Teaching notes for Gobal Games Morris-Shin (2003)

Zhiguo He

University of Chicago, Booth School of Business

Coordination and Multiple Equilibria

- Many economic scenarios feature coordinations among economic agents
  - Bank runs a la Diamond-Dybvig
  - Currency attack a la Morris-Shin
- Basically, strategic complementarity, as agents are happier if they do the same thing
  - Typically in financial market trading game features strategic substitution
  - If other people buy, pushing up price, so you want to sell
- Multiple equilibria emerge easily with strategic complementarity.
- Global games technique help get a unique equilibrium.

Carlsson and van Damme (1993): Setting

- Two players, binary actions "invest" or "not invest," fundamental θ
- Normal form

	Invest	Not Invest
Invest	θ, θ	$\theta - 1, 0$
Not Invest	$0, \theta - 1$	$0, 0$

- Basically for agent i, the payoff of investing is

$$\theta - \mathbf{1}\{j \text{ does not invest}\}$$

- Unique equilibrium if $\theta > 1$ (both invest) and $\theta < 0$ (nobody invest).
- Two equilibria if $\theta \in (0, 1)$: either both invest, or nobody invest.

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- Unique equilibrium if $\theta > 1$ (both invest) and $\theta < 0$ (nobody invest).
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- Problem: common knowledge in equilibrium strategies given complete information
  - Perfect guessing each other's strategies
- Introduce private information about θ to break it

Carlsson and van Damme: Private Signals

- Nobody knows θ exactly, but observe a private signal

$$x_i = \theta + \varepsilon_i,\ i = 1, 2$$

  where $\varepsilon_i \sim \mathcal{N}\left(0, \sigma^2\right)$ iid across agents
- Bayesian updating. For simplicity, say the prior of θ is "improper prior" or noninformative prior (equally likely over real line)
  - Given x_i, the posterior of θ is simply $\mathcal{N}\left(x_i, \sigma^2\right)$
  - Given x_i, the posterior of x_j is $\mathcal{N}\left(x_i, 2\sigma^2\right)$

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- Agent i's conditional payoff

$$\mathbb{E}\left[\theta - \mathbf{1}\left\{j \text{ not invest}\right\} \middle| x_i\right] = x_i - \Pr\left(j \text{ not invest} \middle| x_i\right)$$

- Two distinctive roles played by the signal x_i
  - First term: x_i tells me something about fundamental
  - Second term: x_i tells me something about the distribution of agent j's signal x_j and thus his strategy

Carlsson and van Damme: Strategic component

- The second term captures the idea of "guessing" each other's strategy, in a simple but powerful way
- Suppose that everybody follows a cutoff rule with threshold k

$$\begin{cases} \text{Invest} & \text{if } x > k \\ \text{Not invest} & \text{otherwise} \end{cases}$$

- Because $x_j \sim \mathcal{N}\left(x_i, 2\sigma^2\right)$, the probability of agent j not investing is

$$\Pr\left(x_j \leq k \middle| x_i\right) = \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma}\right)$$

- So, agent i will invest if and only if

$$x_i - \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma}\right) \geq 0$$

Carlsson and van Damme: Unique Equilibrium Threshold (1)

- The equilibrium cutoff k: when $x_i = k$, agent i indifferent between invest and not invest

$$k - \Phi\left(\frac{k-k}{\sqrt{2}\sigma}\right) = 0 \Rightarrow k = \frac{1}{2}$$

- So, the unique equilibrium is that every agent invest if his/her signal $x_i > 1/2$.
- Intuition:
  - Symmetry: when receiving $x_i = k$, the probability of j getting signal x_j below k is 0.5
  - Strategic uncertainty (guessing each other) implies the second term to be 0.5
  - The first fundamental term have to be 0.5 for equilibrium threshold

Carlsson and van Damme: Unique Equilibrium Threshold (2)

- The assumption of threshold strategy can be relaxed
  - Unique equilibrium surviving iterated deletion of strictly dominated strategies
- Magically, it does not depends on how noisy the private signal σ is!
- Say $\sigma \rightarrow 0$ so that the game seems to converge to the full information case, the equilibrium is still unique
  - Although fundamental uncertainty shrinks, the strategic uncertainty effect remains at 0.5
  - As a result, the agent invests only when the fundamental is above 0.5

