payoffs are correlated. Our main results do not change if we assume the payoffs of the two assets are correlated.

$\sigma_{\varepsilon_k}^2$. In addition to the stock, a type I investor is also subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_I \sim \mathbf{N}(0, \sigma_X^2)$ units of a non-traded risky asset on date 0, with \hat{X}_I realized and only directly known to the investor on date 0.⁹ The non-traded asset has a per-unit payoff of $\tilde{N} \sim \mathbf{N}(0, \sigma_N^2)$ that has a covariance of σ_{kN} with \tilde{V}_k and is realized and becomes public on date 1. The correlations between the nontraded asset and asset k result in a liquidity demand for hedging the nontraded asset payoff.

All trades must go through market makers whose market making cost is assumed to be 0. Specifically, given market bid price B_k and ask price A_k , I and U investors sell to market makers at the bid or buy from them at the ask or do not trade at all. We assume that both N_I and N_U are relatively large such that all I and U investors are price takers and there are no strategic interactions among them or with market makers.

For each $i \in \{I, U, M_1, M_2\}$, investors of type i are ex ante identical. Accordingly, we restrict our analysis to symmetric equilibria where all type i investors adopt the same trading strategy. Let \mathcal{I}_i represent a type i investor's information set on date 0 for $i \in \{I, U, M_1, M_2\}$. Given B_k and A_k , for $i \in \{I, U\}$, a type i investor's problem is

$$\max_{\theta_{i1}, \theta_{i2}} E[-e^{-\delta \tilde{W}_i} | \mathcal{I}_i], \quad (1)$$

subject to the budget constraint

$$\tilde{W}_i = \sum_{k=1,2} \left(\theta_{ik}^- B_k - \theta_{ik}^+ A_k + (\bar{\theta}_k + \theta_{ik}) \tilde{V}_k \right) + \hat{X}_i \tilde{N}, \quad (2)$$

where $\hat{X}_U = 0$, $\delta > 0$ is the absolute risk-aversion parameter, θ_{ik} is the signed order size of investor i , $x^+ \equiv \max(0, x)$, and $x^- \equiv \max(0, -x)$.

Since all trades in asset k must go through market makers M_k , market makers can have

⁹Informed investors in our model can be interpreted as a representative broker who combines information-based trades from traders who have private information and liquidity trades from liquidity traders such as mutual funds.

market powers especially when the number of market makers M_k is small. To model the oligopolistic competition among the market makers M_j , we use the notion of the Cournot competition. Specifically, we assume that market makers M_j simultaneously choose the optimal number of shares to sell at ask (A_k) and to buy at bid (B_k), taking into account the price impact of their trades.

For $k = 1, 2$, let $\alpha_k = (\alpha_{1k}, \alpha_{2k}, \dots, \alpha_{N_{M_k k}})^\top$ and $\beta_k = (\beta_{1k}, \beta_{2k}, \dots, \beta_{N_{M_k k}})^\top$ be the vector of the number of shares market makers M_k sell at ask (i.e., ask depth) and buy at bid (i.e., bid depth) respectively. Given the demand schedules of the informed and the uninformed ($\theta_I^*(A_k, B_k)$ and $\theta_U^*(A_k, B_k)$), the bid price $B_k(\beta_k)$ (i.e., the inverse supply function) and the ask price $A_k(\alpha_k)$ (i.e., the inverse demand function) can be determined by the following stock market clearing conditions at the bid and ask prices.

$$\sum_{j=1}^{N_{M_k}} \alpha_{jk} = \sum_{i=I, U} N_i \theta_{ik}^*(A_k, B_k)^+, \quad \sum_{j=1}^{N_{M_k}} \beta_{jk} = \sum_{i=I, U} N_i \theta_{ik}^*(A_k, B_k)^-, \quad (3)$$

where the left-hand sides represent the total sales and purchases by market makers M_k respectively and the right-hand sides represent the total purchases and sales by other investors respectively.

Then for $j = 1, 2, \dots, N_{M_k}$ and $k = 1, 2$, the market maker M_{jk} 's problem is

$$\max_{\alpha_{jk} \geq 0, \beta_{jk} \geq 0} E \left[-e^{-\delta \tilde{W}_{M_{jk}}} | \mathcal{I}_{M_k} \right], \quad (4)$$

subject to the budget constraint

$$\tilde{W}_{M_{jk}} = \alpha_{jk} A_k(\alpha_k) - \beta_{jk} B_k(\beta_k) + (\bar{\theta}_k + \beta_{jk} - \alpha_{jk}) \tilde{V}_k. \quad (5)$$

Note that different from other investors, a market maker takes into account the price impact of her own trades, i.e., recognizing both A_k and B_k will be affected by her trades.

This leads to our definition of the Nash equilibrium of the Cournot competition.

Definition 1 *An equilibrium $(\theta_{I_k}^*(A_k, B_k), \theta_{U_k}^*(A_k, B_k), A_k^*, B_k^*, \alpha_k^*, \beta_k^*)$ is such that*

1. *given any A_k and B_k , $\theta_{i_k}^*(A_k, B_k)$ solves a type i investor's Problem (1) for $i \in \{I, U\}$;*
2. *given $\theta_{I_k}^*(A_k, B_k)$ and $\theta_{U_k}^*(A_k, B_k)$, $\alpha_{j_k}^*$ and $\beta_{j_k}^*$ solve potential market maker M_{j_k} 's Problem (4), for $j = 1, 2, \dots, N_{Mk}$;*
3. *$A_k^* := A_k(\alpha_k^*)$ and $B_k^* := B_k(\beta_k^*)$ clear both the stock and the risk-free asset markets.*

III. The equilibrium with symmetric information

For comparison, we first consider two symmetric information cases for each asset k : the no-information case, where no one observes the private signal \hat{s}_k , and the full-information case, where all agents observe the private signal \hat{s}_k . In both cases, other investors can infer a type I investor's liquidity shock from the equilibrium stock price. The equilibrium illiquidity arises from the market power of market makers. Since the no-information case is a special case of the full-information case when $\sigma_{\varepsilon_k}^2 \rightarrow \infty$, we only need to solve for the equilibrium in the full-information case. The equilibrium prices for the no-information case can be obtained by simply setting $\sigma_{\varepsilon_k}^2$ to ∞ . With two risky assets, we have four symmetric cases because we have no-information and full-information cases for each asset.

A. Perfect competition with symmetric information

With perfect competition, equilibrium bid and ask prices must be the same and thus all investors trade at the same price. Let P_{ks}^* denote the equilibrium price for asset k . For the full-information case, investors' information sets are such that $\mathcal{I}_I = \mathcal{I}_U = \mathcal{I}_{M_k} = \{\hat{X}_I, \hat{s}_1, \hat{s}_2, P_{1s}^*, P_{2s}^*\}$. Therefore, a type i ($i = I, U$) investor's problem is equivalent to

$$\max_{\theta_{i1}, \theta_{i2}} -e^{\sum_{k=1,2} \delta \theta_{ik} P_{ks}} E[e^{-\sum_{k=1,2} \delta(\theta_{ik} + \bar{\theta}_k) \tilde{V}_k - \delta \hat{X}_i \tilde{N}} | \mathcal{I}_i], \quad (6)$$

and a type M_k investor's problem is equivalent to

$$\max_{\theta_{ik}} -e^{\delta \theta_{ik} P_{ks}} E[e^{-\delta(\theta_{ik} + \bar{\theta}_k) \tilde{V}_k - \delta \hat{X}_i \tilde{N}} | \mathcal{I}_{M_k}], \quad (7)$$

where $\hat{X}_U = \hat{X}_{M_k} = 0$. Let $\hat{h}_{ik} = -\frac{\sigma_{kN}}{\text{Var}[\hat{V}_k | \hat{s}_k]} \hat{X}_i$ be a type i investor's hedging demand for asset k and $\hat{H}_{ik} = \delta \text{Var}[\hat{V}_k | \hat{s}_k] \hat{h}_{ik}$ be the premium of asset k that a type i investor is willing to pay for hedging. Then the optimal demand schedule is

$$\theta_{ik}^*(P_{ks}) = \frac{E[\tilde{V}_k | \hat{s}_k] + \hat{H}_{ik} - P_{ks}}{\delta \text{Var}[\hat{V}_k | \hat{s}_k]} - \bar{\theta}_k, \quad i = I, U, M_k, \quad k = 1, 2. \quad (8)$$

Equation (8) then implies that the reservation price of asset k for a type- i investor is

$$P_{ik}^R \equiv E[\tilde{V}_k | \hat{s}_k] + \hat{H}_{ik} - \delta \text{Var}[\hat{V}_k | \hat{s}_k] \bar{\theta}_k, \quad i = I, U, M_k, \quad k = 1, 2. \quad (9)$$

Equation (9) implies that the reservation price of asset k for a type- i investor increases with expected payoff of asset k and the premium of asset k for hedging and decreases with asset k 's payoff volatility. Given the normality and independence of \tilde{V}_k and $\tilde{\varepsilon}_k$, we have

$$E[\tilde{V}_k | \hat{s}_k] = \bar{V}_k + \rho_{I_k} \hat{s}_k, \quad \text{Var}[\tilde{V}_k | \hat{s}_k] = \rho_{I_k} \sigma_{\varepsilon_k}^2, \quad (10)$$

where $\rho_{I_k} = \frac{\sigma_{\tilde{V}_k}^2}{\sigma_{\tilde{V}_k}^2 + \sigma_{\varepsilon_k}^2}$.

Remark 1 As $\sigma_{\varepsilon_k}^2 \rightarrow \infty$, we have $\text{Var}[\tilde{V}_k | \hat{s}_k]$ converge to $\sigma_{\tilde{V}_k}^2$ and $E[\tilde{V}_k | \hat{s}_k]$ converge to \bar{V}_k , and therefore the equilibrium with full-information case converges to the equilibrium with no-information case.

Let ΔRP_{ks} denote the difference in the reservation prices of the I and U investors, i.e.,

$$\Delta RP_{ks} \equiv P_{Ik}^R - P_{Uk}^R = \hat{H}_{Ik}. \quad (11)$$

The following theorem provides the equilibrium prices and equilibrium stock demand.

Theorem 1 *With symmetric information and perfect competition,*

1. *for $k = 1, 2$, the equilibrium price of the asset k is*

$$P_{ks}^* = \frac{N_I}{N_k} P_{Ik}^R + \frac{N_U}{N_k} P_{Uk}^R + \frac{N_{M_k}}{N_k} P_{Mk}^R = P_{Uk}^R + \frac{N_I}{N_k} \Delta RP_{ks}; \text{ and} \quad (12)$$

2. *the equilibrium stock quantities demanded are*

$$\theta_{Ik}^* := \theta_{Ik}^*(P_{ks}^*) = \left(1 - \frac{N_I}{N_k}\right) \frac{\Delta RP_{ks}}{\delta Var[\hat{V}_k | \hat{S}_k]}, \quad \theta_{ik}^* := \theta_{ik}^*(P_{ks}^*) = -\frac{N_I}{N_k} \frac{\Delta RP_{ks}}{\delta Var[\hat{V}_k | \hat{S}_k]}, \quad i = U, M_k. \quad (13)$$

Theorem 1 shows that the equilibrium price for asset k is the population weighted average of the reservation prices of all the investors. The equilibrium prices can also be rewritten as the reservation prices of the uninformed investor plus a fraction of the difference in the reservation prices ΔRP_{ks} . Part 2 of Theorem 1 implies that I investors buy asset k and U investors sell asset k if and only if I investors have a higher reservation price of asset k than U investors. Later we show that this result carries through the cases with imperfect competition and with asymmetric information.

The following Proposition provides correlation between the equilibrium prices and the correlation between stock demand for both stocks.

