Performance Evaluation with High Moments and Disaster Risk

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Background

- Tail risk and rare disasters have been central to the recent meltdown in financial markets.
- Markets were hit by catastrophic events whose ex-ante probabilities were considered negligible.
- Traditional performance evaluation measures (such as the Sharpe ratio) rely on the first two distribution moments.
- Low distribution moments hardly reflect extreme but rare outcomes:

$$\mu_k = \sum_{i=1}^{n} p_i g_i^k.$$

Higher Moments in Asset Pricing

- Investors favor right skewness (e.g., Kraus and Litzenberger (1976), Jean (1971), Kane (1982), and Harvey and Siddique (2000))
- Investors are averse to tail-risk and rare disasters (e.g., Barro (2009) and Gabaix (2008)).
- Normative performance evaluation measures should reflect these preferences as well as aversion to rare disasters.
- To the best of our knowledge (which is a bit limited), there is no performance evaluation measure that incorporates these high moment and disaster risk properties, in a manner consistent with theory.

In This Paper

- Theory
  - We study two performance indices based on a simple reinterpretation of the novel riskiness measures proposed by Aumann and Serrano (JPE 2008) and Foster and Hart (JPE 2009).
  - We follow the unified framework in Hart (2011).
  - We show that these indices reflect *all* distribution moments in a manner that is consistent with the asset pricing literature.
  - We study the magnitudes of the moment effects on performance.
  - We show that the indices differ in how they reflect disaster risk.

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  - We study the magnitudes of the moment effects on performance.
  - We show that the indices differ in how they reflect disaster risk.
- Applications
  - We show how to estimate the indices using GMM.
  - We apply the indices to popular investment strategies and to well-known anomalies and examine their attractiveness accounting for high moments.
  - We apply the indices to the selection of mutual funds.

Reminder: Second Order Stochastic Dominance (SOSD)

- Monotone Concave Dominance
- Assume all investors have increasing and concave utility (risk aversion).
- Consider two investments (gambles) g and g'.
- If for all u

$$\mathbf{E}u\left(g\right) \geq \mathbf{E}u\left(g'\right)$$

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  we say that g dominates g' by SOSD.
- SOSD is easy to check by looking at the distribution of the gambles. No need to know actual utilities.
- Problem: SOSD is an incomplete order. Often - impossible to compare gambles.

Example: Sharpe Ratio Fails SOSD

	g_1			g_2		
	Value	Probability	*CDF*	Value	Probability	*CDF*
	-10	0.001	0.001	-1	0.001	0.001
	1	0.999	1	1	0.9	0.901
				4	0.099	1
μ_1		0.989			1.295	
m_2		0.121			0.808	
m_3		-1.327			1.924	
m_4		14.583			5.335	
S		2.845			1.441	

Properties of AS and FH Performance Indices

(i) Impose a complete order on investments.
(ii) Coincide with SOSD, whenever SOSD can be applied.
(iii) Account for high distribution moments in a manner consistent with the asset pricing literature: increasing in mean and skewness and decreasing in variance and tail-risk of the investment.
(iv) Utility-based measures, and are hence more robust and coherent than using the moments themselves.

Setup - Investments/Gambles

- An investment (or gamble) can be modeled as a random variable, which we generically denote by g.
- Assume that:
  - g admits finitely many values. In particular, all of the moments of g are well defined. Denote the raw moments by μ_k and the central moments by m_k, $k = 1, 2, ...$
  - $\mathbf{E}(g) = \mu_1 > 0$.
  - g admits some negative values with a positive probability.
- Refer to the set of all possible gambles by $\mathcal{G}$.

