Smooth Trading with Overconfidence and Market Power

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Main Idea

We present a dynamic model of informed trading which derives endogenously the speed with which traders trade and study the properties of markets when all traders smooth out their trading.

We model a trading game of oligopolistic traders with overconfidence and market power, where speed of trading is governed by trade-off:

- Reducing market impact motivates traders to trade slowly.
- Decaying private information motivates them to trade quickly.

Practical Implications

- Model explains why dumping large quantities on the market too quickly has large temporary price impact. This is relevant for understanding flash crashes.
- Model explains why even though prices immediately reveal average signal of traders, traders continue to trade in equilibrium.

Key Feature

We develop an equilibrium model where speed matters. For each trader, **price** is linear function of **traders inventory** (permanent price impact) and **derivative of inventory** (temporary impact).

In contrast, the speed of trading usually plays a limited role in most models. Price impact costs of changing inventory levels continuously by a given amount usually do not depend on the derivative of the trader's inventory levels. There is no temporary price impact.

Speed of Trading and Kyle (1985)

- The **informed trader** privately observes the liquidation value of a risky asset and spreads large trades out over time taking his market impact into account.
- **Noise traders** dump quantities into the market.
- In continuous time version, order flow consists of the "smooth" order flow ($\sim dt$) from informed trader and the "diffusion" order flow ($\sim dB_t$) from noise traders.
- Competitive **market makers** provide liquidity for orders of all sizes X by offering linear supply curve $P(X) = P_- + \lambda \cdot X$ with the constant market depth λ and break-even.
- Equilibrium prices follow a martingale.

Speed of Trading and Kyle (1985)

In Kyle (1985), the speed of trading is important neither for informed trader nor noise trader:

- The **informed trader** trades smoothly, but his profits do not depend on the rate of trading.
- The **noise trader** would be better off by smooth out their trading over time, as he would walk up or down the demand curve as a price-discriminating monopolist.

Related Dynamic Theoretical Papers

- Vayanos (1999): Oligopolistic traders receive random uncorrelated shocks to endowments and trade to share the risks. Adverse selection due to trader's private information about hedging needs leads to smoothing of trading. Speed of trading based on risk aversion (impatience).
- Du and Zhu (2013): Traders have private values, smooth out trading. Adverse selection due to private information about private values leads to smoothing of trading.

Our paper differs in that private information is correlated, implying private information decays as other traders acquire more information. It also implies a trade-off between trading slowly due to price impact and trading fast due to information decays.

Model Overview

We consider dynamic oligopoly model of informed trading in continuous time, to make transparent the idea that each trader trades "smoothly."

- **Overconfidence:** Trade based on "relative" agreement to disagree. Each trader believes his flow of information is more precise than other traders believe it to be.
- **Market Power:** Each trader learns from prices and restricts trading to reduce impact.
- **Symmetry:** No noise traders; no market makers; no rational uninformed traders.

Our model is a fully-fledged dynamic version of Kyle (1989), but trading is based on **agreement-to-disagree**, not noise trading.

Continuous-Time Model: Assumptions

- There are N risk-averse oligopolistic traders, who trade a risky asset with a zero net supply against a risk-free asset.
- Risk-free rate is r.
- A risky asset pays out dividends $D(t)$ growing at a rate $G^*(t)$.

$$dD(t) = -\alpha_D \cdot D(t) \cdot dt + G^*(t) \cdot dt + \sigma_D \cdot dB_D,$$

$$dG^*(t) = -\alpha_G \cdot G^*(t) \cdot dt + \sigma_G \cdot dB_G.$$

Dividends D are observable. Growth rate G^* is unobservable.

Continuous-Time Model: Information Flow

- Let $E_t^n\{\dots\}$ and $var_t^n\{\dots\}$ use all information and n's beliefs:

$$G_n(t) := E_t^n\{G^*(t)\}, \quad \Omega := var_t^n\left\{\frac{G^*(t)}{\sigma_G}\right\}$$

- Each trader observes the **public information** flow $dI_0(t)$ from dividends ($\tau_0 = \Omega \cdot \sigma_G^2/\sigma_D^2$):

$$dI_0(t) = \tau_0^{1/2} \cdot \frac{G^*(t)}{\Omega^{1/2}\sigma_G} \cdot dt + dB_0.$$

Continuous-Time Model: Information Flow

- Each trader n has continuous flow of **private information** $dI_n(t)$ about the unobserved growth rate $G^*(t)$:

$$dI_n(t) = \tau_n^{1/2} \cdot \frac{G^*(t)}{\Omega^{1/2}\sigma_G} \cdot dt + dB_n, \qquad n = 1, \ldots, N.$$

- Trader n infers from prices the **average of other signals**

$$I_{-n} := \frac{1}{N-1}\Sigma_{m \neq n} I_m(t).$$

- Each trader n thinks that his own information has high precision $\tau_n = \tau_H$ and other traders have low precision $\tau_m = \tau_L$, $m \neq n$, with $\tau_H > \tau_L \geq 0$.