Proposition 1 *With symmetric information and perfect competition,*

1.

$$Corr(\theta_{i1}^*, \theta_{i2}^*) = 1 \times Sign(\sigma_{1N} \sigma_{2N}), \quad i = I, U, M_k;$$

2.

$$Corr(P_{1s}^*, P_{2s}^*) = \frac{N_I^2 \delta^2 \sigma_X^2 \prod_{k=1,2} \frac{\sigma_{kN}}{N_k}}{\prod_{k=1,2} \sqrt{\frac{\sigma_{V_k}^4}{\sigma_{V_k}^2 + \sigma_{\varepsilon_k}^2} + \frac{N_I^2}{N_k^2} \delta^2 \sigma_{kN}^2 \sigma_X^2}};$$

$$Corr(\tilde{V}_1 - P_{1s}^*, \tilde{V}_2 - P_{2s}^*) = \frac{N_I^2 \delta^2 \sigma_X^2 \prod_{k=1,2} \frac{\sigma_{kN}}{N_k}}{\prod_{k=1,2} \sqrt{\frac{\sigma_{V_k}^2 \sigma_{\varepsilon_k}^2}{\sigma_{V_k}^2 + \sigma_{\varepsilon_k}^2} + \frac{N_I^2}{N_k^2} \delta^2 \sigma_{kN}^2 \sigma_X^2}}.$$

Part 1 of Proposition 1 implies that if the correlation between asset 1 and the non-traded asset has the same (opposite) sign as the correlation between asset 2 and the non-traded asset, then for any investor, the trading directions in the two assets are the same (opposite). Proposition 1 also implies that it is investors' correlated hedging trades in two stocks that lead to comovements in stocks' equilibrium prices and comovements in stocks' returns. For example, if one asset is not correlated with the non-traded asset, then the correlations between the two stocks reduce to zero. In addition, the magnitude of the correlation between the equilibrium stock prices, $|Corr(P_{1s}^*, P_{2s}^*)|$, increases in δ , σ_X , $|\sigma_{kN}|$ (due to the increase of the covariances) and σ_{ε_k} , and decreases in σ_{V_k} . The magnitude of the correlation between the equilibrium stock returns on date 1, $|Corr(\tilde{V}_1 - P_{1s}^*, \tilde{V}_2 - P_{2s}^*)|$, increases in δ , σ_X , and $|\sigma_{kN}|$, and decreases in σ_{V_k} and σ_{ε_k} . This implies that the magnitude of the correlation between the stock prices on date 0 and the correlation between the stock returns on date 1 increases with the volatility of investor I 's hedging premium.

B. Symmetric information with imperfect competition

When the competition among market makers is imperfect, given A_k, B_k ($k = 1, 2$), and $i \in \{I, U\}$, it can be shown the optimal demand schedule for a type i investor is

$$\theta_{i_k}^*(A_k, B_k) = \begin{cases} \frac{P_{i_k}^R - A_k}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]} & A_k < P_{i_k}^R, \\ 0 & B_k \leq P_{i_k}^R \leq A_k, \\ -\frac{B_k - P_{i_k}^R}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]} & B_k > P_{i_k}^R. \end{cases} \quad (14)$$

As in the perfect competition case, we conjecture that I investors buy stock k and U investors sell stock k if and only if I investors have a higher reservation price of stock k than U investors. The following theorem shows that this conjecture is indeed correct.

Theorem 2 *With symmetric information and imperfect competition,*

1. *the equilibrium ask and bid prices are*

$$A_{ks}^* := A(\beta_{ks}^*) = P_{Uk}^R + \frac{N_{M_k} N_I}{(N_k + 1)(N_{M_k} + 1)} \Delta RP_{ks} + \frac{\Delta RP_{ks}^+}{N_{M_k} + 1}, \quad (15)$$

$$B_{ks}^* := B(\alpha_{ks}^*) = P_{Uk}^R + \frac{N_{M_k} N_I}{(N_k + 1)(N_{M_k} + 1)} \Delta RP_{ks} - \frac{\Delta RP_{ks}^-}{N_{M_k} + 1}, \quad (16)$$

and the bid-ask spread of stock k is

$$A_{ks}^* - B_{ks}^* = \frac{|\Delta RP_{ks}|}{N_{M_k} + 1} = \frac{|\hat{H}_{Ik}|}{N_{M_k} + 1};$$

2. *the equilibrium stock quantities demanded for stock k are*

$$\theta_{Ik}^* = \frac{N_{M_k}(N_U + N_{M_k} + 1)}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]} \right), \quad \theta_{Uk}^* = -\frac{N_I N_{M_k}}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]} \right), \quad (17)$$

$$\theta_{Mk}^* = -\frac{N_I}{N_k + 1} \left(\frac{\Delta RP_{ks}}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]} \right); \text{ and}$$

the equilibrium quote depths are

$$\alpha_{ks}^* = \frac{N_I(N_{M_k} + N_U + 1)}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta Var[\tilde{V}_k|\hat{s}_k]} \right)^- + \frac{N_I N_U}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta Var[\tilde{V}_k|\hat{s}_k]} \right)^+,$$

$$\beta_{ks}^* = \frac{N_I(N_{M_k} + N_U + 1)}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta Var[\tilde{V}_k|\hat{s}_k]} \right)^+ + \frac{N_I N_U}{(N_k + 1)(N_{M_k} + 1)} \left(\frac{\Delta RP_{ks}}{\delta Var[\tilde{V}_k|\hat{s}_k]} \right)^-,$$

which implies that the equilibrium trading volume in stock k is

$$N_{M_k}(\alpha_{ks}^* + \beta_{ks}^*) = \frac{N_I N_{M_k} (N_{M_k} + 2N_U + 1)}{(N_{M_k} + 1)(N_k + 1)} \left(\frac{|\Delta RP_{ks}|}{\delta Var[\tilde{V}_k|\hat{s}_k]} \right). \quad (18)$$

Theorem 2 implies that the bid and ask spread of stock k is equal to the absolute value of stock k 's reservation price difference $|\Delta RP_{ks}|$, divided by the number of market makers in stock k plus one. This implies that the spread increases with $|\Delta RP_{ks}|$ and decreases in the competition among market makers of stock k . However, because of the absence of the wealth effect due to the CARA preferences, competition among market makers in one asset market does not affect the prices or the spread of the other asset. Moreover, Theorem 2 shows that the trading volume in stock k also increases with $|\Delta RP_{ks}|$.

The following Proposition provides the correlation between the spreads and the correlation between the trading volume in both stocks.

Proposition 2 *With symmetric information and imperfect competition,*

1. $Corr(A_{1s}^* - B_{1s}^*, A_{2s}^* - B_{2s}^*) = Corr(N_{M_1}(\alpha_{1s}^* + \beta_{1s}^*), N_{M_2}(\alpha_{2s}^* + \beta_{2s}^*)) = 1$;
2. For $k = 1, 2$, $Var(A_{ks}^* - B_{ks}^*) = \frac{1-2/\pi}{(N_{M_k}+1)^2} \delta^2 \sigma_{kN}^2 \sigma_X^2$.

Proposition 2 implies that, in the absence of asymmetric information, investors' correlated trading in two stocks leads to perfectly positively correlated bid-ask spreads of stock 1 and stock 2. This implies that the correlation between the liquidity is always positive under symmetric information, *i.e.*, the liquidity of stock 1 improves when the liquidity

of stock 2 improves and vice versa. We show later that these results no longer hold in the presence of asymmetric information. In addition, Part 3 of Proposition 2 implies that investors' correlated trading in two stocks also leads to co-movements in liquidity fluctuation (measured by the volatility of the spreads) of the two stocks. For example, the volatilities of the bid-ask spreads of both stocks increase in the uncertainty of the liquidity shock, σ_X^2 .

Another measure of illiquidity is the price impact of the informed investors' trading in equilibrium, *i.e.*, Kyle's lambda. As in Vayanos and Wang (2011), we measure the price impact using the regression coefficient of the equilibrium price on the informed investors' trade, *i.e.*,

$$\lambda_{pck} = \frac{Cov(P_{ks}^*, N_I \theta_{Ik}^*)}{Var(N_I \theta_{Ik}^*)}, \quad \lambda_{ick} = \frac{Cov(\frac{A_{ks}^* + B_{ks}^*}{2}, N_I \theta_{Ik}^*)}{Var(N_I \theta_{Ik}^*)}. \quad (19)$$

where λ_{pck} (λ_{ick}) is the price impact for perfect (imperfect) competition case. We have

Proposition 3 *The price impact with perfect (imperfect) competition is*

$$\lambda_{pck} = \frac{\delta \rho_{Ik} \sigma_{\varepsilon_k}^2}{N_U + N_{Mk}}, \quad \lambda_{ick} = \frac{\delta \rho_{Ik} \sigma_{\varepsilon_k}^2 (N_{Mk} N_I + \frac{1}{2}(N_k + 1))}{N_I N_{Mk} (N_U + N_{Mk} + 1)}. \quad (20)$$

Proposition 3 implies that, in the absence of asymmetric information, the price impact of stock k increases in $\delta, \sigma_{\varepsilon_k}$ and σ_{V_k} . In addition, the price impact of stock k does not depend on the information quality and payoff volatility of the *other* stock. We will show later the price impact of stock k changes if the information quality or the payoff volatility of the other stock changes in the presence of asymmetric information.

IV. The equilibrium with asymmetric information

We now assume that only informed investors observe a private signal \hat{s}_1 about asset 1's payoff \tilde{V}_1 , and observe a private signal \hat{s}_2 about asset 2's payoff \tilde{V}_2 .¹⁰ Therefore, informed investors' trades can be motivated by both liquidity shock and private information. If the informed can only observe a private signal about stock 1 but no signal about stock 2, then the uninformed can figure out the liquidity shock from the equilibrium price of stock 2 and then figure out the informed' private signal about stock 1 from the equilibrium price of stock 1. Therefore, the asymmetric information case for stock 1 is reduced to the symmetric information case. As before, we first consider the perfect competition case.

A. Perfect competition with asymmetric information

Let P_{ka} denote the stock price of asset k with asymmetric information in a competitive market. The optimal demand schedule of an informed investor is then

$$\theta_{Ik}^*(P_{ka}) = \frac{\bar{V}_k + \rho_{Ik}\hat{S}_k - P_{ka}}{\delta \text{Var}[\tilde{V}_k|\hat{s}_k]} - \bar{\theta}_k, \quad (21)$$

where $\hat{S}_k = \hat{s}_k + \frac{1}{\rho_{Ik}}\hat{H}_{Ik}$. (21) implies that the reservation price of stock k for I investors is

$$P_{Ika}^R = \bar{V}_k + \rho_{Ik}(\hat{S}_k - \delta\sigma_{\varepsilon_k}^2\bar{\theta}_k). \quad (22)$$

Since the informed investor's demand of asset k is a monotonically increasing function of \hat{S}_k , his order reveals the value of \hat{S}_k to market makers M_k . Thus we conjecture that

¹⁰The assumption that the investors with liquidity shocks observe private signals about both assets is a simple way to keep the private information from fully revealing. For example, if those without liquidity shocks observe a private signal about one of the two assets, then the asymmetric information case for this asset is reduced to symmetric information case. For the more general case where both investors are endowed with some non-traded risky asset and some investors observe a signal about the payoff of asset 1 while others observe a signal about the payoff of asset 2, there are 63 subcases. We can still obtain closed-form solutions. However, the general case does not add new economics and therefore we focus on the simplest case where informed investors observe signals about both assets to get clear intuitions.

the equilibrium prices P_{ka}^* depends on \hat{S}_k . Since the uninformed investors can then infer the value of \hat{S}_k from the market price P_{ka}^* , the information sets for the informed, the uninformed investors and market makers are $\mathcal{I}_I = \{\hat{S}_1, \hat{S}_2, \hat{X}_I, P_{1a}^*, P_{2a}^*\}$ and $\mathcal{I}_U = \mathcal{I}_M = \{P_{1a}^*, P_{2a}^*\} = \{\hat{S}_1, \hat{S}_2\}$ respectively. Let $\sigma_{Hk} \equiv \delta\sigma_{kN}\sigma_X$ be non-zero, with σ_{Hk}^2 being the variance of the premium of stock k for hedging, \hat{H}_{Ik} . Then the conditional expectation and variance of \tilde{V}_k are

$$E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] = \bar{V}_k + \frac{\sigma_{V_k}^2 \left(\sigma_{\hat{S}_{-k}}^2 \hat{S}_k - Cov(\hat{S}_1, \hat{S}_2) \hat{S}_{-k} \right)}{\sigma_{\hat{S}_1}^2 \sigma_{\hat{S}_2}^2 - Cov^2(\hat{S}_1, \hat{S}_2)}, \quad (23)$$

$$Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] = \sigma_{V_k}^2 \left(1 - \frac{\sigma_{\hat{S}_{-k}}^2 \sigma_{V_k}^2}{\sigma_{\hat{S}_1}^2 \sigma_{\hat{S}_2}^2 - Cov^2(\hat{S}_1, \hat{S}_2)} \right), \quad (24)$$

where $\sigma_{\hat{S}_k}^2 = \sigma_{V_k}^2 + \sigma_{\varepsilon_k}^2 + \frac{\sigma_{Hk}^2}{\rho_{I_k}^2}$ and $Cov(\hat{S}_1, \hat{S}_2) = \frac{\delta^2 \sigma_X^2 \sigma_{1N} \sigma_{2N}}{\rho_{I_1} \rho_{I_2}}$. It can be shown that

$$Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] = \frac{\sigma_{V_k}^2 \sigma_{\varepsilon_k}^2}{\sigma_{V_k}^2 + \sigma_{\varepsilon_k}^2} + \frac{\sigma_{Hk}^2}{1 + \sum_{j=1,2} \frac{\sigma_{Hj}^2 (\sigma_{V_j}^2 + \sigma_{\varepsilon_j}^2)}{\sigma_{V_j}^4}}. \quad (25)$$

Therefore, $Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]$ increases in σ_{ε_k} but decreases in $\sigma_{\varepsilon, -k}$ ($-k$ indicates the other stock). Intuitively, as $\sigma_{\varepsilon, -k}$ increases, the trades from the informed in the other asset convey more information about the hedging demand (i.e., the uninformed attribute more of \hat{S}_{-k} to \hat{X}_I), which helps the uninformed increase the precision of the estimation of the asset k payoff. Therefore, $\tau_{-k} = \frac{1}{\sigma_{\varepsilon, -k}^2}$ is a measure of information asymmetry of stock k .