Setup - Utilities

- Assume that investors have Von Neumann-Morgenstern utility functions over wealth denoted by $u(\cdot)$, which are differentiable as many times as needed.
- We assume further that $u' > 0$ and $u'' < 0$.
- Additional conditions:
  - Non-increasing absolute risk aversion, i.e. $-\frac{u''(w)}{u'(w)}$ is weakly decreasing.
  - Non-decreasing relative risk aversion, i.e. $-w\frac{u''(w)}{u'(w)}$ is weakly increasing.
  - $\lim_{w \downarrow 0} u(w) = -\infty$.
- Denote the class of all such utility functions by $\mathcal{U}^*$.
- Example, all CRRA utility functions of the form $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ with $\gamma \geq 1$.

Traditional Approach to Comparing Gambles

- Let w_0 denote the initial wealth of an investor, to which we refer as her "status-quo."
- Suppose you'd like to compare two gambles g and g'.
- Traditional approach is to compare $\mathbf{E}u\left(w_0+g\right)$ with $\mathbf{E}u\left(w_0+g'\right)$.
- Unless we have SOSD, this requires knowing the utility function and wealth.

New Approach to Comparing Gambles (AS and FH)

- Rather than comparing two gambles directly, compare each gamble to the status quo.
  - Is $Eu(w_0 + g) \leq u(w_0)$?
  - Is $Eu(w_0 + g') \leq u(w_0)$?
- If g is uniformly rejected less often than g', then g is deemed more attractive than g'.
- Uniform rejection can take two forms: across all wealth levels or across all utility functions.

Wealth Uniform Rejection

Definition

Say that an investor with utility u and initial wealth w_0 rejects a gamble g if $\mathbf{E}u\left(w_0 + g\right) \leq u\left(w_0\right)$.

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Definition

Say that a gamble g wealth-uniformly dominates gamble g' if whenever g is wealth-uniformly rejected by a utility function u, g' is also wealth-uniformly rejected by u.

A Bit More Intuition

- g wealth-uniformly dominates g' if whenever an investor with utility function u prefers the status-quo to g for all wealth levels, she also prefers the status-quo to g' for all wealth levels.
- In other words, g is preferred to g', if g' is "more often" wealth-uniformly rejected than g is.

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- This makes sense, but sounds very abstract. What can we do with this?

The AS Result

Theorem

(Aumann and Serrano (2008), Hart (2011)). Wealth-uniform dominance induces a complete order on $\mathcal{G}$ that extends SOSD. This order can be represented by a performance index $P^{AS}(g)$ assigned to any gamble g, which is given by the unique positive solution to the implicit equation

$$\mathbf{E}\left[\exp\left(-P^{AS}(g)g\right)\right] = 1.$$

That is, for any two gambles g and g', g wealth-uniformly dominates g' if and only if $P^{AS}(g) \geq P^{AS}(g')$.

Economic Interpretation of the AS Index

- The condition
$$\mathbf{E}\left[\exp\left(-P^{AS}(g)g\right)\right]=1$$
is equivalent to
$$\mathbf{E}\left[-\exp\left(-P^{AS}(g)(w_0+g)\right)\right]=-\exp\left(-P^{AS}(g)\,w_0\right),$$
regardless of w_0.
- Interpretation: $P^{AS}(g)$ is the level of absolute risk aversion that makes an investor with CARA utility indifferent between taking g and the status quo, regardless of the initial wealth w_0.
- An investor with CARA utility would accept g when $ARA < P^{AS}(g)$, and would reject g when $ARA \geq P^{AS}(g)$

Economic Interpretation of the AS Index (cont.)

- An investor with non-increasing absolute risk aversion would wealth-uniformly reject g if and only if $ARA_{w \to \infty} \geq P^{AS}(g)$.
- A higher $P^{AS}(g)$ implies a smaller set of utility functions that wealth-uniformly reject g.
- The AS index corresponds to wealth-uniform dominance.

Utility Uniform Rejection

Definition

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Say that a gamble g utility-uniformly dominates gamble g' if whenever g is utility-uniformly rejected at an initial wealth level w_0 , g' is also utility-uniformly rejected at w_0.

Intuition: g utility-uniformly dominates g' if whenever all investors with initial wealth level w_0 prefer the status-quo to g, they also prefer the status-quo to g'.

