Portability, salary and asset price risk: a continuous-time expected utility comparison of DB and DC pension plans

An Chen

University of Ulm

joint with Filip Uzelac (University of Bonn)

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Defined benefit and defined contribution plans

- Occupational pension plans
  - DB plan: the employee's pension benefit is determined by a formula which takes into account years of service and wages (or salaries) of the employee.
  - DC plan: sponsoring companies (and often also their employees) pay a promised contribution to an external pension fund. The pension payment is simply determined as the market value of the backing assets.
- Developing trend
  - US: the division between assets held in DB plans and DC plans was 60% versus 40% in 1987, and in 2007 this division was reversed
  - UK: final salary DB plans constituted 92% of all pension funds in 1979 and this number was reduced to 41% in 2005.

Tradeoffs between DB and DC plans

Portability risk: the risk not to have the ability to fully transfer years of credited service or accumulated benefits from one employer to another.

- cash equivalent loss, backloading loss
- playing a minor role in DC plans: a DC plan can be easily ported between job switchings
- driving source of risk in single employer DB pension plans: DB plan holders lose part of their benefits after changing jobs
  - US: workers in the US hold 10 or 11 jobs during their working lives (Hall (1982))
  - UK: fewer than 5% of workers remain with the same employer and that the average worker in the UK changes jobs about six times in a life time (Blake (2000))

Tradeoffs between DB and DC plan

- Investment risk
  - DB plan: the sponsoring company is responsible for providing promised (future) pension benefits to the employees
    - the company decides about investment policies in a pension fund
    - the company bears the entire investment risks in a DB plan
  - DC plan: The company does not ensure a promised pension payment to the employees
    - The employees bear the entire investment risks
- Income risk:
  - DB plans: The benefits are directly linked to the salary
  - DC plans: Contributions are usually a fraction of the salary

Contributions of the paper

- We formally model the above tradeoffs between DB and DC plans in the presence of stochastic wages, job moving and asset price risk
- We compare the DB with the DC plan in an expected utility-based framework
  - power utility
  - mean-shortfall
  - mean-downside deviation

⇒ The expected utilities for DB plans are determined analytically and for DC plans by Euler discretization scheme

Main results

- We compute the critical job switching intensity (from the DB plan) such that the beneficiary is indifferent between the DB and the DC plan.
- We confirm some results in existing literature (e.g. Cocco and Lopes (2004), Samwick and Skinner (2004), Poterba et al. (2006) and Siegman (2010)):
  - DB plan is preferred by an older beneficiary.
  - Adjusting the contribution of the beneficiary to a higher level makes the DC plan more attractive.
  - A rise in the salary growth rate increases the attractiveness of the DB plan, while a higher salary volatility decreases its attractiveness
  - Equity holding in a DC plan plays a substantial role in the relative attractiveness of the retirement plans, but there does not exist a clear dominating strategy for all the preferences.

...Main results

- One striking result which is inconsistent with the existing literature
  - The attractiveness of the DB plan can decrease in the level of risk aversion
  - The DC plan can become most attractive for the most risk-averse power-beneficiary.
- Rationales behind this result:
  - Portability risk is modelled as a jump risk which generates much disutility for very risk-averse beneficiaries.
  - A DC plan can offer better diversification because it is not purely driven by the income risk (asset risk plays a decisive role too).

Agenda

- Introductions and Motivations ($\surd$)
- Pension benefits of DB and DC plans
  - Portability risk, salary and asset price risk
- Expected utility from DB and DC plans
- Numerical results
- Concluding remarks

Salary process and job moves

- Final salary DB plan with a retirement date T
- The salary process is assumed to follow a geometric Brownian motion (GBM) (c.f. Topel and Ward (1992))

$$dS_t = \mu_S(t)S(t)dt + \sigma_S S(t)dW^S(t), \quad S_0 = s$$

  - $\mu_S(t)$: deterministic and possibly time-varying drift (trend in the salary)
  - $\sigma_S > 0$: the constant volatility
  - W^S is a standard Brownian motion under the real world probability measure P

