Teaching notes for "Dynamic Debt Runs"

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Introduction: Halmilton-Jacobi-Bellman Equation (1)

- Ignore optimization problem: $V(y) = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} f(y_s)\, ds \,|y_t = y \right]$ s.t. $dy_t = \mu(y_t)\, dt + \sigma(y_t)\, dZ_t$
- Discrete-time Bellman equation

$$V(y) = \frac{1}{1+\rho} \left(f(y) + \mathbb{E}\left[V(y') \,|y \right] \right) \text{ s.t. } y' = y + \mu(y) + \sigma(y)\varepsilon$$

- Continuous-time, $V(y)$ can be written as

$$\begin{aligned}
V(y) &= \mathbb{E}_t \left[f(y_t)\, dt + \int_{t+dt}^\infty e^{-\rho(s-t)} f(y_s)\, ds \,|y_{t+dt} = y_t + \mu(y_t)\, dt + \sigma(y_t)\, dZ_t \right] \\
&= f(y)\, dt + e^{-\rho \cdot dt} \mathbb{E}_t \left[\int_{t+dt}^\infty e^{-\rho(s-t-dt)} f(y_s)\, ds \,|y_{t+dt} = y_t + \mu(y_t)\, dt + \sigma(y_t)\, dZ_t \right] \\
&= f(y)\, dt + e^{-\rho \cdot dt} \mathbb{E}_t \left[\mathbb{E}_{t+dt} \left(\int_{t+dt}^\infty e^{-\rho(s-t-dt)} f(y_s)\, ds \,|y_{t+dt} = y_t + \mu(y_t)\, dt + \sigma(y_t)\, dZ_t \right) \right] \\
&= f(y)\, dt + (1-\rho dt)\, \mathbb{E}_t \left[V(y_t + \mu(y)\, dt + \sigma(y_t)\, dZ_t) \right] \\
&= f(y)\, dt + (1-\rho dt)\, \mathbb{E}_t \left[V(y_t) + V'(y_t)\, \mu(y_t)\, dt + V'(y_t)\, \sigma(y_t)\, dZ_t + \frac{1}{2} V''(y_t)\, \sigma^2(y_t)\, dt \right] \\
&= f(y)\, dt + (1-\rho dt) \left[V(y) + V'(y)\, \mu(y)\, dt + \frac{1}{2} V''(y)\, \sigma(y)\, dt \right]
\end{aligned}$$

- $V(y)$ cancels. Collecting terms with order dt gives HJB equation.

Halmilton-Jacobi-Bellman Equation (2)

$$\underbrace{\rho V(y)}_{\text{required return}} = \underbrace{f(y)}_{\text{flow (dividend) payoff}} + \underbrace{V'(y)\,\mu(y) + \frac{1}{2}\sigma^2(y)\,V''(y)}_{\text{local change of value function (capital gain, long-term payoffs)}}$$

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- Introduce "control" x_t, choosing x to maximize value
- HJB with optimization (**local optimization** is enough!) is

$$\underbrace{\rho V(y)}_{\text{required return}} = \max_x \underbrace{f(y;x)}_{\text{flow (dividend) payoff}} + \underbrace{V'(y)\,\mu(y;x) + \frac{1}{2}\sigma^2(y;x)\,V''(y)}_{\text{local change of value function (capital gain, long-term payoffs)}}$$

  - Merton problem, $x = (c, \alpha)$: consumption c and portfolio α, affecting evolution of wealth W (state y)
  - $f(W;c) = u(c)$, $dW_t / W_t = -c_t dt + r dt + \alpha_t ((\mu - r)\,dt + \sigma dZ_t)$

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- Will see "stopping" control, and Poisson event (say intensity ϕ)

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$$\underbrace{+\phi(V(y;x,\text{post Poisson}) - V(y))}_{\text{local change of value function due to Poisson}}$$

Runs on Non-bank Financial Institutions

- Runs on the non-bank financial institutions was one of the main causes of the credit crisis of 2007-2008.
  - e.g., Bernanke (2008), Cox (2008), Geithner (2008), Brunnermeier (2009), Gorton (2008), Krishnamurthy (2009), and Shin (2009).
- The classic Diamond-Dybvig model on bank runs:
  - The simultaneous coordination problem among depositors leads to a self-fulfilling bank-run equilibrium.
- Global-games models of bank runs:
  - Depositors observe noisy private signals about bank fundamental: Rochet-Vives (2004) and Goldstein-Pauzner (2005)
  - Signal noise leads to strategic uncertainty and prevents multiple equilibria: Carlsson-van Damme (1993) and Morris-Shin (2003).
- Many questions about debt runs involve time-varying fundamentals:
  - How does a firm's asset price volatility affect its debt run risk?
  - Do credit lines mitigate runs?
  - Do longer debt maturities mitigate runs?