The FH Result

Theorem

(Foster and Hart (2009), Hart (2011)). Utility-uniform dominance induces a complete order on $\mathcal{G}$ that extends SOSD. This order can be represented by a performance index $P^{FH}(g)$ assigned to any gamble g, which is given by the unique positive solution to the implicit equation

$$\mathbf{E}\left[\log\left(1+P^{FH}(g)g\right)\right]=0.$$

That is, for any two gamble g and g', g utility-uniformly dominates g' if and only if $P^{FH}(g) \geq P^{FH}(g')$.

Economic Interpretation of the FH Index

- The condition
$$\mathbf{E}\left[\log\left(1+P^{FH}(g)g\right)\right]=0,$$
is equivalent to
$$\mathbf{E}\left[\log\left(\frac{1}{P^{FH}(g)}+g\right)\right]=\log\left(\frac{1}{P^{FH}(g)}\right).$$
- Interpretation: $\frac{1}{P^{FH}(g)}$ can be interpreted as the level of wealth that renders an investor with log utility indifferent between taking g or staying with the status quo.
- A log investor with higher initial wealth than $\frac{1}{P^{FH}(g)}$ would accept g, whereas a log investor with lower initial wealth than $\frac{1}{P^{FH}(g)}$ would reject g.

Economic Interpretation of the FH Index (cont.)

- Non-decreasing relative risk aversion and $\lim_{w \downarrow 0} u(w) = -\infty$ imply $RRA \geq 1$ at all wealth levels.
- The log utility is the least risk averse utility in $\mathcal{U}^*$.
- A gamble g is utility-uniformly rejected at w_0 if and only if the log utility rejects g at w_0, i.e., $w_0 < \frac{1}{P^{FH}(g)}$.
- A higher $P^{FH}(g)$ implies a smaller set of wealth levels at which g is utility-uniformly rejected.
- The FH index corresponds to utility-uniform dominance.

Our Interpretation vs. AS and FH's Interpretation

- Aumann and Serrano (2008) and Foster and Hart (2009) present their measures as "riskiness indices" rather than "performance indices."
- Their interpretation: If investors uniformly reject one gamble more often than another, then the former is more risky.
- Our interpretation: If investors uniformly reject one gamble more often than another, then the latter is more attractive to investors (has better performance).
- The mapping: $P^{AS} = 1/R^{AS}$ and $P^{FH} = 1/R^{FH}$, where R^{AS} and R^{FH} are the relevant riskiness measures.

Moment Properties

- Using a Taylor expansion around zero, rewrite the implicit equation:

$$\mathbf{E}\left[\exp\left(-P^{AS}(g)g\right)\right] = 1$$

as

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left(P^{AS}(g)\right)^n \mu_n(g) = 0.$$

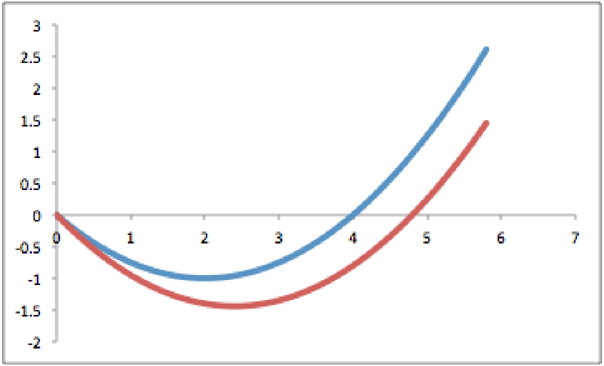
- Similarly, using a Taylor expansion around μ_1, we can write the same equation as,

$$1 + \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \left(P^{AS}(g)\right)^n m_n(g) = \exp\left(P^{AS}(g)\,\mu_1(g)\right).$$

- Similar expansions are available for the FH index.

Moment Properties (cont.)

- Any two gambles may differ in several of their moments.
- Consider the hypothetical exercise of changing one moment at a time while keeping all other moments unchanged.
- The effect on P^{AS} depends on whether the moment is odd or even.