Note that in the presence of asymmetric information, the conditional distribution of the stock payoff now depends on the characteristics of the other market, including the information about the other stock, although these two stocks are independent. For example, suppose $Cov(\hat{S}_1, \hat{S}_2) > 0$, then the conditional expected payoff of stock 1 decreases with the signal \hat{S}_2 because the higher \hat{S}_2 is, the more likely \hat{X}_I is high and thus the contribution

of \tilde{V}_1 in driving \hat{S}_1 is more likely to be small. In addition, $\text{Var}[\tilde{V}_k|\hat{S}_1, \hat{S}_2] > \text{Var}[\tilde{V}_k|\hat{S}_k]$. Let

$$\nu_k = \frac{\text{Var}[\tilde{V}_k|\hat{S}_1, \hat{S}_2]}{\text{Var}[\tilde{V}_k|\hat{S}_k]} > 1 \quad (26)$$

be the ratio of the conditional variance of stock k payoff of the uninformed to that of the informed. Then, the optimal demand schedule for stock k of a U -investor is:

$$\theta_{Uk}^*(P_{ka}) = \frac{E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - P_{ka}}{\delta \text{Var}[\tilde{V}_k|\hat{S}_1, \hat{S}_2]} - \bar{\theta}_k. \quad (27)$$

As in the symmetric information case with perfect competition, market makers solve exactly the same problem as the uninformed and have the same reservation price as the uninformed. Equation (27) then implies that the reservation price of stock k for a U investor and an M_k investor is now

$$P_{Uka}^R = P_{Mka}^R = E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - \delta \text{Var}[\tilde{V}_k|\hat{S}_1, \hat{S}_2] \bar{\theta}_k. \quad (28)$$

Thus the difference in the reservation prices for stock k is:

$$\Delta RP_{ka} = P_{Ika}^R - P_{Uka}^R = \hat{H}_{Ik} + (E[\tilde{V}_k|\hat{S}_k] - E[\tilde{V}_k|\hat{S}_1, \hat{S}_2]) + \delta \bar{\theta}_k (\text{Var}[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - \text{Var}[\tilde{V}_k|\hat{S}_k]),$$

which can be simplified into

$$\Delta RP_{ka} = \frac{\sigma_{Hk} \left(\sum_{j=1,2} \frac{\sigma_{Hj}}{\sigma_{Vj}^2} \hat{S}_j + \delta \sigma_{Hk} \bar{\theta}_k \right)}{1 + \sum_{j=1,2} \frac{\sigma_{Hj}^2 (\sigma_{Vj}^2 + \sigma_{\varepsilon j}^2)}{\sigma_{Vj}^4}}. \quad (29)$$

As in the symmetric information case, we conjecture that I investors buy stock k and U and M_k investors sell stock k if and only if $\Delta RP_{ka} > 0$. The following theorem provides the equilibrium price and equilibrium stock demand, confirming our conjecture.

Theorem 3 *In the presence of asymmetric information, there exists a unique competitive equilibrium with stock price P_{ka} being linear in \hat{S}_1 and \hat{S}_2 ,*

1. *the equilibrium price is*

$$P_{ka}^* = \frac{\nu_k N_I}{N_{ka}} P_{Ika}^R + \frac{N_U}{N_{ka}} P_{Uka}^R + \frac{N_{M_k}}{N_{ka}} P_{Mka}^R = P_{Uka}^R + \frac{\nu_k N_I}{N_{ka}} \Delta R P_{ka}, \quad (30)$$

2. *the equilibrium stock quantities demanded are*

$$\theta_{Ik}^* = \left(1 - \frac{\nu_k N_I}{N_{ka}}\right) \frac{\Delta R P_{ka}}{\delta \text{Var}[\tilde{V}_k | \hat{S}_k]}, \quad \theta_{Uk}^* = \theta_{Mk}^* = -\frac{\nu_k N_I}{N_{ka}} \frac{\Delta R P_{ka}}{\delta \text{Var}[\tilde{V}_k | \hat{S}_1, \hat{S}_2]}, \quad (31)$$

where $N_{ka} \equiv \nu_k N_I + N_{M_k} + N_U > N_k$ is the information weighted total population.

Theorem 3 implies that the equilibrium price P_{ka}^* is linear in both \hat{S}_1 and \hat{S}_2 and therefore, in equilibrium, all investors can indeed infer the unique value of \hat{S}_1 and \hat{S}_2 from observing the market prices. As shown by (30), similar to the symmetric information case, the equilibrium price is again a weighted average of the reservation prices of the investors in the economy. However, different from the symmetric information case, the equilibrium price of stock k depends on both private signals \hat{s}_1 and \hat{s}_2 . Since $E[\Delta R P_{ka}] > 0$ for any $k = 1, 2$, the informed buy both assets in expectation due to the estimation risk premium required by the uninformed which lowers their reservation prices for both assets.

Proposition 4 *With asymmetric information and perfect competition, we have*

1.

$$\text{Corr}(\theta_{i1}, \theta_{i2}) = 1 \times \text{Sign}(\sigma_{1N} \sigma_{2N}), i = I, U, M_k;$$

2.

$$\text{Corr}(P_{1a}^*, P_{2a}^*) = \frac{(a_1 a_2 + b_1 b_2) \text{Cov}(\hat{S}_1, \hat{S}_2) + a_1 b_2 \sigma_{\hat{S}_1}^2 + b_1 a_2 \sigma_{\hat{S}_2}^2}{\prod_{k=1,2} \sqrt{a_k^2 \sigma_{\hat{S}_k}^2 + b_k^2 \sigma_{\hat{S}_{-k}}^2} + 2a_k b_k \text{Cov}(\hat{S}_1, \hat{S}_2)}; \quad (32)$$

3.

$$\begin{aligned} \text{Cov}(\tilde{V}_1 - P_{1a}^*, \tilde{V}_2 - P_{2a}^*) &= C_1 \sigma_{H1} \sigma_{H2} \left(\frac{N_U + N_{M1}}{N_{1a}} + \frac{N_U + N_{M2}}{N_{2a}} \right) \\ &+ C_1^2 \sigma_{H1} \sigma_{H2} \frac{(a_1 a_2 + b_1 b_2) \text{Cov}(\hat{S}_1, \hat{S}_2) + a_1 b_2 \sigma_{\hat{S}_1}^2 + b_1 a_2 \sigma_{\hat{S}_2}^2}{N_{1a} N_{2a} \sigma_{V1}^4 \sigma_{V2}^4 \text{Cov}^2(\hat{S}_1, \hat{S}_2)}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} a_k &= N_{ka} \sigma_{V_k}^2 \sigma_{\hat{S}_{-k}}^2 \sigma_{H_{-k}} + \nu_k N_I \sigma_{H_k} \sigma_{V_{-k}}^2 \text{Cov}(\hat{S}_1, \hat{S}_2), \\ b_k &= -(N_U + N_{M_k}) \sigma_{H_{-k}} \sigma_{V_k}^2 \text{Cov}(\hat{S}_1, \hat{S}_2), \\ C_1 &= \left(1 + \sum_{j=1,2} \frac{\sigma_{H_j}^2 (\sigma_{V_j}^2 + \sigma_{\varepsilon_j}^2)}{\sigma_{V_j}^4} \right)^{-1}. \end{aligned} \quad (34)$$

Similar to the symmetric information case, Proposition 4 implies investors' correlated trading in two stocks lead to comovements in stocks' equilibrium prices and comovements in stocks' returns. However, as we can see from Figure 1, the magnitude of the correlation between the equilibrium stock prices, $|\text{Corr}(P_{1a}^*, P_{2a}^*)|$ is not monotonic in any of the parameters considered in Figure 1. In addition, we have

Corollary 1 1. $\text{Corr}(P_{1a}^*, P_{2a}^*) < 0$, if and only if $0 < \delta^2 \sigma_X^2 < C_2$, where C_2 is defined in (50) in the Appendix;

2. (a) If $\delta^2 \sigma_X^2 > C_2$ and $\sigma_{1N} \sigma_{2N} < 0$, then $\text{Corr}(P_{1a}^*, P_{2a}^*) > 0 > \text{Corr}(P_{1s}^*, P_{2s}^*)$;

(b) If $0 < \delta^2 \sigma_X^2 < C_2$ and $\sigma_{1N} \sigma_{2N} > 0$, then $\text{Corr}(P_{1s}^*, P_{2s}^*) > 0 > \text{Corr}(P_{1a}^*, P_{2a}^*)$;

3. $\text{Corr}(\tilde{V}_1 - P_{1a}^*, \tilde{V}_2 - P_{2a}^*) < 0$ if and only if $\sigma_{1N} \sigma_{2N} < 0$.

Parts 1 and 2 of Corollary 1 implies that the presence of asymmetric information can change the sign of the correlation between the prices. This is because, for example, when the correlation of hedging trades is negative ($\sigma_{1N} \sigma_{2N} < 0$), the correlation between prices in the absence of asymmetric information is negative while the information filtering effect

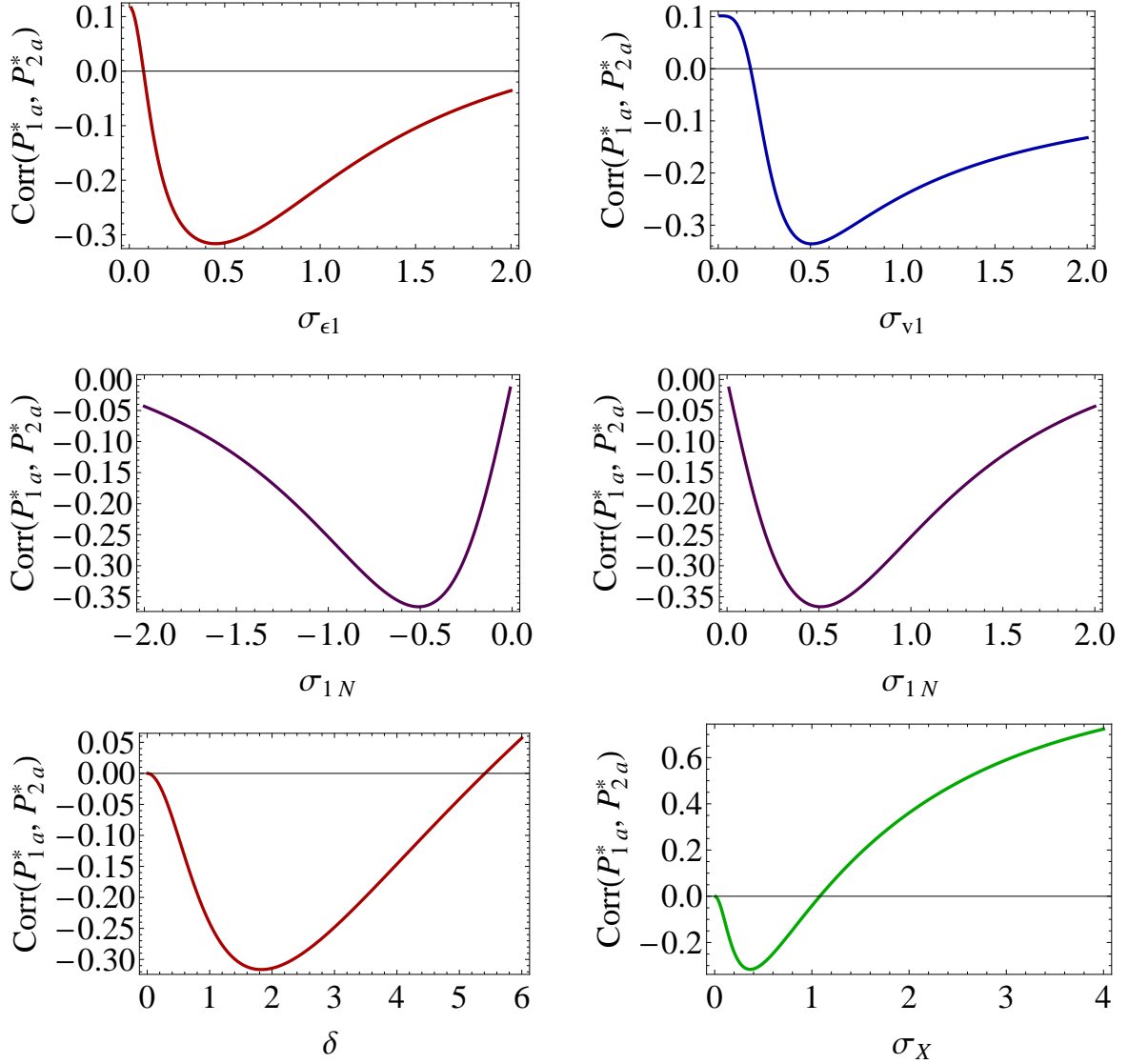


Figure 1: The correlation coefficient between the equilibrium price for asset 1 and asset 2 against $\sigma_{\epsilon 1}, \sigma_{v1}, \sigma_{1N}, \delta$ and σ_X . The figures of the correlation against $\sigma_{\epsilon 2}, \sigma_{v2}$, and σ_{2N} are similar to those against $\sigma_{\epsilon 1}, \sigma_{v1}$, and σ_{1N} , respectively. The default parameter values are: $\bar{\theta}_1 = \bar{\theta}_2 = 2$, $\delta = 2$, $\sigma_{v1} = \sigma_{v2} = 0.4$, $\sigma_{\epsilon 1} = \sigma_{\epsilon 2} = 0.4$, and $\sigma_{1N} = -0.8, \sigma_{2N} = 0.4, \sigma_X = 0.4, N_I = 100, N_{M1} = 10, N_{M2} = 10, N_U = 1000$.