Dynamic Debt Runs

- A model with time-varying fundamental without noisy private signals:
  - Fundamental is time-varying and all creditors share the same public information about fundamental.
  - The firm uses a staggered debt structure, i.e., creditors make rollover decisions at different times.
- A unique threshold equilibrium:
  - each creditor rolls over or not based on current fundamental;
  - a "rat race" among creditors in choosing rollover thresholds.
- Results similar to static global-games models:
  - Severe runs on firms with weaker fundamentals, greater illiquidity.
- New results:
  - Higher volatility increases strategic uncertainty and thus exacerbates runs.
  - When fundamental volatility is sufficiently high, stronger credit lines and longer debt maturities can exacerbate runs.

A Brief Literature Review

- Static global games models:
  - Carlsson and van Damme (1993), Morris and Shin (1998)
  - Rochet and Vives (2004), Goldstein and Pauzner (2005)
- Dynamic coordination games based on higher order beliefs:
  - Abreu and Brunnermeier (2003), Chamley (2003), Angeletos, Hellwig, and Pavan (2007)
- Dynamic coordination games based on observable fundamentals:
  - Frankel and Pauzner (2000)
- Growing literature on modeling rollover risk:
  - Acharya, Gale, and Yorulmzer (2011, JF), Morris and Shin (2009), Brunnermeier and Oehmke (2013, JF), He and Xiong (2012, JF)

The Model Structure

- A firm holds a long-term risky asset by rolling over short-term debt.
- The environment of illiquid/imperfect capital markets:
  - The firm cannot find a single creditor (with deep pockets) to finance all the debt, and has to rely on a continuum of small creditors.
  - When some creditors choose to run, the firm needs to draw on unreliable credit lines.
  - The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.
- Two key assumptions:
  - The asset fundamental is time-varying and publicly observable
  - A staggered debt structure: coordination problem between creditors maturing at different times

Long-Term Asset

- The firm holds a long-term asset:
  - The asset generates constant cash flow rdt over a period dt.
  - At a Poisson arrival time τ_ϕ with intensity ϕ, the asset matures with a final payoff equal to τ_ϕ value of a publicly observable process:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dZ_t.$$

  - Why Poisson? Memoryless, time elapsed does not matter.
- Risk-neutral agents with discount rate ρ. Asset fundamental value:

$$\begin{aligned} F(y_t) &= \mathbb{E}_t\left[\int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi - t)} y_{\tau_\phi}\right] = \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y_t \\ \rho F(y) &= r + \mu y F'(y) + \frac{\sigma^2}{2} y^2 F''(y) + \phi(y - F(y)) \quad \text{HJB equation} \end{aligned}$$

  - y_t is the firm fundamental, hit by persistent shocks.
  - Our model ignores complications from private information.

Staggered Debt Financing

- Staggered structure and debt contract are exogenously given.

Staggered Debt Financing

- Staggered structure and debt contract are exogenously given.
- We assume a unit measure of short-term creditors (discount rate ρ).
  - Coupon flow payment rdt over $(t, t + dt)$.
  - $r > \rho$.
- A staggered debt structure:
  - Each contract matures with a probability of δdt, a la Calvo (1983).
  - In aggregate, δdt fraction of debt matures over $(t, t + dt)$.
  - This fraction is small and thus avoiding the Diamond-Dybvig type simultaneous coordination problem.
  - Rollover risk: during a contract lock-in period (say 3-month), other creditors might run.
- At τ_δ, an individual creditor decides to run or roll over.
  - Threshold strategy y_*: roll over if and only if $y \geq y_*$.
  - Assumption: only consider threshold strategies.

Debt Run and Liquidation

- Over $(t, t + dt)$, δdt fraction of contracts matures.
- If they choose to run, the firm needs to draw on its credit lines.
  - With prob $\theta \delta dt$, the credit lines fail, causing the firm to fail.
    - θ: unreliability of credit lines, or uncertainty of cash reserves.
    - Can also be interpreted as imperfect government bailout.
  - With prob $1 - \theta \delta dt$, the firm raises new fund and pays the creditors.
    - New debt takes the same contract form.