Public versus Private Information: Setting

- Consider a more elegant Normal framework with public information
- A continuum of agents
  - Not investing: 0; Investing: $\theta + l - 1$ where l is the proportion of people investing
- θ is fundamental, with prior

$$\theta \sim \mathcal{N}\left(y, \tau^2\right)$$

  where y is the public signal (everybody observes it)
- Previous setting, improper prior, it is as if $\tau^2 \to \infty$
- Private signal

$$x_i = \theta + \varepsilon_i \text{ with } \varepsilon_i \sim \mathcal{N}\left(0, \sigma^2\right)$$

- The posterior of θ given x_i and y is

$$\theta \sim \mathcal{N}\left(\frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} \equiv \overline{\theta}, \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}\right)$$

- Consider threshold strategy k so that investing iff $\overline{\theta} > k$

Strategic Uncertainty

- Given x_i and y, what is his belief about other agent's signal $x' = \theta + \varepsilon'$?
  - Obviously this gives his guessing of other agent's actions
- The posterior of x' given x_i and y

$$x' \sim \mathcal{N}\left(\overline{\theta}, \frac{\sigma^2\tau^2}{\sigma^2+\tau^2} + \sigma^2 = \frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}\right)$$

- The posterior probability of $\overline{\theta}' = \frac{\sigma^2 y + \tau^2 x'}{\sigma^2+\tau^2} > k$ is

$$1 - \Phi\left(\frac{k + \frac{\sigma^2}{\tau^2}(k-y) - \overline{\theta}}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right)$$

- Apply this to every one, law of large numbers implies this gives the population of investing I

Equilibrium Characterization

- At equilibrium threshold $\overline{\theta} = k$, indifference $\overline{\theta} + l - 1 = k + l - 1 = 0$ implies

$$k = \Phi\left(\frac{k + \frac{\sigma^2}{\tau^2}(k-y) - k}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right) = \Phi\left(\frac{\sigma^2/\tau^2}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}(k-y)\right)$$

- Again, LHS is fundamental k, LHS is strategic uncertainty
- Define

$$\gamma(\sigma,\tau) \equiv \frac{\sigma^2}{\tau^4}\left(\frac{\sigma^2+\tau^2}{\sigma^2+2\tau^2}\right)$$

  the equilibrium threshold k satisfies

$$k = \Phi\left(\sqrt{\gamma}(k-y)\right)$$

- The only difference from previous example: introduce τ (precision of public signal) and σ (precision of private signal)
  - When $\tau \to \infty$, $\gamma \to 0$, so $k = \Phi(0 \cdot (k-y)) = 0.5$ always

Multiplicity of Equilibria (1)

- With public signal, not always true that we have unique equilibrium
- Key equation

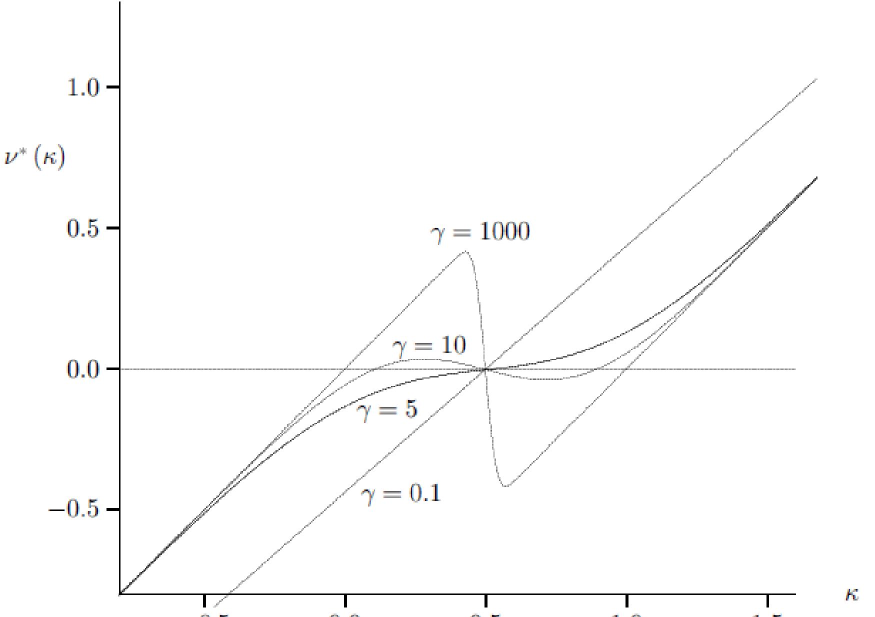
$$v(k) = k - \Phi\left(\sqrt{\gamma}\,(k - y)\right) = 0$$