makes the correlation between prices positive in the presence of asymmetric information when $\delta^2 \sigma_X^2 > C_2$. Figure 1 illustrates some cases where this reversal occurs. For example, the right subfigure on σ_{1N} in Figure 1 shows that, in contrast to the symmetric information case, even when the correlation of the hedging demands in the two markets is positive ($\sigma_{1N}\sigma_{2N} > 0$), the correlation of prices can be negative. This is because of the effect of the information asymmetry on the estimation of the conditional distribution of the stock payoffs. More specifically, for example, as signal \hat{S}_1 increases, it increases the conditional expected payoff of stock 1 but decreases the conditional expected payoff of stock 2 if \hat{S}_1 and \hat{S}_2 has a positive correlation (i.e., if $\sigma_{1N}\sigma_{2N} > 0$), as explained above. This information filtering effect may dominate the effect of positively correlated hedging demands and make the correlation between the prices negative. As another example, for the last subfigure, the hedging demands are negatively correlated, however, for large σ_X^2 the effect of information filtering dominates and the price correlation becomes positive. The non-monotonicity of the price correlation in the parameters considered in Figure 1 comes from the fact that these parameters affect both the effect of correlated hedging and the effect of the information filtering.

Part 3 of Corollary 1 implies that, as in the symmetric information case, the sign of the correlation between the stock returns on date 1 is the same as the sign of the correlation between the hedging demands in the two markets, *i.e.*, $Sign(\sigma_{1N}\sigma_{2N})$.

B. Asymmetric information with imperfect competition

Let B_{ka}^* and A_{ka}^* be the equilibrium bid price and ask price for asset k . Define

$$C_{Ik} \equiv \frac{N_{M_k}(N_U + N_{M_k} + 1)}{(N_{M_k} + 1)(N_{ka} + 1)}, \quad C_{Uk} \equiv \frac{\nu_k N_{M_k} N_I}{(N_{M_k} + 1)(N_{ka} + 1)}. \quad (35)$$

The following theorem provides the equilibrium bid and ask prices and equilibrium stock holdings in the presence of asymmetric information and market power.

Theorem 4 *In the presence of asymmetric information and market power, we have that*

1. *the equilibrium bid and ask prices are*

$$A_{ka}^* = P_{Uka}^R + C_{Uk} \Delta RP_{ka} + \frac{\Delta RP_{ka}^+}{N_{M_k} + 1},$$

$$B_{ka}^* = P_{Uka}^R + C_{Uk} \Delta RP_{ka} - \frac{\Delta RP_{ka}^-}{N_{M_k} + 1},$$

which implies that the bid and ask spread is

$$A_{ka}^* - B_{ka}^* = \frac{|\Delta RP_{ka}|}{N_{M_k} + 1}, \quad (36)$$

2. *the equilibrium stock quantities demanded are*

$$\theta_{Ik}^* = C_{Ik} \frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{s}_k]}, \quad \theta_{Uk}^* = -C_{Uk} \frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{S}_1, \hat{S}_2]}, \quad \theta_{Mk}^* = \frac{(N_{M_k} + 1)}{N_{M_k}} \theta_{Uk}^*; \quad (37)$$

the equilibrium quote depths are

$$\alpha_{ka}^* = \frac{N_I C_{Ik}}{N_{M_k}} \left(\left(\frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{s}_k]} \right)^+ + \frac{\nu_k N_U}{N_U + N_{M_k} + 1} \left(\frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{S}_1, \hat{S}_2]} \right)^- \right), \quad (38)$$

and

$$\beta_{ka}^* = \frac{N_I C_{Ik}}{N_{M_k}} \left(\frac{\nu_k N_U}{N_U + N_{M_k} + 1} \left(\frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{S}_1, \hat{S}_2]} \right)^+ + \left(\frac{\Delta RP_{ka}}{\delta Var[\tilde{V}_k | \hat{s}_k]} \right)^- \right), \quad (39)$$

which implies that the equilibrium trading volume is

$$N_{M_k} (\alpha_{ka}^* + \beta_{ka}^*) = \frac{N_{M_k} N_I (N_{M_k} + 2N_U + 1)}{(N_{M_k} + 1)(N_{ka} + 1)} \left(\frac{|\Delta RP_{ka}|}{\delta Var[\tilde{V}_k | \hat{s}_k]} \right). \quad (40)$$

Theorem 4 implies that, as in the symmetric information case, the bid and ask spread of

stock k is equal to the absolute value of the reservation price difference of stock k , divided by $N_{M_k} + 1$. Since $|\Delta RP_{ka}|$ depends on both \hat{S}_1 and \hat{S}_2 , the bid and ask spread of stock k depends on both \hat{S}_1 and \hat{S}_2 . Because correlated hedging demands lead to correlated \hat{S}_1 and \hat{S}_2 , Theorem 4 implies that, similar to the symmetric information case, investors' correlated trading in two stocks leads to correlated bid-ask spreads of stock 1 and stock 2. More specifically, we have

$$\text{Corr}(A_{1a}^* - B_{1a}^*, A_{2a}^* - B_{2a}^*) = \text{Corr}(|\Delta RP_{1a}|, |\Delta RP_{2a}|), \quad (41)$$

and

$$\text{Corr}(N_{M_1}(\alpha_{1a}^* + \beta_{1a}^*), N_{M_2}(\alpha_{2a}^* + \beta_{2a}^*)) = \text{Corr}(|\Delta RP_{1a}|, |\Delta RP_{2a}|). \quad (42)$$

Then we have

Proposition 5 *With asymmetric information,*

$$\text{Corr}(A_{1a}^* - B_{1a}^*, A_{2a}^* - B_{2a}^*) < 0,$$

if $-4\sigma_{2N}\bar{\theta}_2 < \sigma_{1N}\bar{\theta}_1 < -\sigma_{2N}\bar{\theta}_2$, and

$$0 < \sigma_X^2 < \min \left\{ \frac{\delta^2 \sigma_{1N}^2 \bar{\theta}_1^2 - 4C_3}{4\delta^2 C_3^2}, \frac{\delta^2 \sigma_{2N}^2 \bar{\theta}_2^2 - 4C_3}{4\delta^2 C_3^2} \right\}, \quad (43)$$

where

$$C_3 = \sum_{j=1,2} \frac{\sigma_{jN}^2 (\sigma_{Vj}^2 + \sigma_{\varepsilon j}^2)}{\sigma_{Vj}^4}. \quad (44)$$

Different from the symmetric information case, Proposition 5 implies that, as illustrated in Figure 2, the correlation between the liquidity of two assets can be negative, *i.e.*, the liquidity of stock 1 may improve when the liquidity of stock 2 deteriorates and vice versa. This happens usually when the correlation between hedging demands in the two stocks is

negative. To understand the intuition, note that by (29), we have for $k = 1, 2$,

$$|\Delta RP_{ka}| = C_1 |\sigma_{Hk}| |z + \delta \sigma_{Hk} \bar{\theta}_k|,$$

where

$$z = \frac{\sigma_{H1}}{\sigma_{V1}^2} \hat{S}_1 + \frac{\sigma_{H2}}{\sigma_{V2}^2} \hat{S}_2$$

is normally distributed with mean 0 and variance

$$\sigma_z^2 = \frac{1 - C_1}{C_1^2}, \quad (45)$$

and C_1 is as defined in (34).

Therefore for $z \in \left(\min(-\delta \sigma_{H1} \bar{\theta}_1, -\delta \sigma_{H2} \bar{\theta}_2), \max(-\delta \sigma_{H1} \bar{\theta}_1, -\delta \sigma_{H2} \bar{\theta}_2) \right)$, $|\Delta RP_{1a}|$ and $|\Delta RP_{2a}|$ move in the opposite directions as z changes, and move in the same directions outside this range. This implies that if $\sigma_{1N} \sigma_{2N} < 0$ or equivalently, $\sigma_{H1} \sigma_{H2} < 0$ and the variance of z is small then $|\Delta RP_{1a}|$ and $|\Delta RP_{2a}|$ and thus the spreads are negatively correlated. For the main intuition, suppose $\sigma_{H1} < 0$ and $\sigma_{H2} > 0$ and the informed have a positive liquidity shock \hat{X}_I and small signal $|\hat{s}_1|$ and $\hat{s}_2 = 0$ so that the informed buy asset 1 and also buy asset 2 due to the estimation risk premium of the uninformed, i.e., $\Delta RP_{1a} > 0$ and $\Delta RP_{2a} > 0$ (or equivalently, $-\delta \sigma_{H2} \bar{\theta}_2 < z < -\delta \sigma_{H1} \bar{\theta}_1$). When \hat{S}_1 increases, relative to the informed, the uninformed underestimate more the expected payoff of stock 1, which implies that the reservation price difference increases and so does the spread of asset 1. The uninformed also overestimate more that of asset 2 which in turn offsets more of the estimation risk premium and thus drives down the spread of asset 2. Therefore, the spreads move in the opposite directions. Since the expected value of z is zero and the spreads move in the opposite directions around $z = 0$, if the volatility of z is small, then most of the states concentrate around 0 and therefore the spreads are overall negatively correlated.