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- Early liquidation recovers $\alpha \in (0,1)$ of the fundamental value:

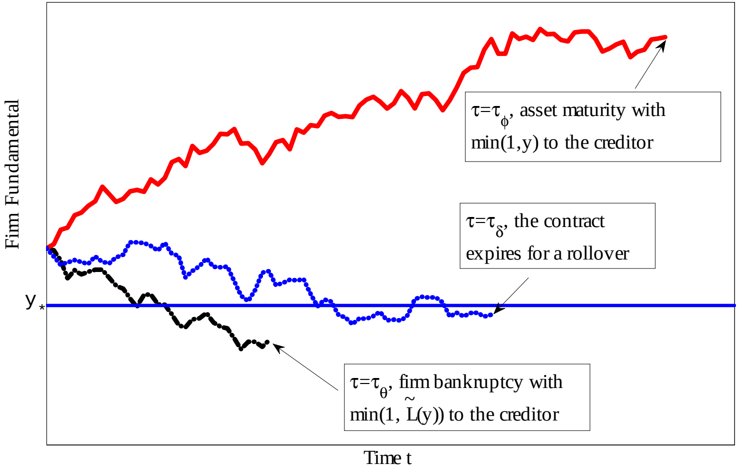
$$\widetilde{L}(y_t) = \alpha F(y_t).$$

  - Liquidation decision is irreversible, no partial liquidation.
- The firm's liquidation value, $\widetilde{L}(y)$, is equally divided among all creditors, including the running ones.
- The probability of firm failing by one's own run is tiny $(\theta\delta dt)\Rightarrow$ expected payoff from running is

$$\theta\delta dt \cdot \alpha F(y_t) + (1-\theta\delta dt)\cdot 1 = 1$$

Three Possible Paths for An Individual Creditor

- A creditor receives r until a random time $\tau = \min\left(\tau_\phi, \tau_\delta, \tau_\theta\right)$;
- Other creditors' rollover threshold y_*: rollover when $y > y_*$, run otherwise.



An Individual Creditor's Problem

- Given other creditors' threshold y_*, his value function is

$$
\begin{aligned}
V\left(y_t; y_*\right) \quad = \quad & \mathbb{E}_t \left\{ \int_t^{\tau} e^{-\rho(s-t)} r ds \right. \\
& + \underbrace{e^{-\rho(\tau-t)} \min\left(1, y_\tau\right) \mathbf{1}_{\{\tau=\tau_\phi\}}}_{\text{Top path, the asset matures and pays off}} \\
& + \underbrace{e^{-\rho(\tau-t)} \underset{\text{rollover or run}}{\max} \left\{1, V\left(y_\tau; y_*\right)\right\} \mathbf{1}_{\{\tau=\tau_\delta\}}}_{\text{Middle path, make the rollover decision when contract expires}} \\
& + \left. \underbrace{e^{-\rho(\tau-t)} \min\left(1, L + l y_\tau\right) \mathbf{1}_{\{\tau=\tau_\theta\}}}_{\text{Bottom path, the firm fails due to other creditors' run}} \right\}.
\end{aligned}
$$

Derivation of Equilibrium (1)

- How to derive $V(y; y_*)$, i.e., the individual creditor's continuation value, given y and **others** running at y_*?
- HJB equation

$$\begin{aligned}\rho V(y; y_*) &= \mu y V_y + \frac{\sigma^2}{2} y^2 V_{yy} + r + \phi\left[\min(1, y) - V(y; y_*)\right] \\ &\quad + \theta\delta \mathbf{1}_{\{y<y_*\}}\left[\min(L + ly, 1) - V(y; y_*)\right] \\ &\quad + \delta \max_{\text{rollover or run}}\left\{0, 1 - V(y; y_*)\right\}.\end{aligned}$$

- Potentially we need to solve for $V(y; y_*', y_*)$ where y_*' is the individual optimal response to y_*. We avoid this.