Portability loss

- The number of job moves is modeled as an (in)homogenous Poisson process $N(t)_{t\geq 0}$ with intensity $\lambda_t \geq 0$.
  - $N(t)$ assumed to be independent of W_t^S
- Pension adjusted salary process $(\tilde{S}_t)_{t\geq 0}$ as a jump diffusion

$$d\tilde{S}_t = \mu_S(t)\tilde{S}_t dt + \sigma_S \tilde{S}_t dW_t^S + \tilde{S}_{t-} dQ_t, \quad \tilde{S}_0 = S_0,$$

  - $t-$: time immediately before a job move
  - $Q(t) = \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process, where $Y_i, i = 1, ...N(t)$ are percentage changes in the pension adjusted salary process when the employee changes his job and assumed to be deterministic

...Portability loss

- Pension adjusted salary process (see e.g. Shreve (2004))

$$\tilde{S}_T = s \exp\left\{\left(\int_0^T \mu_S(u)\,du - \frac{1}{2}\sigma_S^2 T\right) + \sigma_S W_T^S\right\}$$
$$\exp\left\{\sum_{j=1}^{J} (N(t_j) - N(t_{j-1})) \ln(1 + Y_j)\right\},$$

  - $1 + Y_i = \beta_i$, $0 < \beta_i < 1$
  - β is a piecewise constant but time-varying function (with $t_J = T$)

$$\beta = \left\{\beta_j, \quad t_j \leq t \leq t_{j+1}, j = 1...J\right\}$$

  where J denotes the number of career periods.

Final payment of a DB plan

- The DB plan we consider is a final salary DB plan.
  - We assume that the employee receives a continuous annuity $b(T) = \alpha \tilde{S}_T$.
  - To make the DB plan and the DC plan comparable we convert the life annuity of the DB plan into a lump sum $B(T)$

$$B(T) = \int_T^{\infty} b(T)\, e^{-r\,(\tau - T)}\, p_\tau \, d\tau,$$

    - p_τ is a continuous survival distribution function and τ the time of death. Under exponential distribution, $B(T)$ becomes

$$\begin{aligned} B(T) &= \int_T^{T^1} b(T)\, e^{-r\,(\tau - T)}\, e^{-\mu(\tau - T)} d\tau \\ &= \frac{b(T)}{r + \mu} \left[1 - e^{-(r+\mu)\,(T^1 - T)}\right] := b(T)\, a(T), \end{aligned}$$

    where $a(T)$ can be interpreted as the annuity factor.

DC plans: underlying backing assets

- The pension benefit of a DC plan is simply determined as the market value of the backing assets.
- Two assets in our economy:
  - a riskless asset F with price process $(F(t))_{t\geq 0}$

$$dF(t) = rF(t)dt, \quad F_0 = 1$$

  - a risky non-dividend-paying asset A with price process $(A_t)_{t\geq 0}$

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t), \quad A_0 = a$$

  where $d[W, W^S]_t = \rho\, dt$ and $d[W, N]_t = 0$.

DC plans: pension income process

- Pension income process

$$dX_t = X(t)\left[(r + \pi\,\sigma\,\theta)\,dt + \pi\,\sigma\,dW(t)\right] + cS(t)dt,\ X_0 = c\,S_0,$$

where $\theta = \frac{(\mu - r)}{\sigma}$ denotes the market price of risk.

  - Employee's investment follows a rebalancing strategy: a constant fraction π, $0 \leq \pi \leq 1$ which will be invested in the risky asset A and the remaining fraction $(1 - \pi)$ is invested in the riskless asset F.
  - Employee and employer contribute continuously the amount $c(t)\,S(t)\,dt$ to the employee's pension account and these contributions are also invested continuously over time.

- The pension benefit coincides with the terminal value of the DC account X_T

Matching contribution

- In order to make the pension outcomes comparable we need to ensure that the employee bears the same costs in the two pension retirement plans.
  - A way to achieve this is to assume that the employee contributes continuously the amount $q\, S_t\, dt$, where $q \leq c$, in either retirement plan.
  - Assumption: the replacement rate in DB plan is split into a replacement rate α^{ER}, $0 \leq \alpha^{ER} \leq 1$ and a replacement rate α^{EE}, $0 \leq \alpha^{EE} \leq 1$: $\alpha = \alpha^{ER} + \alpha^{EE}(q)$.
  - We link the employee's contribution rate to his replacement rate by requiring that

$$E\left[q \int_0^T S_u\, du\right] = a(T)\, \alpha^{EE}\, E\left[S_T\right],$$

...Matching contribution

- The total contribution rate c in the DC plan is determined by
  - taking the above specified employee's contribution rate q
  - assuming that the employer simply matches the employee's contribution in the DC plan,
  - $c = \delta\, q$, where $\delta \geq 1$ denotes the matching factor.