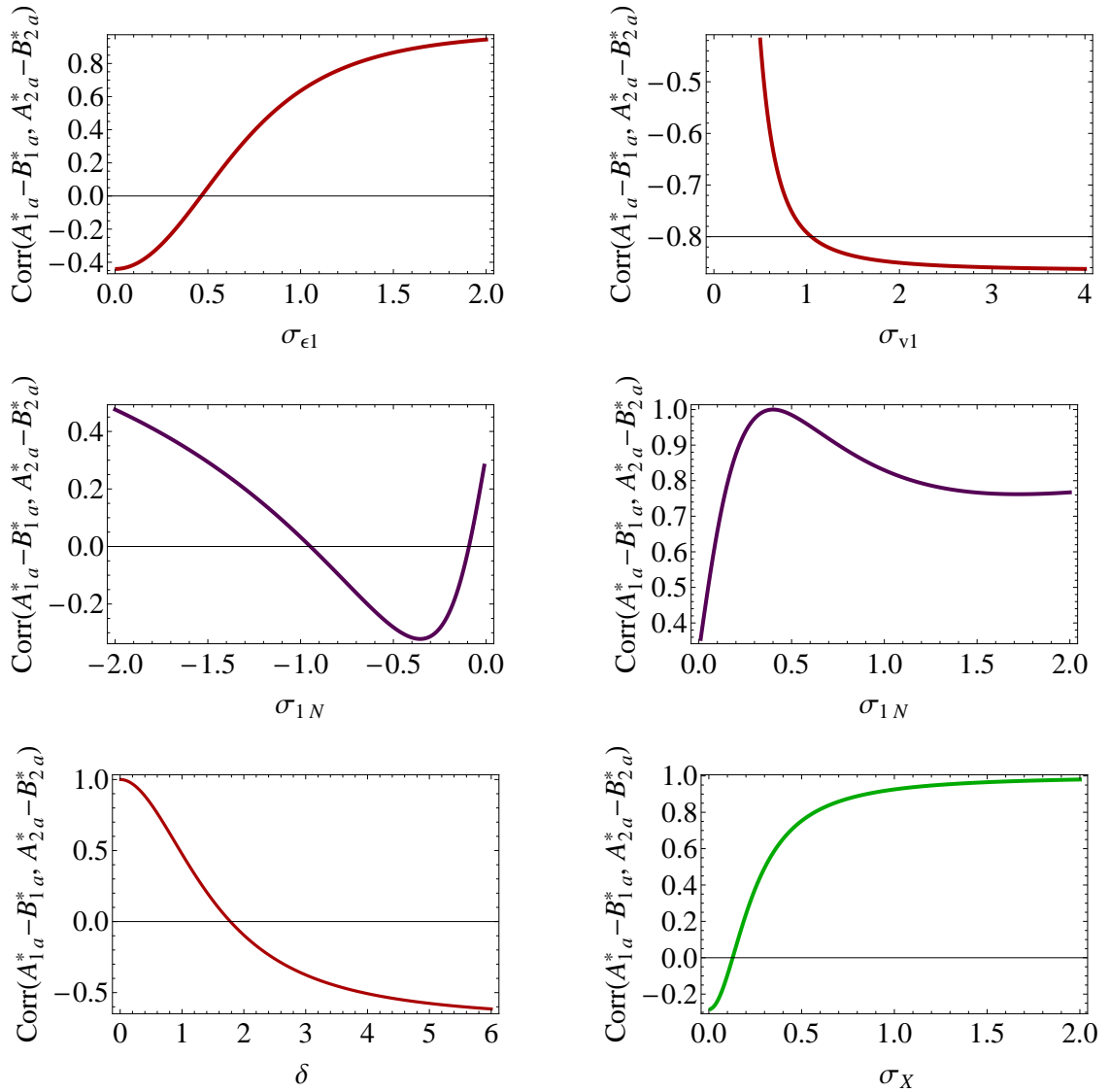


Figure 2: The correlation coefficient between the equilibrium bid and ask spreads for asset 1 and asset 2 against $\sigma_{\epsilon 1}$, $\sigma_{V 1}$, $\sigma_{1 N}$, δ and σ_X . The figures of the correlation against $\sigma_{\epsilon 2}$, $\sigma_{V 2}$, and $\sigma_{2 N}$ are similar to those against $\sigma_{\epsilon 1}$, $\sigma_{V 1}$, and $\sigma_{1 N}$, respectively. The default parameter values are: $\bar{\theta}_1 = \bar{\theta}_2 = 2$, $\delta = 2$, $\sigma_{V 1} = 0.4$, $\sigma_{V 2} = 0.4$, $\sigma_{\epsilon 1} = \sigma_{\epsilon 2} = 0.4$, and $\sigma_{1 N} = -0.8$, $\sigma_{2 N} = 0.4$, $\sigma_X = 0.1$, $N_I = 100$, $N_{M 1} = 10$, $N_{M 2} = 10$, $N_U = 1000$.

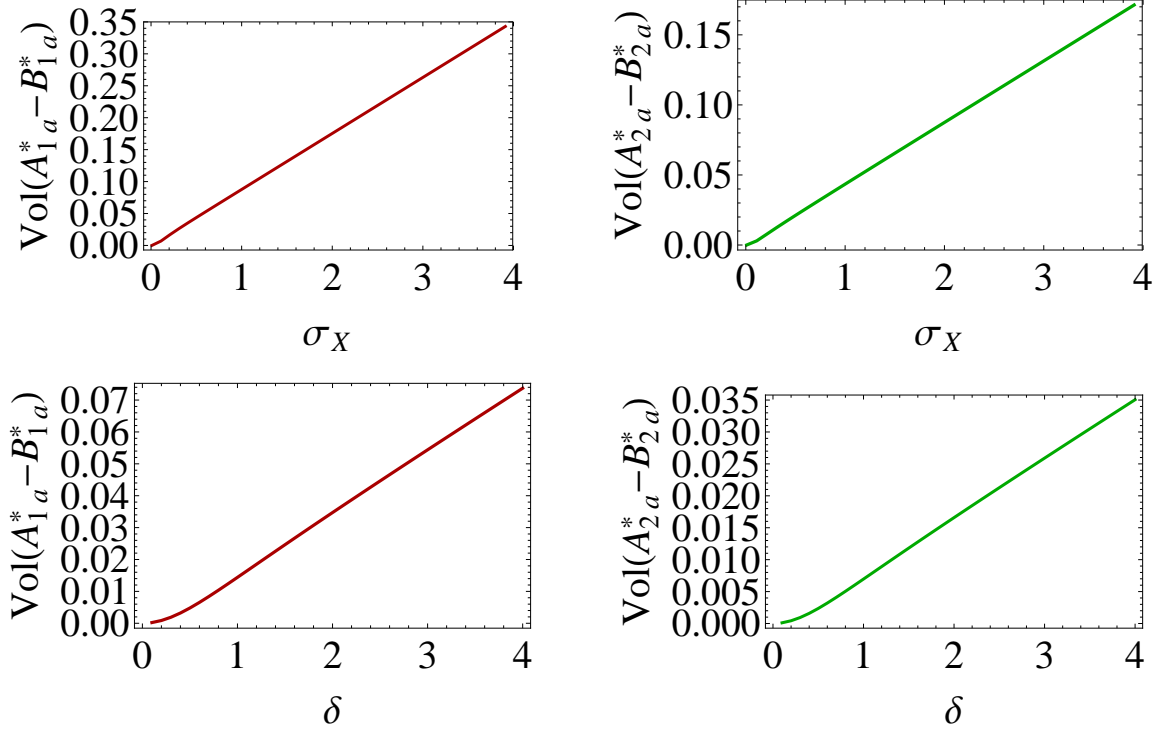


Figure 3: The volatility of bid-ask spreads of asset 1 and asset 2 against σ_X and δ . The default parameter values are: $\bar{\theta}_1 = \bar{\theta}_2 = 2$, $\delta = 2$, $\sigma_{V_1} = \sigma_{V_2} = 0.4$, $\sigma_{\varepsilon_1} = \sigma_{\varepsilon_2} = 0.4$, and $\sigma_{1N} = -0.8$, $\sigma_{2N} = 0.4$, $\sigma_X = 0.4$, $N_I = 100$, $N_{M_1} = 10$, $N_{M_2} = 10$, $N_U = 1000$.

Consistent with the above intuition, Figure 2 shows that if $\sigma_{1N}\sigma_{2N} < 0$, the correlation between the spreads can be negative when σ_{ε_1} or σ_X is small, δ or σ_{V_1} is large. A decrease in σ_{ε_1} or σ_X or an increase in σ_{V_1} decrease σ_z^2 and an increase in δ widens the range within which the two spreads move in the opposite directions. Proposition 5 provides an explicitly sufficient condition for the correlation between the spreads to be negative. In addition, the magnitude of the correlation between the spreads is higher with symmetric information than with asymmetric information. Figure 2 also shows that the magnitude of the correlation between the spreads can decrease with information asymmetry of either stock 1 or stock 2 ($\frac{1}{\sigma_{\varepsilon_k}^2}$ is a measure of information asymmetry of stock $-k$).

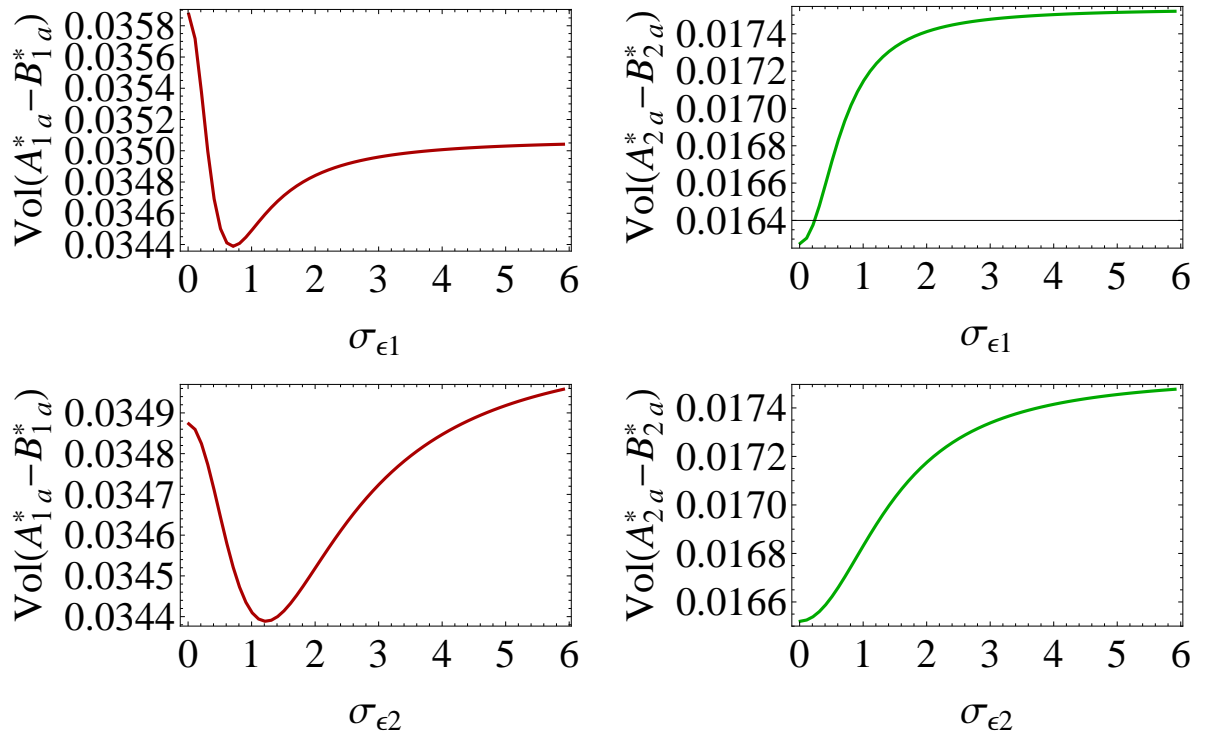


Figure 4: The volatility of bid-ask spreads of asset 1 and asset 2 against $\sigma_{\epsilon 1}$, and $\sigma_{\epsilon 2}$. The default parameter values are: $\theta_1 = \theta_2 = 2$, $\delta = 2$, $\sigma_{V_1} = \sigma_{V_2} = 0.4$, $\sigma_{\epsilon 1} = \sigma_{\epsilon 2} = 0.4$, and $\sigma_{1N} = -0.8$, $\sigma_{2N} = 0.4$, $\sigma_X = 0.4$, $N_I = 100$, $N_{M1} = 10$, $N_{M2} = 10$, $N_U = 1000$.

We now study the co-movements in liquidity risks of the two stocks. We have

$$Var(A_{ka}^* - B_{ka}^*) = \frac{C_1^2 \sigma_{Hk}^2 \sigma_z^2}{(N_{M_k} + 1)^2} \left(1 + z_k^2 - z_k^2 \left(\frac{2\mathbf{n}(z_k)}{z_k} + 2\mathbf{N}(z_k) - 1 \right)^2 \right), \quad (46)$$

where $n(\cdot)$ and $N(\cdot)$ are respectively the *pdf* and *cdf* of the standard normal distribution and σ_z is as defined in (45), C_1 is as defined in (34) and $z_k = \frac{\delta \sigma_{Hk} \bar{\theta}_k}{\sigma_z}$. As in the symmetric information case, investors' correlated trading may lead to co-movements in liquidity risks measured by volatilities of bid-ask spreads. As illustrated in Figure 3, for example, the volatilities of the bid-ask spreads of both stocks increase in σ_X^2 and δ . In addition, in contrast to the symmetric information case, because σ_z^2 depends on the parameter values of both stocks, the liquidity risk of one stock is affected by the characteristics of the other stock. For example, Figure 4 shows that the liquidity risk of stock 2 is affected by the quality of the private information about stock 1's final payoff. Figure 4 also illustrates that the liquidity risk of stock k may be non-monotonic in $\sigma_{\varepsilon k}$ if $\sigma_{kN} < 0$. Figure 4 shows that the liquidity risk of stock 1 may decrease when the liquidity risk of stock 2 increases. This happens usually when the correlation between hedging demands in the two stocks is negative.