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- Potentially we need to solve for $V(y; y'_*, y_*)$ where y'_* is the individual optimal response to y_*. We avoid this.
- We show any (threshold) equilibrium must be symmetric, i.e., in equilibrium this individual creditor runs at y_* as well.
- So what is any individual's value if **everybody** runs at y_*, given y?
- We find an equilibrium if and only if
  - $V(y_*; y_*) = 1$;
  - $V(y; y_*)$, as a function of y, crosses 1 only once.
- Finally we show uniqueness of y_* s.t. $V(y_*; y_*) = 1$

Derivation of Equilibrium (2)

- From now, I ignore y_* in $V(y; y_*)$. only focus on y.
- So we have two regions and y can move back and forth.
  - If $y < y_*$, higher discount rate $\rho + \phi + (\theta + 1)\delta$, higher payoff flow

$$\begin{aligned} 0 &= \frac{\sigma^2}{2} y^2 V_{yy} + \mu y V_y - [\rho + \phi + (\theta + 1)\delta] V(y) \text{ (below)} \\ &\quad + \phi \min(1, y) + \theta\delta \min(L + ly, 1) + r + \delta; \end{aligned}$$

  - If $y \geq y_*$, lower discount rate $\rho + \phi$, lower payoff flow

$$0 = \frac{\sigma^2}{2} y^2 V_{yy} + \mu y V_y - (\rho + \phi) V(y) + \phi \min(1, y) + r. \quad \text{(above)}$$

- The solutions $V(y)$ have to be pasted together at y_*:

$$\begin{aligned} V(y = y_*^+) &= V(y = y_*^-) : \text{ value matching} \\ V'(y = y_*^+) &= V'(y = y_*^-) : \text{ smooth pasting} \end{aligned}$$

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- The solutions $V(y)$ have to be pasted together at y_*:

$$\begin{aligned} V\left(y = y_*^+\right) &= V\left(y = y_*^-\right) : \text{ value matching} \\ V'\left(y = y_*^+\right) &= V'\left(y = y_*^-\right) : \text{ smooth pasting} \end{aligned}$$

- Note: sometimes you see smooth-pasting as optimality condition (say, option exercising; default in Leland)
  - Here, smooth-pasting has nothing to do with optimality.
  - Value matching because y is continuous at y_*; smooth pasting because y can move back and forth at y_*

Derivation of Equilibrium (3)

- Fundamental eq. of (below): $\frac{1}{2}\sigma^2 x(x-1) + \mu x - [\rho + \phi + (1+\theta)\delta] = 0$, roots:

$$-\gamma_1 = -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 [\rho + \phi + (1+\theta)\delta]}}{\sigma^2} < 0,$$

$$\eta_1 = -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 [\rho + \phi + (1+\theta)\delta]}}{\sigma^2} > 1$$

and

$$V(y) = A_{below} + B_{below} y + C_{below} y^{-\gamma_1} + D_{below} y^{\eta_1}$$

- Fundamental eq. of (above): $\frac{\sigma^2}{2} x(x-1) + \mu x - (\rho + \phi) = 0$, roots

$$-\gamma_2 = -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 (\rho + \phi)}}{\sigma^2} < 0,$$

$$\eta_2 = -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 (\rho + \phi)}}{\sigma^2} > 1$$

and

$$V(y) = A_{above} + B_{above} y + C_{above} y^{-\gamma_2} + D_{above} y^{\eta_2}$$

Derivation of Equilibrium (4)

- A and B can be determined directly by setting $C = D = 0$, plugging, matching functions (just as I calculate the fundamental value $F(y)$)
- Two usual tricks to pin down C & D.
  - For below, $C_{below} = 0$ because $y^{-\gamma_1} \longrightarrow \infty$ when $y \longrightarrow 0$
  - For above, $D_{above} = 0$ because $y^{\eta_2} \longrightarrow \infty$ when $y \longrightarrow \infty$
- We need to determine D_{below} and C_{above}. But we have value-matching and smooth-pasting. Done!
  - D_{below} and C_{above} are solved as functions of y_*,
  - In paper, $\min(L + ly, 1)$ and $\min(y, 1)$ create a bit more complication

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  - D_{below} and C_{above} are solved as functions of y_*,
  - In paper, $\min(L + ly, 1)$ and $\min(y, 1)$ create a bit more complication
- So far, standard and useful (Leland models, investment models, real options)
- A great (intuitive) book: The art of smooth pasting by Dixit

Derivation of Equilibrium (5)

- Equilibrium y_* is defined as $V(y_*; y_*) = 1$.
- What is hard and requires certain technical skills is to show:
  - It is an equilibrium, i.e., if $V(y_*; y_*) = 1$, then $V(y; y_*)$, as a function of y, crosses 1 only once.
  - It is unique, i.e., there exists only ONE y_* so that $W(y_*) \equiv V(y_*; y_*) = 1$
    - We show $W(0) < 1$, $W(\infty) > 1$ and $W'(\cdot) > 0$
- Conclusion: exists a **unique** equilibrium threshold y_* s.t. $V(y_*; y_*) = 1$.
- Intuition:
  - Equilibrium uniquely defined in upper and lower dominance regions.
  - Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.
  - This is similar to global games, replacing y by private signal.