Continuous-Time Model: Bayesian Updating

- Trader n constructs signals $H_i, i = 0, ..N$ as weighted average of information flow:

$$H_n(t) := \int_{u=-\infty}^{t} e^{-(\alpha_G+\tau)\cdot(t-u)} \cdot dI_n(u),$$

$$\textit{where} \quad \tau = \tau_0 + \tau_H + (N-1)\cdot\tau_L, \qquad \Omega^{-1} = 2\cdot\alpha_G + \tau.$$

$$G_n(t) = \sigma_G \cdot \Omega^{1/2} \cdot \left(\tau_0^{1/2} \cdot H_0(t) + \tau_H^{1/2} \cdot H_n(t) + \sum_{m \neq n} \tau_L^{1/2} \cdot H_m(t) \right)$$

- The importance of each bit of information about G^* decays exponentially at a rate $\alpha_G + \tau$, the same for every trader.

Continuous-Time Model: Optimization

Each trader n chooses consumption path c_t and trading rate x_t to maximize CARA utility function $U(c_t) = -e^{-A \cdot c_t}$,

$$V(M, S, D, H_0, H_n, H_{-n}) = \max_{\{c_t, x_t\}} \; E_t \left[\int_{s=t}^{\infty} -e^{-\rho(s-t)} \cdot U(c_s) \cdot ds \right].$$

Inventory: $dS(t) = x_t \cdot dt.$

Cash: $dM(t) = (r \cdot M(t) + S(t) \cdot D(t) - c_t - P(x_t) \cdot x_t) \cdot dt.$

Dividends: $dD(t) = -\alpha_D \cdot D(t) \cdot dt + G^*(t) \cdot dt + \sigma_D \cdot dB_D.$

Information Flow Dynamics: $d\hat{H}_n$ and $d\hat{H}_{-n} = \sum_{m \neq n} d\hat{H}_m$ incorporate H_0 into H_n.

The Transversality Condition:

$$\lim_{t \to +\infty} E[e^{-\rho t} V(M(t), S(t), D(t), H_0(t), H_n(t), H_{-n}(t))] = 0.$$

Conjectured Linear Strategies

- Trader n conjectures the other $N-1$ traders, $m = 1, \ldots N, m \neq n$, submit symmetric linear demand schedules of the form

$$X_m(t) = \gamma_D \cdot D(t) + \gamma_H \cdot \hat{H}_m(t) - \gamma_S \cdot S_m(t) - \gamma_P \cdot P(t).$$

- Let $x_n(t) := dS_n(t)/dt$. Assume continuous single price auction like Kyle (1989). Market clearing quantities are *derivatives* of inventories.

$$P(x_n(t)) = \frac{\gamma_D}{\gamma_P} \cdot D(t) + \frac{\gamma_H}{\gamma_P} \cdot \hat{H}_{-n}(t) + \frac{\gamma_S}{\gamma_P} \frac{1}{N-1} \cdot S_n(t) + \frac{1}{(N-1)\gamma_P} \cdot x_n(t).$$

- All traders exercise monopoly power optimally, with "no regret pricing," taking into account how their own trading affects other traders' beliefs about H_{-m} inferred from "fully revealing" prices.

Conjecture: Quadratic Value Function

Value function depends on nine *psi*-parameters:

$$
\begin{aligned}
V(M_n, S_n, D, \hat{H}_n, \hat{H}_{-n}) = \\
&- \exp\Big[\psi_0 + \psi_M \cdot M_n + \psi_{SD} \cdot S_n \cdot D \\
&\quad + \frac{1}{2}\psi_{SS} \cdot S_n^2 + \psi_{Sn} \cdot S_n \cdot \hat{H}_n + \psi_{Sx} \cdot S_n \cdot \hat{H}_{-n} \\
&+ \frac{1}{2}\psi_{nn} \cdot \hat{H}_n^2 + \frac{1}{2}\psi_{xx} \cdot \hat{H}_{-n}^2 + \psi_{nx} \cdot \hat{H}_n \cdot \hat{H}_{-n}\Big].
\end{aligned}
$$

Note: Wealth does not enter value function; instead have separate components cash $M(t)$ and security holdings $S(t)$.