Next we examine the price impact of the informed investors' trading in equilibrium, we have

Proposition 6 1. *The price impact with perfect (imperfect) competition is*

$$\lambda_{pck} = \frac{\delta \rho_{Ik} \sigma_{\varepsilon k}^2}{N_I (N_U + N_{M_k})} \left(\frac{N_{ka} \rho_{I1} \rho_{I2} \sigma_{V1}^2 \sigma_{V2}^2}{\rho_{I1} \sigma_{V1}^2 \sigma_{H2}^2 + \rho_{I2} \sigma_{V2}^2 \sigma_{H1}^2} + \nu_k N_I \right), \quad (47)$$

$$\lambda_{ick} = \frac{\delta \rho_{Ik} \sigma_{\varepsilon k}^2}{N_I C_{Ik}} \left(\frac{\rho_{I1} \rho_{I2} \sigma_{V1}^2 \sigma_{V2}^2}{\rho_{I1} \sigma_{V1}^2 \sigma_{H2}^2 + \rho_{I2} \sigma_{V2}^2 \sigma_{H1}^2} + C_{Uk} + \frac{1}{2(N_{M_k} + 1)} \right). \quad (48)$$

2. (a)

$$\frac{\partial \lambda_{pck}}{\partial \sigma_{\varepsilon k}} > 0 \quad \text{iff} \quad 0 < \sigma_{\varepsilon k}^2 < C_{\varepsilon k},$$

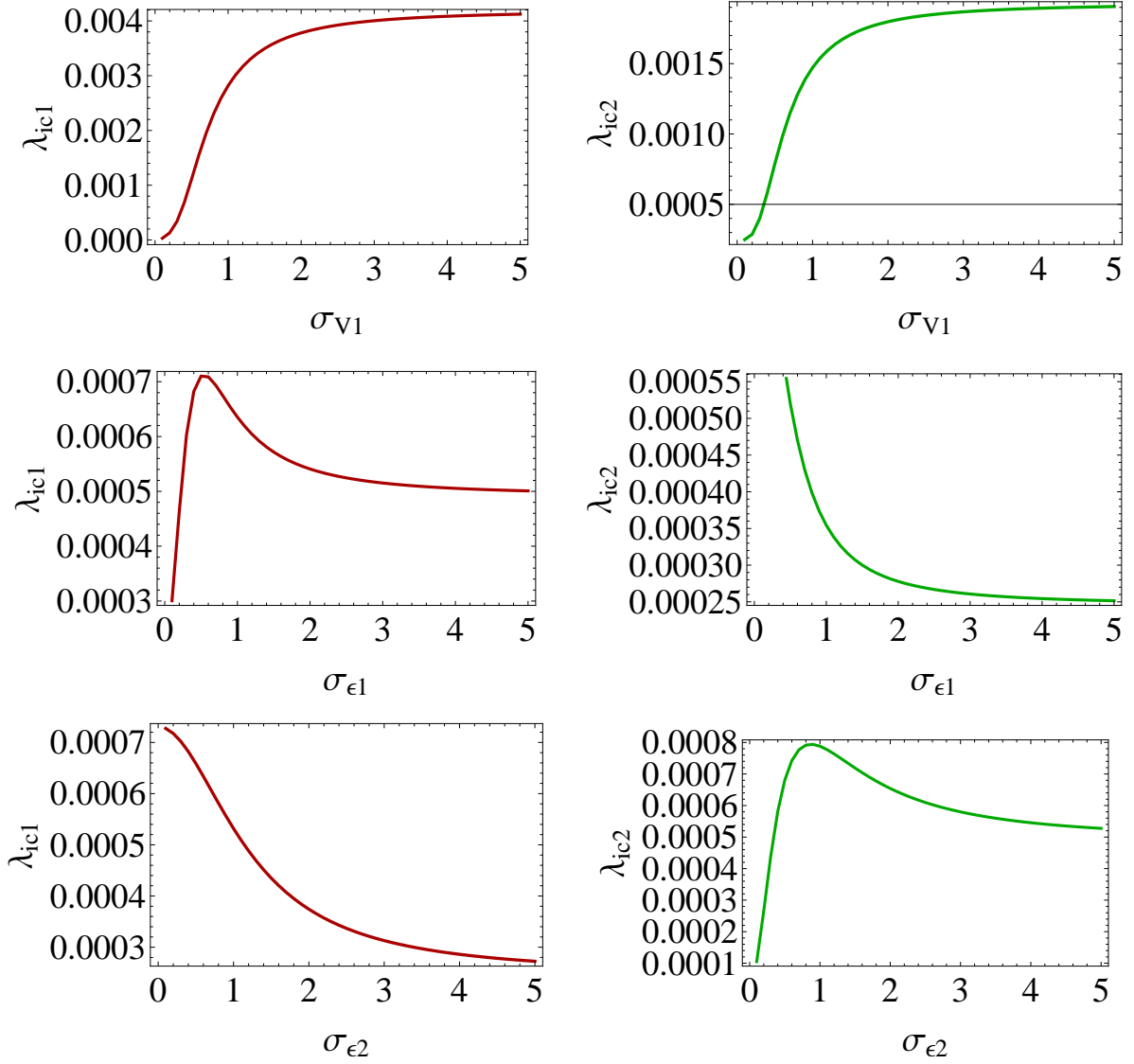


Figure 5: The price impact of asset 1 and asset 2 against σ_{V1} , $\sigma_{\epsilon 1}$, and $\sigma_{\epsilon 2}$. The default parameter values are: $\bar{\theta}_1 = \bar{\theta}_2 = 2$, $\delta = 2$, $\sigma_{V1} = \sigma_{V2} = 0.4$, $\sigma_{\epsilon 1} = \sigma_{\epsilon 2} = 0.4$, and $\sigma_{1N} = -0.8$, $\sigma_{2N} = 0.4$, $\sigma_X = 0.4$, $N_I = 100$, $N_{M1} = 10$, $N_{M2} = 10$, $N_U = 1000$.

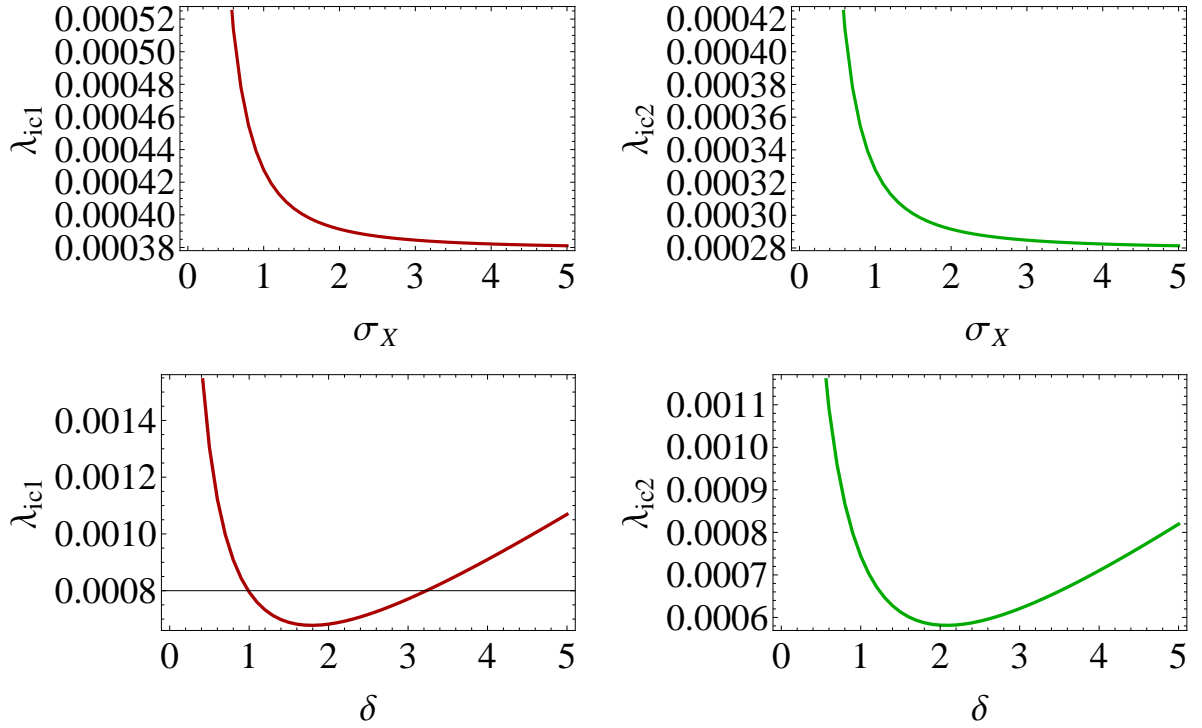


Figure 6: The price impact of asset 1 and asset 2 against σ_X , and δ . The default parameter values are: $\bar{\theta}_1 = \bar{\theta}_2 = 2$, $\delta = 2$, $\sigma_{V_1} = \sigma_{V_2} = 0.4$, $\sigma_{\varepsilon_1} = \sigma_{\varepsilon_2} = 0.4$, and $\sigma_{1N} = -0.8$, $\sigma_{2N} = 0.4$, $\sigma_X = 0.4$, $N_I = 100$, $N_{M1} = 10$, $N_{M2} = 10$, $N_U = 1000$.

where $C_{\varepsilon k}$ is defined in (63) in the Appendix.

(b)

$$\frac{\partial \lambda_{pck}}{\partial \sigma_{\varepsilon, -k}} < 0, \quad \frac{\partial \lambda_{pck}}{\partial \sigma_X^2} < 0.$$

As illustrated in Figures 5 and 6, in contrast to the symmetric information case, the price impact for stock k may be nonmonotonic in $\sigma_{\varepsilon k}$ and δ . In addition, the price impact for stock k decreases in $\sigma_{\varepsilon, -k}$ and σ_X . The non-monotonicity of the price impact of stock k in $\sigma_{\varepsilon k}^2$ is driven by two effects: an information quality effect and an information filtering effect. The price impact of stock k decreases in the quality of the information about stock k 's payoff and increases in the asymmetric information filtering effect (across the informed and the uninformed). As $\sigma_{\varepsilon k}$ increases, the information quality decrease and the asymmetric information filtering effect also decreases. Therefore, as $\sigma_{\varepsilon k}^2$ increases, the price impact of stock k increases due to the information quality effect and decreases due to the asymmetric information filtering effect. When $\sigma_{\varepsilon k}^2$ is small, the information quality effect dominates and thus the price impact of stock k increases in $\sigma_{\varepsilon k}^2$. When $\sigma_{\varepsilon k}^2$ is large, the asymmetric information filtering effect dominates and thus the price impact of stock k decreases in $\sigma_{\varepsilon k}^2$. Interestingly, the price impact of stock k decreases in the quality of the information about the payoff of the other stock. This is because uninformed investors can estimate the hedging demand better from the equilibrium price when informed investors' private signal about the other stock's payoff is less precise and thus uninformed investors' uncertainty about stock k 's payoff decreases. This implies that the price impact of stock 1 may increase when the price impact of stock 2 decreases.

V. Conclusions

We propose a tractable equilibrium model with asymmetric information, imperfect competition among market makers, risk averse investors and multiple risky assets to study

the widely documented commonality in stock illiquidity and returns. In the absence of asymmetric information, illiquidity correlation is always positive even when hedging demands are negatively correlated and in addition, price correlations always have the same sign as the hedging demand correlation. In the presence of asymmetric information, however, both illiquidity correlation (measured by the correlation of bid-ask spreads) and stock price correlation can become negative if hedging demands are negatively correlated due to the information filtering effect. The magnitude of the correlation between the spreads can decrease with information asymmetry. Our analysis shows that the presence of asymmetric information can qualitatively change the impact of correlated demand on stock commonality. In addition, information about one asset can affect the price and the liquidity of another asset even though they have independent payoffs. Moreover, as the precision of the private information about a stock improves, the uninformed's estimation precision of another stock's payoff worsens, and therefore the price impact (resp. liquidity risk) for one stock may increase as the price impact (resp. liquidity risk) for the other stock decreases.

Appendix

Proof of Theorem 1: The proof is as the proof of Theorem 3, where $E[\tilde{V}_k|\hat{S}_1, \hat{S}_2]$ is replaced by $E[\tilde{V}_k|\hat{s}_k]$ and $Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]$ is replaced by $Var[\tilde{V}_k|\hat{s}_k]$. Q.E.D.

Proof of Proposition 1: We have

$$Cov(P_{1s}^*, P_{2s}^*) = Cov(\tilde{V}_1 - P_{1s}^*, \tilde{V}_2 - P_{2s}^*) = N_I^2 \delta^2 \sigma_X^2 \prod_{k=1,2} \frac{\sigma_{kN}}{N_k};$$

$$Cov(\theta_{I1}^*, \theta_{I2}^*) = \sigma_X^2 \prod_{k=1,2} \frac{\sigma_{kN}(N_k - N_I)}{N_k Var[\tilde{V}_k|\hat{s}_k]}; \quad Cov(\theta_{i1}^*, \theta_{i2}^*) = N_I^2 \sigma_X^2 \prod_{k=1,2} \frac{\sigma_{kN}}{N_k Var[\tilde{V}_k|\hat{s}_k]}, \quad i = U, M_k;$$

$$Var(P_{ks}^*) = \frac{\sigma_{V_k}^4}{\sigma_{V_k}^2 + \sigma_{\varepsilon_k}^2} + \frac{N_I^2}{N_k^2} \delta^2 \sigma_{kN}^2 \sigma_X^2, \quad Var(\tilde{V}_k - P_{ks}^*) = (1 - \rho_{I_k})^2 \sigma_{V_k}^2 + \rho_{I_k}^2 \sigma_{\varepsilon_k}^2 + \frac{N_I^2}{N_k^2} \delta^2 \sigma_{kN}^2 \sigma_X^2.$$

Q.E.D.