The Unique Monotone Equilibrium

- Strategic uncertainty originates from time-varying fundamental.
  - e.g., Frankel and Pauzner (2000).
  - In contrast to Carlsson and van Damme (1993) and Morris and Shin (1998), strategic uncertainty arises from noise in private signals.
- Requires a well spread-out fundamental process.
  - Does not rely on specific information structure and immune from information revealed by market prices, e.g., Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006).
- Difference from Frankel and Pauzner (2000).
  - Strategic complemetary exists in continuation values not in flow payoffs; deletion of dominated strategies not applicable.
  - We use a guess-and-verify approach.

Debt run externality

Each creditor's run imposes an externality on the other creditors who are locked in

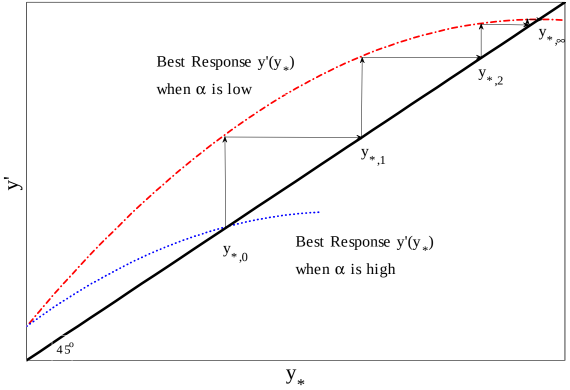
	Choice of maturing creditors		
	Run		Rollover
Possible firm outcomes	failed	survived	survived
Probability	$\theta\delta dt$	$1-\theta\delta dt$	1
Payoff of current maturing creditors	$\widetilde{L}(y)$	1	$V(y)$
Payoff of future maturing creditors	$\widetilde{L}(y)$	$V(y)$	$V(y)$

Static-Rollover Benchmark

- Suppose that the firm's debt all matures at time 0, and each creditor simultaneously decides whether to rollover into a perpetual debt contract.
  - Multiple self-fulfilling equilibria emerge if the current fundamental is in an intermediate region.
  - Runs cannot occur when the firm's current liquidation value is higher than its liability, i.e., if $F(y_t) > 1/\alpha$.

Rat Race after a Drop in Liquidation Value

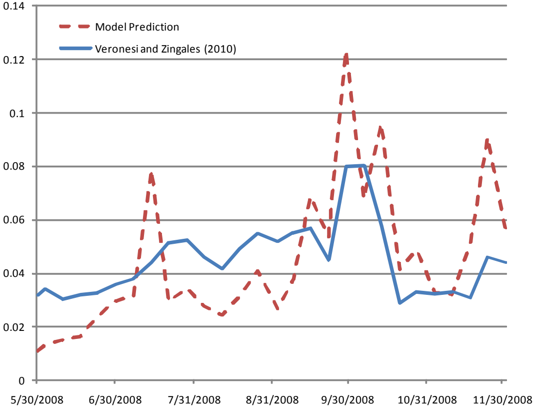
- Consider an unexpected drop in liquidation recovery rate α.
- A creditor's optimal response y' to other creditors' threshold y_*.



Calibrating Model Parameters

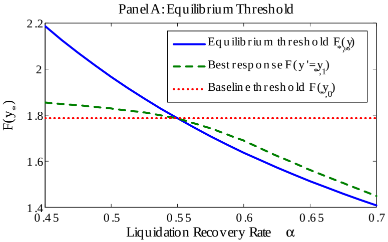
- We use a set of parameters for illustration.
  - Discount rate $\rho = 1.5\%$.
  - Asset cashflow $r = 7\%$; asset duration $1/\phi = 13$.
  - Asset's liquidation recovery rate $\alpha = 55\%$.
  - Asset's volatility $\sigma = 20\%$, growth rate $\mu = 1.5\%$, and current fundamental $y_0 = 1.4$.
  - Debt rollover frequency $\delta = 10$.
  - Unreliability of credit lines $\theta = 5$.