Moment Properties

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- Increasing in mean and skewness.
- Decreasing in variance and tail risk.

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- Let $\hat{\mu}_k \equiv \sqrt[k]{\mu_k}$ for $k = 1, 2, ...$ and $\hat{m}_k \equiv \sqrt[k]{m_k}$ be the standardized moments.
- Define

$$\eta_k(g) \equiv \left| \frac{\partial P(g)}{\partial \hat{\mu}_k} \cdot \frac{\hat{\mu}_k}{P(g)} \right|$$

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- **Result #2:** $\eta_k^{AS}(g)$ and η_k^{FH} can be either decreasing or increasing in k.
- Conclusion: High moments can be important for performance evaluation. No reason to neglect high moments.

Rare Disasters

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- Small probability events are not likely to affect low moments, but may become dominant when high moments are taken into account.
- P^{FH} is extremely sensitive to rare disasters.
- Let $g_0 \in \mathcal{G}$ be a gamble and choose $L > 0$ very large.
- One can think of g_0 as a "business as usual" gamble that involves some gains and losses but no disastrous events, whereas $-L$ is a very big and unusual loss.
- Consider the composite gamble g_α that assigns probability $1 - \alpha$ to g_0 and α to $-L$, where α is some small probability.
- The gamble g_α reflects both "business-as-usual" realizations and the rare disaster.

Rare Disasters and the FH Index

- **Result #3:** Let $g_0 \in \mathcal{G}$ be a gamble and $L > 0$ such that $P^{FH}(g_0) > 1/L$. Let $\alpha \in (0, 1)$ and let g_α denote a composite gamble that assigns probability $1 - \alpha$ to g_0 and α to $-L$. Then, $\lim_{\alpha \to 0} P^{FH}(g_\alpha) = 1/L$.

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- To get intuition recall that:

$$\mathbf{E}\left[\log\left(\frac{1}{P^{FH}(g)} + g\right)\right] = \log\left(\frac{1}{P^{FH}(g)}\right).$$

- From the properties of log,

$$P^{FH}(g) < \frac{1}{-\min g}.$$

- Conclusion: P^{FH} is consumed by rare disasters.

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$$P^{FH}(g) < \frac{1}{-\min g}.$$

- Conclusion: P^{FH} is consumed by rare disasters.
- The key - lack of continuity of P^{FH}.

Statistical Estimation - GMM

- The two indices are naturally estimated using GMM through the moment conditions:

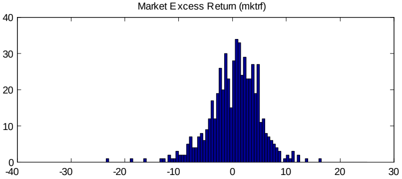
$$\mathbf{E}\left[\exp\left(-P^{AS}(g)g\right)-1\right]=0$$

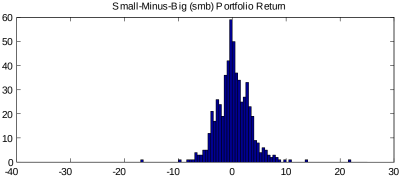
and

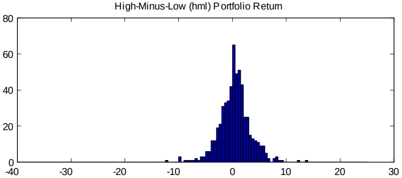
$$\mathbf{E}\left[\log\left(1+P^{FH}(g)g\right)\right]=0.$$

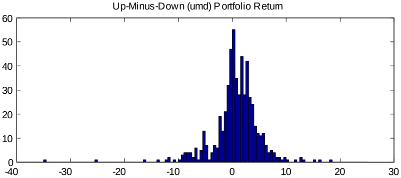
- GMM works out a distribution for the parameters that is consistent, efficient, and asymptotically normal.
- The model is "just identified" (number of moment conditions is equal to the number of estimated parameters).
- We use GMM standard errors to test hypothesis regarding the underlying population.