Proof of Theorem 2: The proof is as the proof of Theorem 4, where $E[\tilde{V}_k|\hat{S}_1, \hat{S}_2]$ is replaced by $E[\tilde{V}_k|\hat{s}_k]$ and $Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]$ is replaced by $Var[\tilde{V}_k|\hat{s}_k]$. Q.E.D.

Proof of Proposition 2:

$$Cov(A_{1s}^* - B_{1s}^*, A_{2s}^* - B_{2s}^*) = \delta^2 Cov(|\hat{X}_I|, |\hat{X}_I|) \prod_{k=1,2} \frac{|\sigma_{kN}|}{N_{M_k} + 1} = \delta^2 \sigma_X^2 \left(1 - \frac{2}{\pi}\right) \prod_{k=1,2} \frac{|\sigma_{kN}|}{N_{M_k} + 1}.$$

It follows that $Corr(A_{1s}^* - B_{1s}^*, A_{2s}^* - B_{2s}^*) = 1$. Similarly we can show that $Corr(N_{M_1}(\alpha_{1s}^* + \beta_{1s}^*), N_{M_2}(\alpha_{2s}^* + \beta_{2s}^*)) = 1$. For Part 3, $Var(A_{ks}^* - B_{ks}^*) = E(A_{ks}^* - B_{ks}^*)^2 - E^2(A_{ks}^* - B_{ks}^*) = \frac{1-2/\pi}{(N_{M_k}+1)^2} \delta^2 \sigma_{kN}^2 \sigma_X^2$.

Q.E.D.

Proof of Theorem 3: The optimal stock holding of an uninformed investor is given in (27), and similarly, for an market maker we get:

$$\theta_{M_k}^* = \frac{E[\tilde{V}_k | \hat{S}_1, \hat{S}_2] - P_a}{\delta Var[\tilde{V}_k | \hat{S}_1, \hat{S}_2]} - \bar{\theta}_k. \quad (49)$$

Substituting (21), (27) and (49) into the market clearing condition $N_I \theta_I^* + N_U \theta_U^* + N_M \theta_M^* = 0$, we get the equilibrium stock price P_{ka}^* . Substituting P_{ka}^* into (21), (27) and (49), we can get I , U and M investors' optimal stock holdings. *Q.E.D.*

Proof of Proposition 4: We have $Corr(P_{1a}^*, P_{2a}^*) =$

$$\begin{aligned} & Corr\left((N_{1a}\sigma_{V_1}^2\sigma_{\hat{S}_2}^2\sigma_{H2} + \nu_1 N_I \sigma_{H1}\sigma_{V_2}^2 Cov(\hat{S}_1, \hat{S}_2))\hat{S}_1 - (N_{M_1} + N_U)\sigma_{H2}\sigma_{V_1}^2 Cov(\hat{S}_1, \hat{S}_2)\hat{S}_2, \right. \\ & \quad \left. (N_{2a}\sigma_{V_2}^2\sigma_{\hat{S}_1}^2\sigma_{H1} + \nu_2 N_I \sigma_{H2}\sigma_{V_1}^2 Cov(\hat{S}_1, \hat{S}_2))\hat{S}_2 - (N_{M_2} + N_U)\sigma_{H1}\sigma_{V_2}^2 Cov(\hat{S}_1, \hat{S}_2)\hat{S}_1\right) \\ & = \frac{(a_1 a_2 + b_1 b_2)Cov(\hat{S}_1, \hat{S}_2) + a_1 b_2 \sigma_{\hat{S}_1}^2 + b_1 a_2 \sigma_{\hat{S}_2}^2}{\sqrt{a_1^2 \sigma_{\hat{S}_1}^2 + b_1^2 \sigma_{\hat{S}_2}^2 + 2a_1 b_1 Cov(\hat{S}_1, \hat{S}_2)} \sqrt{a_2^2 \sigma_{\hat{S}_2}^2 + b_2^2 \sigma_{\hat{S}_1}^2 + 2a_2 b_2 Cov(\hat{S}_1, \hat{S}_2)}}. \\ & \quad Cov(\tilde{V}_1 - P_{1a}^*, \tilde{V}_2 - P_{2a}^*) = -Cov(\tilde{V}_1, P_{2a}^*) - Cov(\tilde{V}_2, P_{1a}^*) + Cov(P_{1a}^*, P_{2a}^*) \\ & = C_1 \sigma_{H1} \sigma_{H2} \left(\frac{N_U + N_{M1}}{N_{1a}} + \frac{N_U + N_{M2}}{N_{2a}} \right) + C_1^2 \sigma_{H1} \sigma_{H2} \frac{(a_1 a_2 + b_1 b_2)Cov(\hat{S}_1, \hat{S}_2) + a_1 b_2 \sigma_{\hat{S}_1}^2 + b_1 a_2 \sigma_{\hat{S}_2}^2}{N_{1a} N_{2a} \sigma_{V_1}^4 \sigma_{V_2}^4 Cov^2(\hat{S}_1, \hat{S}_2)}. \end{aligned}$$

Q.E.D.

Proof of Corollary 1: $Cov(P_{1a}^*, P_{2a}^*) < 0$ is equivalent to

$$c_1 \delta^4 \sigma_X^4 + d_1 \delta^2 \sigma_X^2 + e_1 < 0,$$

where

$$\begin{aligned}
c_1 &= (\sigma_{1N}^2(\sigma_{\varepsilon 1}^2 + \sigma_{V1}^2)^2\sigma_{V2}^4 + \sigma_{2N}^2\sigma_{\varepsilon 1}^2(\sigma_{\varepsilon 2}^2 + \sigma_{V2}^2)\sigma_{V1}^4) (\sigma_{1N}^2\sigma_{\varepsilon 2}^2(\sigma_{\varepsilon 1}^2 + \sigma_{V1}^2)\sigma_{V2}^4 + \sigma_{2N}^2\sigma_{V1}^4(\sigma_{\varepsilon 2}^2 + \sigma_{V2}^2)^2), \\
d_1 &= \sigma_{V1}^4\sigma_{V2}^4 (\sigma_{1N}^2\sigma_{\varepsilon 2}^2(\sigma_{\varepsilon 1}^2 + \sigma_{V1}^2)^2\sigma_{V2}^4 + \sigma_{2N}^2\sigma_{\varepsilon 1}^2(\sigma_{\varepsilon 2}^2 + \sigma_{V2}^2)^2\sigma_{V1}^4) \\
&+ \sigma_{V1}^4\sigma_{V2}^4\sigma_{\varepsilon 1}^2\sigma_{\varepsilon 2}^2 \left(1 - \frac{(N_{M1} + N_U)(N_{M2} + N_U)}{N_I^2} \right) (\sigma_{V1}^4\sigma_{2N}^2(\sigma_{\varepsilon 2}^2 + \sigma_{V2}^2) + \sigma_{1N}^2\sigma_{V2}^4(\sigma_{\varepsilon 1}^2 + \sigma_{V1}^2)), \\
e_1 &= \sigma_{\varepsilon 1}^2\sigma_{\varepsilon 2}^2\sigma_{V1}^8\sigma_{V2}^8 \left(1 - \frac{(N_{M1} + N_U)(N_{M2} + N_U)}{N_I^2} \right),
\end{aligned}$$

Therefore, $Cov(P_{1a}^*, P_{2a}^*) < 0$ if and only if $0 < \delta^2\sigma_X^2 < C_2$, where

$$C_2 = \frac{-d_1 + \sqrt{d_1^2 - 4c_1e_1}}{2c_1}. \quad (50)$$

Q.E.D.

Proof of Proposition 3: From Theorem 1 and (19), we get the price impact with perfect competition is $\lambda_{pck} = \frac{\delta\rho_{Ik}\sigma_{\varepsilon k}^2}{N_U + N_{Mk}}$. From Theorem 2 and (19), we get the price impact with imperfect competition is $\lambda_{ick} = \frac{Cov(\frac{A_{ks}^* + B_{ks}^*}{2}, N_I\theta_{Ik}^*)}{Var(N_I\theta_{Ik}^*)}$. Q.E.D.

Proof of Theorem 4: We prove the case when $\Delta RP_{ka} < 0$. In this case, we conjecture that I investors sell stock k at the bid and U investors buy stock k at the ask. Given bid price B_k and ask price A_k , the optimal demand of I and U are:

$$\theta_{Ik}^* = \frac{E[\tilde{V}_k|\hat{s}_k] + \hat{H}_{Ik} - B_k}{\delta Var[\tilde{V}_k|\hat{s}_k]} - \bar{\theta}_k \quad \text{and} \quad \theta_{Uk}^* = \frac{E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - A_k}{\delta Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]} - \bar{\theta}_k. \quad (51)$$

Substituting (51) into the market clearing conditions (3), we get that the market clear-

ing bid and ask prices are:

$$\begin{aligned}
A_k &= E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - \delta Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]\bar{\theta}_k - \frac{\delta Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2]}{N_U} \sum_{j=1}^{N_{Mk}} \alpha_{jk}, \\
B_k &= E[\tilde{V}_k|\hat{s}_k] + \hat{H}_{Ik} - \delta Var[\tilde{V}_k|\hat{s}_k]\bar{\theta}_k + \frac{\delta Var[\tilde{V}_k|\hat{s}_k]}{N_I} \sum_{j=1}^{N_{Mk}} \beta_{jk},
\end{aligned} \tag{52}$$

where β_{jk} and α_{jk} are the optimal shares of stock M_{jk} choose to buy from I investors and sell to U investors respectively. Market maker M_{jk} 's problem is:

$$\min_{\alpha_{jk}, \beta_{jk}} -\delta(\alpha_{jk}A_k - \beta_{jk}B_k) - \delta(\bar{\theta}_k + \beta_{jk} - \alpha_{jk})E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] + \frac{1}{2}\delta^2 Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2](\bar{\theta}_k + \beta_{jk} - \alpha_{jk})^2, \tag{53}$$

where A_k and B_k are the market clearing prices given in (52). F.O.C with respect to β_{jk} gives us:

$$\begin{aligned}
&E[\tilde{V}_k|\hat{s}_k] + \hat{H}_{Ik} - E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] + \delta \left(Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - Var[\tilde{V}_k|\hat{s}_k] \right) \bar{\theta}_k \\
&+ \frac{\delta Var[\tilde{V}_k|\hat{s}_k]}{N_I} \sum_{j=1}^{N_{Mk}} \beta_{jk} + \left(\frac{Var[\tilde{V}_k|\hat{s}_k]}{N_I} + Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] \right) \delta \beta_{jk} - \delta Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] \alpha_{jk} = 0.
\end{aligned} \tag{54}$$

Summing all, we get:

$$\begin{aligned}
&N_{Mk} \left(E[\tilde{V}_k|\hat{s}_k] + \hat{H}_{Ik} - E[\tilde{V}_k|\hat{S}_1, \hat{S}_2] + \delta \left(Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] - Var[\tilde{V}_k|\hat{s}_k] \right) \bar{\theta}_k \right) \\
&+ \frac{\delta Var[\tilde{V}_k|\hat{s}_k] N_{Mk}}{N_I} \sum_{j=1}^{N_{Mk}} \beta_{jk} + \left(\frac{Var[\tilde{V}_k|\hat{s}_k]}{N_I} + Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] \right) \delta \sum_{j=1}^{N_{Mk}} \beta_{jk} - \delta Var[\tilde{V}_k|\hat{S}_1, \hat{S}_2] \sum_{j=1}^{N_{Mk}} \alpha_{jk} = 0.
\end{aligned} \tag{55}$$

Using the F.O.C with respect to α_{jk} , we get:

$$\frac{\delta}{N_U} \sum_{j=1}^{N_{Mk}} \alpha_{jk} - \delta(\beta_{jk} - \alpha_{jk}) + \frac{\delta}{N_U} \alpha_{jk} = 0. \quad (56)$$

Summing all, we get:

$$\sum_{j=1}^{N_{Mk}} \beta_j = \frac{N_U + N_{Mk} + 1}{N_U} \sum_{j=1}^{N_{Mk}} \alpha_{jk}. \quad (57)$$

Substituting (57) into (55), we get

$$\sum_{j=1}^{N_{Mk}} \alpha_{jk} = -\frac{N_{Mk} N_I N_U}{(N_{Mk} + 1)(N_{ka} + 1)} \frac{\Delta R P_{ka}}{\delta \text{Var}[\tilde{V}_k | \hat{s}_k]}. \quad (58)$$

Substituting (58) into (52), we can get the equilibrium ask and bid price A_{ka}^* and B_{ka}^* . And then substituting A_{ka}^* and B_{ka}^* into (51), we can get the optimal stock holdings of I and U investors as stated in Theorem 4. Similarly, we can prove the other case of this Theorem when I investors buy and U investors sell. *Q.E.D.*