Equilibrium problem is to solve for nine ψ-parameters and four γ-parameters which sustain a symmetric equilibrium.

Theorem: Symmetric Linear Flow Equilibrium

- Equilibrium defined by solution to six mostly quadratic polynomials in six unknowns. Numerical results consistent with "derived" existence condition

$$\frac{\tau_H^{1/2}}{\tau_L^{1/2}} > 2 + \frac{2}{N-2}.$$

- Market clearing quantities are derivatives of inventories, implying partial adjustment towards "target inventory", depending on constant C_L,

$$x_n(t) = dS_n(t)/dt = \gamma_S \cdot \Big(C_L \cdot (H_n(t) - H_{-n}(t)) - S_n(t) \Big).$$

- Equilibrium price is dampened average, i.e., $0 < C_G < 1$, of buy and hold valuations based on Gordon's growth formula:

$$P^*(t) = \frac{D(t)}{r + \alpha_D} + \frac{C_G \cdot \frac{1}{N} \sum_{n=1}^{N} G_n(t)}{(r+\alpha_D)(r+\alpha_G)}.$$

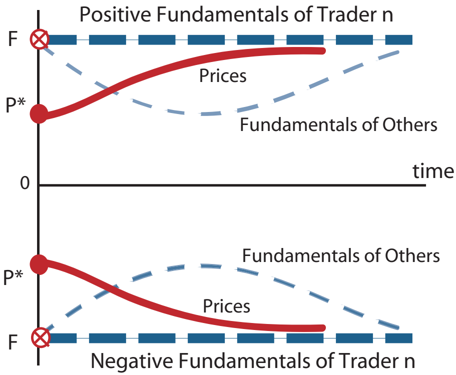
Implications: Prices Adjust Immediately

- Equilibrium price immediately reveals average private signal.
- Equilibrium price is similar to Gordon's formula with growth rate being the average of all estimates G_n:

$$P^*(t) = \frac{D(t)}{r + \alpha_D} + \frac{C_G \cdot \frac{1}{N} \sum_{n=1}^{N} G_n(t)}{(r + \alpha_D)(r + \alpha_G)}.$$

- Since $C_G < 1$, prices are not equal to the average estimates of fundamentals. In the model with agreement to disagree, prices are less sensitive to average signal than if there were a representative agent who observed the average of all signals.

$C_G < 1$: "Dampening Effect"



Even if all traders agree on the fundamental value today, each trader believes that other traders will find out their information is incorrect in the future. Despite fundamentals, each trader "agrees" on a dampened equilibrium price. This effect does not exist in one-period model.

Keynes (1936) and "Dampening Effect"

Keynes:

- *"It is not sensible to pay 25 for an investment of which you believe the prospective yield to justify value of 30, if you also believe that the market will value it at 20 three months hence."*
- When a trader purchases a stock, he is *"attaching his hopes, not so much to its prospective yield, as to a favorable change in the conventional basis of valuation."*

In our dynamic model, traders also seem to be preoccupied with **"short-term price dynamics"** rather than **"hold-to-maturity"** values. The market internalizes overconfidence of traders and corrects equilibrium prices, so instead price deviations are dampened.

Implications: Quantities Adjust Slowly

- Market power with private information makes quantities adjust slowly. Trading on information continues after signals revealed in prices.
- Trading strategy is "partial adjustment" towards "steady-state" target inventory.

$$x_n^*(t) = \gamma_S \cdot \Big(C_L \cdot (H_n(t) - H_{-n}(t)) - S_n(t) \Big).$$

- The speed of execution of trades (γ_S) depends on the degree of competition and disagreement. Size of target inventories C_L depends risk-aversion, the degree of competition and disagreement.

Price Impact Functions and Execution Cost

- From perspective of each trader, price is linear function of traders inventory (permanent price impact) and derivative of inventory (temporary impact).

$$P(S(t), x(t)) = \lambda_0 + \lambda_S \cdot S(t) + \lambda_x \cdot x(t),$$

  where constants $\lambda_S = \gamma_s \cdot \lambda_x$ and $dS(t) = x(t) \cdot dt$.