  - Obviously $v(\pm\infty) = \pm\infty$ so there must exist solution
- Whether the solution is unique depends on whether $\gamma \leq 2\pi$
- Say $y = 0.5$. Then $k = 0.5$ is an equilibrium. Its derivative at that point is

$$v'(k = 0.5) = 1 - \sqrt{\gamma}\phi\left(\sqrt{\gamma}\,(0.5 - 0.5)\right) = 1 - \sqrt{\gamma}\frac{1}{\sqrt{2\pi}}$$

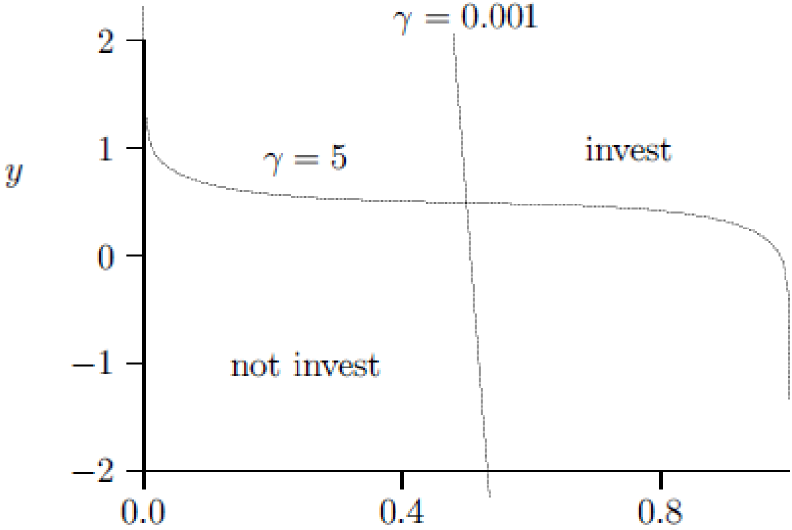
  so if $v'(k = 0.5) < 0$ then there must be at least three equilibria

Multiplicity of Equilibria (2)



Public versus Private Information: "Publicity Effect"

- The equilibrium satisfies $y = k - \frac{1}{\sqrt{\gamma}}\Phi^{-1}(k)$



The Publicity Multiplier

- The public signal, as observed by everybody, carry more weight in coordination and thus the equilibrium
- How much a player's private signal must adjust to compensate for a given change in public signal, so that he is indifferent between investing and not?
- Without strategic uncertainty effect, $\bar{\theta} = \frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} = k$ so

$$\frac{dx}{dy} = -\frac{\sigma^2}{\tau^2}$$

- With strategic effect, $\frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} = \Phi\left(\sqrt{\gamma}\left(\frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2} - y\right)\right)$ so

$$\frac{dx}{dy} = -\frac{\frac{\sigma^2}{\tau^2} + \sqrt{\gamma}\phi\left(\cdot\right)}{1 - \sqrt{\gamma}\phi\left(\cdot\right)}$$

- The smaller the τ (more precise public signal), the greater the γ, the larger the publicity multiplier
- When the publicity multiplier is too large, the multiplicity of equilibria comes back!

The Publicity Multiplier: Implications

- Public signals play a role in coordinating outcomes beyond its mere information content
  - Financial market "overreact" to announcement for Fed. Could be rational
  - Wall street journals or Washington post can affect the agent's belief on others
- Interesting empirical paper: Chwe (1998) on Superbowl advertisement
  - The price of advertisement increases more than linearly in the number of viewers
  - The premium may reflect "coordination value" of consumers' purchase decisions
  - Indeed, the premium mainly concentrate on coordination goods like "Apple Macintosh" or "Beer," but not in "Batteries"

Concluding Remarks

- The common critique of global games is the delicate information structure
  - There is some unobservable fundamental, and everybody gets a private signal about it
  - How about financial market which aggregates all the private information? (Angelatos-Werning)
  - Long-lived private signal is extremely hard to be used in other settings
- To its heart, global games work because it breaks "common knowledge"
- Private signal introduces heterogeneity to individual agents, and that goes a long way to pin down a unique equilibrium
- Is there another simple way to introduce heterogeneity without introducing private signal?
- He-Xiong Dynamic Debt Runs (RFS, 12), unique equilibrium in a run-like model
  - Time-varying fundamentals,
  - Debt holders are taking rollover actions at different times
  - Because fundamentals are changing at different times, you get heterogeneity!