Proof of Proposition 5: Assuming $\sigma_{H1} < 0$ and $\sigma_{H2} > 0$, we have

$$\text{Cov}(|z + \delta \sigma_{H1} \bar{\theta}_1|, |z + \delta \sigma_{H2} \bar{\theta}_2|) = E(|z + \delta \sigma_{H1} \bar{\theta}_1| |z + \delta \sigma_{H2} \bar{\theta}_2|) - E(|z + \delta \sigma_{H1} \bar{\theta}_1|) E(|z + \delta \sigma_{H2} \bar{\theta}_2|), \quad (59)$$

where the first term equals to

$$\begin{aligned} & E(z^2 + \delta^2 \sigma_{H1} \sigma_{H2} \bar{\theta}_1 \bar{\theta}_2) - 2 \int_{-\delta \sigma_{H2} \bar{\theta}_2}^{-\delta \sigma_{H1} \bar{\theta}_1} (z^2 + \delta(\sigma_{H1} \bar{\theta}_1 + \sigma_{H2} \bar{\theta}_2)z + \delta^2 \sigma_{H1} \sigma_{H2} \bar{\theta}_1 \bar{\theta}_2) f(x) dx \\ &= \sigma_z^2 + \delta^2 \sigma_{H1} \sigma_{H2} \bar{\theta}_1 \bar{\theta}_2 + \frac{2\delta \sigma_z \sigma_{H2} \bar{\theta}_2}{\sqrt{2\pi}} e^{-\frac{\delta^2 \sigma_{H1}^2 \bar{\theta}_1^2}{2\sigma_z^2}} - \frac{2\delta \sigma_z \sigma_{H1} \bar{\theta}_1}{\sqrt{2\pi}} e^{-\frac{\delta^2 \sigma_{H2}^2 \bar{\theta}_2^2}{2\sigma_z^2}} \\ & \quad - 2(\sigma_z^2 + \delta^2 \sigma_{H1} \sigma_{H2} \bar{\theta}_1 \bar{\theta}_2) \left(N\left(\frac{\delta \sigma_{H2} \bar{\theta}_2}{\sigma_z}\right) - N\left(\frac{\delta \sigma_{H1} \bar{\theta}_1}{\sigma_z}\right) \right), \end{aligned}$$

and we have

$$E|z + \delta\sigma_{Hk}\bar{\theta}_k| = \delta\sigma_{Hk}\bar{\theta}_k \left(2N \left(\frac{\delta\sigma_{Hk}\bar{\theta}_k}{\sigma_z} \right) - 1 \right) + \frac{2\sigma_z}{\sqrt{2\pi}} e^{-\frac{\delta^2\sigma_{Hk}^2\bar{\theta}_k^2}{2\sigma_z^2}}, \quad k = 1, 2.$$

It can be shown that

$$E(|z + \delta\sigma_{H1}\bar{\theta}_1|)E(|z + \delta\sigma_{H2}\bar{\theta}_2|) > -\delta\sigma_{H1}\bar{\theta}_1 \times \delta\sigma_{H2}\bar{\theta}_2 = -\delta^2\sigma_{H1}\sigma_{H2}\bar{\theta}_1\bar{\theta}_2.$$

Therefore, a sufficient condition for $Cov(|z + \delta\sigma_{H1}\bar{\theta}_1|, |z + \delta\sigma_{H2}\bar{\theta}_2|) < 0$ is

$$\begin{aligned} &= \sigma_z^2 + 2\delta^2\sigma_{H1}\sigma_{H2}\bar{\theta}_1\bar{\theta}_2 + \frac{2\delta\sigma_z\sigma_{H2}\bar{\theta}_2}{\sqrt{2\pi}} e^{-\frac{\delta^2\sigma_{H1}^2\bar{\theta}_1^2}{2\sigma_z^2}} - \frac{2\delta\sigma_z\sigma_{H1}\bar{\theta}_1}{\sqrt{2\pi}} e^{-\frac{\delta^2\sigma_{H2}^2\bar{\theta}_2^2}{2\sigma_z^2}} \\ &\quad - 2(\sigma_z^2 + \delta^2\sigma_{H1}\sigma_{H2}\bar{\theta}_1\bar{\theta}_2) \left(N \left(\frac{\delta\sigma_{H2}\bar{\theta}_2}{\sigma_z} \right) - N \left(\frac{\delta\sigma_{H1}\bar{\theta}_1}{\sigma_z} \right) \right) < 0. \end{aligned}$$

We use the fact that $\frac{x}{1+x^2}n(x) < 1 - \mathbf{N}(x) < \frac{n(x)}{x}$, for $x \geq 0$, where $n(x)$ is the *pdf* for standard normal distribution. We have:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{\delta^2\sigma_{H1}^2\bar{\theta}_1^2}{2\sigma_z^2}} < N \left(\frac{\delta\sigma_{H1}\bar{\theta}_1}{\sigma_z} \right) \left(-\frac{\sigma_z}{\delta\sigma_{H1}\bar{\theta}_1} - \frac{\delta\sigma_{H1}\bar{\theta}_1}{\sigma_z} \right),$$

and

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{\delta^2\sigma_{H2}^2\bar{\theta}_2^2}{2\sigma_z^2}} < N \left(-\frac{\delta\sigma_{H2}\bar{\theta}_2}{\sigma_z} \right) \left(\frac{\sigma_z}{\delta\sigma_{H2}\bar{\theta}_2} + \frac{\delta\sigma_{H2}\bar{\theta}_2}{\sigma_z} \right).$$

Therefore, a sufficient condition for $Cov(|z + \delta\sigma_{H1}\bar{\theta}_1|, |z + \delta\sigma_{H2}\bar{\theta}_2|) < 0$ is

$$1 - \frac{2\sigma_{H1}\bar{\theta}_1}{\sigma_{H2}\bar{\theta}_2} + 2 \left(1 - \frac{\sigma_{H2}\bar{\theta}_2}{\sigma_{H1}\bar{\theta}_1} \right) N \left(\frac{\delta\sigma_{H1}\bar{\theta}_1}{\sigma_z} \right) - 2 \left(1 - \frac{\sigma_{H1}\bar{\theta}_1}{\sigma_{H2}\bar{\theta}_2} \right) N \left(\frac{\delta\sigma_{H2}\bar{\theta}_2}{\sigma_z} \right) < 0. \quad (60)$$

A sufficient condition for inequality (60) to hold is

$$\frac{\delta\sigma_{H1}\bar{\theta}_1}{\sigma_z} < -2, \quad \frac{\delta\sigma_{H2}\bar{\theta}_2}{\sigma_z} > 2, \quad \text{and} \quad \frac{\sigma_{1N}\bar{\theta}_1}{\sigma_{2N}\bar{\theta}_2} + \frac{\sigma_{2N}\bar{\theta}_2}{\sigma_{1N}\bar{\theta}_1} > -8.$$

Therefore, $Corr(A_{1a}^* - B_{1a}^*, A_{2a}^* - B_{2a}^*) < 0$, if $-4\sigma_{2N}\bar{\theta}_2 < \sigma_{1N}\bar{\theta}_1 < -\sigma_{2N}\bar{\theta}_2$, and

$$0 < \sigma_X^2 < \min \left\{ \frac{\delta^2 \sigma_{1N}^2 \bar{\theta}_1^2 - 4C_3}{4\delta^2 C_3^2}, \frac{\delta^2 \sigma_{2N}^2 \bar{\theta}_2^2 - 4C_3}{4\delta^2 C_3^2} \right\}. \quad (61)$$

Q.E.D.

Proof of Proposition 6: From Theorem 3 and (19), we get the price impact with perfect competition is

$$\begin{aligned} \lambda_{pck} &= \frac{\delta N_{ka} Var[\tilde{V}_k | \hat{s}_k] Cov \left(E[\tilde{V}_k | \hat{S}_1, \hat{S}_2] + \frac{\nu_k N_I}{N_{ka}} \Delta RP_{ka}, \Delta RP_{ka} \right)}{N_I (N_U + N_{Mk}) Var(\Delta RP_{ka})} \\ &= \frac{\delta \rho_{Ik} \sigma_{\varepsilon k}^2}{N_I (N_U + N_{Mk})} \left(\frac{N_{ka} \rho_{I1} \rho_{I2} \sigma_{V1}^2 \sigma_{V2}^2}{\rho_{I1} \sigma_{V1}^2 \sigma_{H2}^2 + \rho_{I2} \sigma_{V2}^2 \sigma_{H1}^2} + \nu_k N_I \right). \end{aligned} \quad (62)$$

From Theorem 4 and (19), we get the price impact with imperfect competition is

$$\begin{aligned} \lambda_{ick} &= \frac{\delta Var[\tilde{V}_k | \hat{s}_k] Cov \left(E[\tilde{V}_k | \hat{S}_1, \hat{S}_2] + \left(C_{Uk} + \frac{1}{2(N_{Mk} + 1)} \right) \Delta RP_{ka}, \Delta RP_{ka} \right)}{N_I C_{Ik} Var(\Delta RP_{ka})} \\ &= \frac{\delta \rho_{Ik} \sigma_{\varepsilon k}^2}{N_I C_{Ik}} \left(\frac{\rho_{I1} \rho_{I2} \sigma_{V1}^2 \sigma_{V2}^2}{\rho_{I1} \sigma_{V1}^2 \sigma_{H2}^2 + \rho_{I2} \sigma_{V2}^2 \sigma_{H1}^2} + C_{Uk} + \frac{1}{2(N_{Mk} + 1)} \right). \end{aligned}$$

From (62), we have

$$\begin{aligned} \frac{\partial \lambda_{pck}}{\partial \sigma_X^2} &= - \frac{N_k \sigma_{\varepsilon k}^2 \sigma_{V_k}^2}{C_3 N_I (N_{Mk} + N_U) \delta \sigma_X^4 (\sigma_{\varepsilon k}^2 + \sigma_{V_k}^2)}, \\ \frac{\partial \lambda_{pck}}{\partial \sigma_{\varepsilon, -k}^2} &= - \frac{\sigma_{-k, N}^2 \sigma_{V_k}^4 (N_k \sigma_{\varepsilon k}^2 \sigma_{V_k}^2 + N_I \sigma_{Hk}^2 (\sigma_{\varepsilon k}^2 + \sigma_{V_k}^2))}{C_3^2 N_I (N_{Mk} + N_U) \delta \sigma_X^2 (\sigma_{\varepsilon k}^2 + \sigma_{V_k}^2) \sigma_{V1}^4 \sigma_{V2}^4}, \\ \frac{\partial \lambda_{pck}}{\partial \sigma_{\varepsilon k}^2} &= \frac{\sigma_{V_k}^6 (-N_k \sigma_{V, -k}^8 \sigma_{kN}^2 \sigma_{\varepsilon k}^4 + 2N_I \sigma_{H, -k}^2 \sigma_{V1}^2 \sigma_{V2}^2 \sigma_{V, -k}^2 \sigma_{kN}^2 (\sigma_{\varepsilon, -k}^2 + \sigma_{V, -k}^2) \sigma_{\varepsilon k}^2 + C_4)}{N_I (N_{Mk} + N_U) \delta \sigma_X^2 (\sigma_{\varepsilon k}^2 + \sigma_{V_k}^2)^2 C_3^2 \sigma_{V1}^8 \sigma_{V2}^8}, \end{aligned}$$

where C_3 is as defined in (44) and

$$C_4 = N_k \sigma_{V1}^4 \sigma_{V2}^4 \sigma_{V,-k}^4 \sigma_{kN}^2 + (N_k \sigma_{V_k}^2 \sigma_{-k,N}^2 + 2N_I \sigma_{H,-k}^2 \sigma_{kN}^2) \sigma_{V1}^4 \sigma_{V2}^4 (\sigma_{\varepsilon,-k}^2 + \sigma_{V,-k}^2) \\ + N_I \sigma_{H,-k}^2 \sigma_{-k,N}^2 \sigma_{V_k}^6 (\sigma_{\varepsilon,-k}^2 + \sigma_{V,-k}^2)^2.$$

Therefore, $\frac{\partial \lambda_{pc_k}}{\partial \sigma_{\varepsilon k}} > 0$ iff $0 < \sigma_{\varepsilon k}^2 < C_{\varepsilon k}$, where

$$C_{\varepsilon k} = \frac{N_I \sigma_{H,-k}^2 \sigma_{V1}^2 \sigma_{V2}^2 \sigma_{kN}^2 (\sigma_{\varepsilon,-k}^2 + \sigma_{V,-k}^2) + \sqrt{N_I^2 \sigma_{H,-k}^4 \sigma_{V1}^4 \sigma_{V2}^4 \sigma_{kN}^4 (\sigma_{\varepsilon,-k}^2 + \sigma_{V,-k}^2)^2 + N_k \sigma_{V,-k}^4 \sigma_{kN}^2 C_4}}{N_k \sigma_{V,-k}^6 \sigma_{kN}^2}. \quad (63)$$

Q.E.D.

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