Utility functions

- Power utility (with a constant CRRA γ)

$$u(x) = \begin{cases} \frac{1}{1-\gamma} x^{1-\gamma}, & \gamma \neq 1 \\ \ln x, & \gamma = 1 \end{cases}$$

- Mean-shortfall (MS)

$$u(x) = \begin{cases} x, & x \geq R\,, \\ -\eta_1 (R - x), & x < R, \end{cases}$$

- Mean-downside-deviation (MDD)

$$u(x) = \begin{cases} x, & x \geq R \\ -\eta_2 (R - x)^2, & x < R, \end{cases}$$

Certainty equivalents

- Power utility (with a constant CRRA γ)

$$CE(x) = \begin{cases} (1-\gamma)\Big(E[u(x)]\Big)^{\frac{1}{1-\gamma}}, & \gamma \neq 1 \\ \exp\Big\{E[u(x)]\Big\}, & \gamma = 1 \end{cases}$$

- Mean-shortfall (MS)

$$CE(x) = \begin{cases} E[u(x)] + R, & E[u(x)] \geq 0\,, \\ R - \Big(-\frac{E[u(x)]}{\eta_1}\Big), & E[u(x)] < 0. \end{cases}$$

- Mean-downside-deviation (MDD)

$$CE(x) = \begin{cases} E[u(x)] + R, & E[u(x)] \geq 0\,, \\ R - \sqrt{-\frac{E[u(x)]}{\eta_2}}, & E[u(x)] < 0. \end{cases}$$

Expected utility of DB plan under power utility

Expected utilities for the DB plans can be determined analytically.

The expected utility for the power utility function is given by

$$E[u(B_T^{DB})] = \frac{1}{1-\gamma}\,(\alpha\, s\, a(T))^{1-\gamma}\ \exp\left\{(1-\gamma)\,(\sum_{j=1}^{J}\mu_{S,j}(t_j - t_{j-1}) - \frac{1}{2}\sigma_S^2\, T) + \frac{1}{2}(1-\gamma)^2\,\sigma_S^2\, T\right\}$$

$$\cdot\ \exp\left\{\sum_{j=1}^{J}\lambda_j\,(t_j - t_{j-1})\,(e^{(1-\gamma)\,\ln(\beta_j)} - 1)\right\}.$$

For $\gamma = 1$ (log utility), we obtain

$$E[u(B_T^{DB})] = \ln(a(T)\,\alpha\, s) + \left(\sum_{j=1}^{J}\mu_S(t_j - t_{j-1}) - \frac{1}{2}\,\sigma_S^2\, T\right) + \sum_{j=1}^{J}\lambda_j(t_j - t_{j-1})\,\ln(\beta_j)\,.$$

Numerical results

- Parameter choices

$$\alpha = 0.2,\ \alpha^{ER} = 0.15,\ a = 0.0005,\ \delta = \frac{3}{2},\ S_0 = 1000,$$
$$T = 25,\ T^1 = 30,\ \mu_S = 0.015,\ \sigma_S = 0.13,\ \beta = 0.95,$$
$$r = 0.02,\ \mu = 0.055,\ \sigma = 0.25,\ \rho = 0,\ R = 5\,c\,s\,e^{rT}$$

- The employee enters the retirement plan at the age of 40 and he retires at 65.
- The replacement rate coming from the employee's contributions is $\alpha^{EE} = 0.05$. $\rightarrow$ the employee's contribution rate q is approximately 5.2%.
- λ^*: Indifference job switching intensity
  - $\lambda \leq \lambda^*$: DB plan more attractive than DC
  - A higher value of λ^* implies that the DB plan becomes more attractive

Indifference job switching intensity

Utility	Risk aversion	$\pi = 0.4$	$\pi = 0.57$	$\pi = 0.75$	$\pi = 0.9$
CRRA	$\gamma = 1$	0.3423	0.3169	0.3058	0.3088
	$\gamma = 2$	0.2540	0.2690	0.3087	0.3673
	$\gamma = 4$	0.0897	0.1724	0.3052	0.4309
LA	$\eta_1 = 2.25$	0.3068	0.2532	0.1929	0.1349
	$\eta_1 = 5$	0.2965	0.2504	0.1890	0.1399
DD	$\eta_2 = 2.25$	0.3526	0.3121	0.2831	0.2958
	$\eta_2 = 5$	0.3061	0.2801	0.2821	0.3442

Values of λ^* for the benchmark case.