- The cost of buying $\tilde{B}$ over time T at uniform rate:

$$E\{\tilde{C}\} = \left(\lambda_S + \frac{\lambda_x}{T/2}\right) \cdot \frac{\tilde{B}^2}{2}.$$

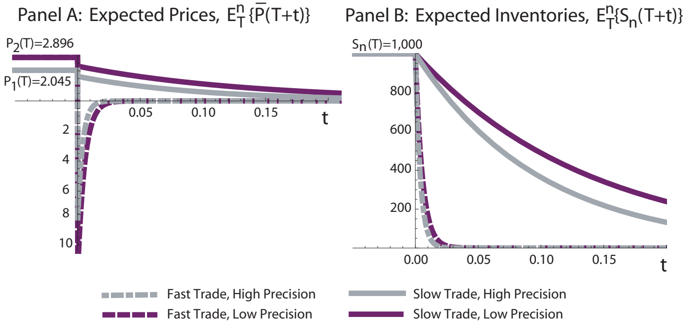
- The cost of buying $\tilde{B}$ at a constant rate γ:

$$E\{\tilde{C}\} = \left(\lambda_S + \gamma \cdot \lambda_x\right) \cdot \frac{\tilde{B}^2}{2}.$$

  Papers on optimal execution study optimal strategies given exogenously specified price impact functions, with a lot of attention on costs due to temporary price impact(Grinold and Kahn (1999) and Almgren and Chriss (2000)).

Fast Trading Can Lead to Flash Crashes

Price path and inventories deviate from the equilibrium if the speed of trading γ deviates from the equilibrium rate γ_S:



$$E^n_T\{\bar{P}(T+t)\} = -\frac{\gamma - \gamma_S}{(N-1)\gamma_P} \cdot e^{-\gamma t} \cdot S_n(T), \qquad E^n_T\{\bar{S}_n(T+t)\} = e^{-\gamma \cdot t} \cdot S_n(T).$$

Speed of Trading and Price Patterns

- Speeding up selling leads to sharp price decline following by V-shaped recovery.
- Slowing down leads to price increase and then convergence to equilibrium level.
- Speeding up execution five times leads to five times bigger price decline relative to the equilibrium price change.
- Speed of recovery depends on price resilience, $\alpha_G + \tau$.
- Selling may occur after prices crash, while the market recovers.

Black (1995)

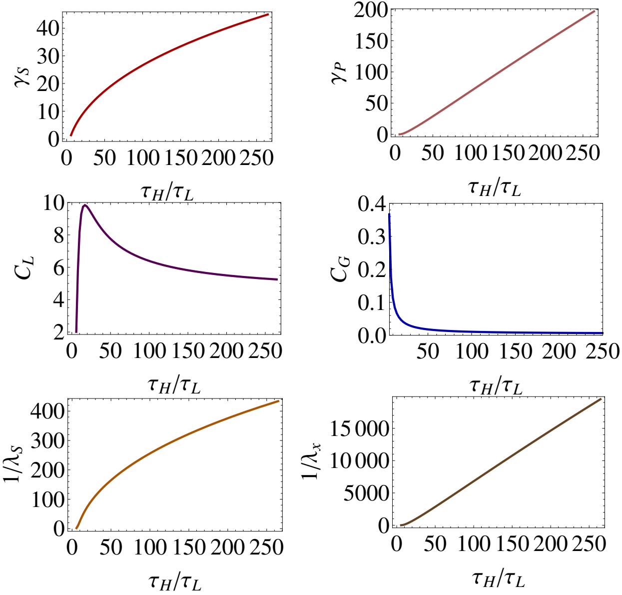
We formulate in a precise mathematical model ideas that Fischer Black has formulated intuitively in his last paper called "Equilibrium Exchanges," where he outlined his thoughts about how would people trade in the equilibrium, if there are no restrictions on trading strategies or on exchanges.

There is no conventional liquidity available for market orders and conventional limit orders. Traders use indexed limit orders at different levels of urgency. Price moves by an amount increasing in level of urgency. In our model of smooth trading, we formally prove the intuition of Black (1995).

Conclusions

- We presented a dynamic oligopoly model of informed trading in continuous time with imperfect competition and agreement to disagree.
- In flow equilibrium, speed of trading is derived endogenously. Transaction costs depend on the speed of trading.
- Trading modest quantities much faster than consistent with equilibrium strategies results in flash-crash patterns with price spikes followed by price reversals.

Numerical Comparative Statics Results



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