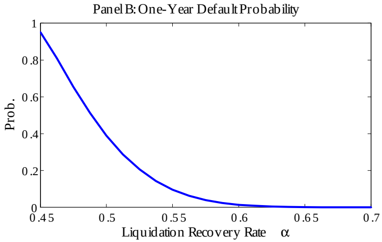
Predicting One Year Default Probability of Merrill Lynch



Effects of Liquidation Value

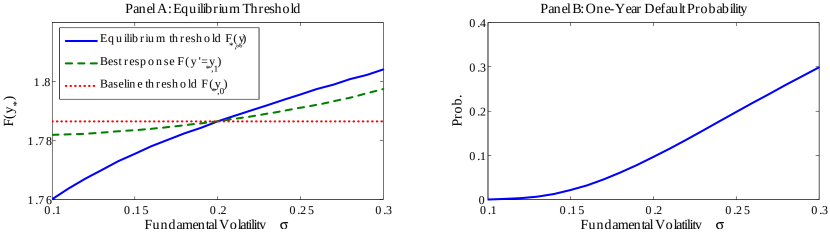
- Illiquidity exacerbates runs.
  - Similar to Rochet and Vives (2004).
- Threshold y_* sensitive to α.
  - Amplification effect by the rat race.





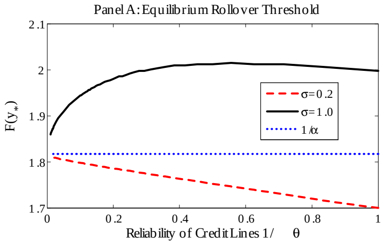
Effects of Fundamental Volatility

- Volatility affects each creditor in three channels:
  - Insolvency risk, causing y_* to increase with σ;
  - Rollover risk (strategic uncertainty), causing y_* to increase with σ;
  - Embedded option, causing y_* to decrease with σ.



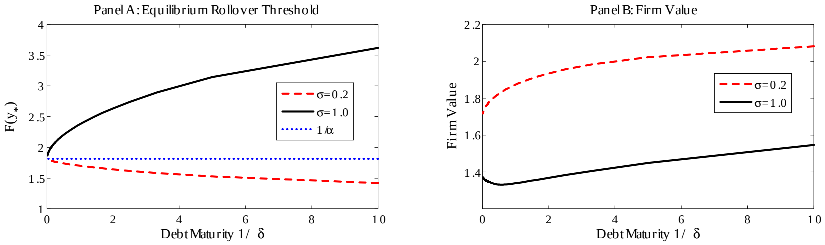
Effects of Credit Lines

- Credit lines can temporarily sustain a firm under runs.
  - Common intuition: stronger credit lines should deter runs.
- When volatility is sufficiently large, credit lines exacerbate runs because fundamental can deteriorates during the period the firm lives on credit lines.
  - Uncertain government bailouts can be counter productive.



Effects of Debt Maturity

- Common intuition: longer debt maturities mitigate runs.
- Two offsetting effects of longer maturities:
  - 1) the firm faces less frequent rollover with other creditors and thus less likely to fail under runs.
  - 2) internally, longer lock-in effect for each creditor, which motivates runs, especially severe when volatility is high.
- Longer maturities exacerbate runs when volatility is sufficiently high.
  - consistent with experience of runs on ABCP, e.g., Covitz, Liang, and Suarez (2009).



Further Discussion

- Synchronous vs Asynchronous Debt Structure
  - It is common for firms to spread out debt expirations.
  - The synchronous structure leads to more severe runs than the static-rollover benchmark when volatility is sufficiently high.
  - Which structure is optimal?
- Optimal Debt Maturity
  - Cheng and Milbradt (2010) extends our model to allow the firm switching b/w two projects: one with high growth and low volatility, the other with low growth and high volatility.
  - The optimal debt maturity trades off discipline on risk shifting and debt run risk.
- Spillover and Systemic Risk
  - When firms hold similar assets and face a downward sloping curve, runs on one firm can spill over to other firms.
  - Each firm's optimal debt structure and debt maturity depend on its own characteristics (fundamental volatility and asset illiquidity) and peer characteristics.

Conclusion

- We develop a dynamic model of debt runs.
  - Two basic ingredients: time-varying fundamental and staggered debt structure.
- The model highlights effects of volatility, debt maturity, and credit lines.
  - Fundamental volatility triggers strategic uncertainty (rollover risk.)
  - When volatility is sufficiently high, stronger credit lines and longer debt maturities exacerbate runs.
- Is readily extendable to address other dynamic corporate finance issues.
- Can be a building block in standard macro models to capture dynamic externalities.