Indifference job switching intensity: Effect of β

Utility	Risk aversion	$\pi = 0.4$	$\pi = 0.57$	$\pi = 0.75$	$\pi = 0.9$
CRRA	$\gamma = 1$	0.2721	0.2533	0.2413	0.2441
	$\gamma = 2$	0.1989	0.2089	0.2431	0.2857
	$\gamma = 4$	0.0631	0.1268	0.2288	0.3374
LA	$\eta_1 = 2.25$	0.2503	0.2032	0.1531	0.1102
	$\eta_1 = 5$	0.2501	0.2019	0.1528	0.1145
DD	$\eta_2 = 2.25$	0.2742	0.2447	0.2260	0.2321
	$\eta_2 = 5$	0.2360	0.2147	0.2179	0.2583

Values of λ^* for a piecewise constant and U-shaped portability loss size with $\beta = [0.95 \quad 0.9 \quad 0.99]$ (original $\beta = 0.95$).

Indifference job switching intensity: Effect of μ_S

Utility	Risk aversion	$\pi = 0.4$	$\pi = 0.57$	$\pi = 0.75$	$\pi = 0.9$
CRRA	$\gamma = 1$	0.3500	0.3257	0.3118	0.3148
	$\gamma = 2$	0.2601	0.2733	0.3164	0.3692
	$\gamma = 4$	0.0974	0.1797	0.3068	0.4370
LA	$\eta_1 = 2.25$	0.3247	0.2695	0.2032	0.1506
	$\eta_1 = 5$	0.3195	0.2687	0.2021	0.1595
DD	$\eta_2 = 2.25$	0.3773	0.3324	0.2972	0.2917
	$\eta_2 = 5$	0.3344	0.3050	0.2895	0.3442

Values of λ^* for a piecewise constant and decreasing salary trend with $\mu_S = [0.0225 \quad 0.0175 \quad 0.01]$ (original $\mu_S = 0.015$).

Certainty equivalent ratio

Utility	Risk aversion	$\pi = 0.4$	$\pi = 0.57$	$\pi = 0.75$	$\pi = 0.9$
CRRA	$\gamma = 1$	1.1707	1.1339	1.1216	1.1185
	$\gamma = 2$	1.0430	1.0615	1.1254	1.2099
	$\gamma = 4$	0.8257	0.9379	1.1188	1.3342
LA	$\eta_1 = 2.25$	1.1179	1.0388	0.9632	0.9088
	$\eta_1 = 5$	1.1105	1.0256	0.9604	0.9150
DD	$\eta_2 = 2.25$	1.2814	1.1812	1.1223	1.1583
	$\eta_2 = 5$	1.2350	1.1616	1.1494	1.3493

Values for the certainty equivalent ratio $\frac{CE^{DB}}{CE^{DC}}$ for a piecewise constant and decreasing job switching intensity $\lambda = [0.3 \quad 0.2 \quad 0.1]$.

Conclusions

- The present paper models some main risks born in DB and DC plans: portability, salary and asset price risk
- We make comparisons between DB and DC plans by analyzing the expected utility of the pension beneficiary under three preferences: power utility, mean-shortfall and mean-downward-deviation preferences.
  - We confirm some results in the existing literature, particularly our model further indicates that portability losses considerably reduce the relative attractiveness of the DB plan.
  - More surprisingly, the attractiveness of the DB plan can decrease in the level of risk aversion.

Possible extensions

- One could further model the decumulation phase
- One could also include endogenous or strategic job moves and unemployment in our setup by allowing the salary process to have jumps.
- One could allow the beneficiary to have a combination of both a DC and DB pension plan or to change the pension plan at some time in his career.