Application #1: Anomalies (Fama-French Factors 1962-2009)









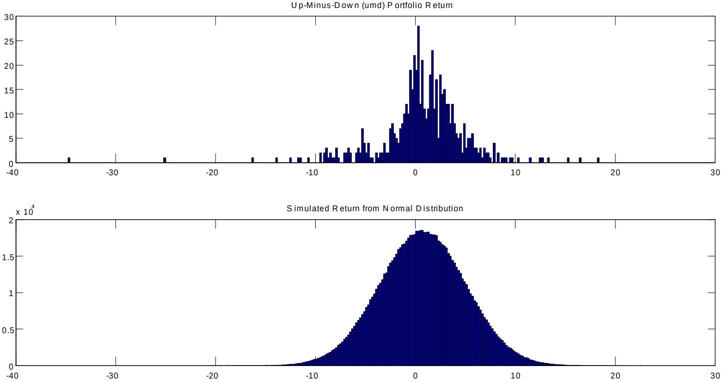
Performance Indices - Fama-French Factors

Columns (1)-(4) report GMM estimates for various moments and performance indices of different portfolios, and columns (5)-(10) report difference test results. The t-statistics are included in the parentheses below the corresponding estimates. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (***) levels.

Panel A: Various Moments										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	mktrf	*smb*	*hml*	*umd*	*mktrf − smb*	*mktrf − hml*	*mktrf − umd*	*smb − hml*	*smb − umd*	*hml − umd*
μ_1	0.4059	0.2298	0.4391	0.7257	0.1761	-0.0332	-0.3198	-0.2093	-0.4959	-0.2866
	(2.15)**	(1.75)*	(3.62)***	(4.02)***	(0.91)	(-0.13)	(-1.14)	(-1.05)	(-2.20)**	(-1.23)
m_2	20.4828	9.9792	8.4912	18.7827	10.5036	11.9917	1.7001	1.4880	-8.8035	-10.2915
	(12.02)***	(8.74)***	(11.42)***	(6.71)***	(5.53)***	(7.24)***	(0.57)	(1.44)	(-3.28)***	(-3.85)***
m_3	-50.9774	16.6665	-0.6708	-116.1113	-67.6439	-50.3066	65.1339	17.3373	132.7778	115.4405
	(-2.02)**	(0.87)	(-0.11)	(-1.46)	(-2.19)**	(-1.91)*	(0.77)	(0.76)	(1.66)*	(1.44)
m_4	2092.19	850.61	390.76	4862.86	1241.58	1701.42	-2770.67	459.85	-4012.25	-4472.10
	(3.45)***	(2.03)**	(4.27)***	(1.73)*	(1.71)*	(2.81)***	(-0.97)	(1.21)	(-1.42)	(-1.59)

Panel B: Performance Measures										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	mktrf	*smb*	*hml*	*umd*	*mktrf − smb*	*mktrf − hml*	*mktrf − umd*	*smb − hml*	*smb − umd*	*hml − umd*
S	0.0897	0.0728	0.1507	0.1675	0.0169	-0.0610	-0.0778	-0.0779	-0.0947	-0.0168
	(2.09)**	(1.77)*	(3.57)***	(3.49)***	(0.35)	(-0.89)	(-1.13)	(-1.19)	(-1.46)	(-0.25)
P^{AS}	0.0382	0.0468	0.1014	0.0630	-0.0086	-0.0632	-0.0248	-0.0545	-0.0162	0.0384
	(2.08)**	(1.74)*	(3.44)***	(3.15)***	(-0.32)	(-1.63)	(-0.86)	(-1.22)	(-0.47)	(1.03)
η_1^{AS}	0.9602	1.0081	0.9638	0.7716						
ς_2^{AS}	1.8263	2.0320	1.8101	1.2041						
ς_3^{AS}	0.0869	0.0795	0.0072	0.2345						
ς_4^{AS}	0.0454	0.0634	0.1427	0.2063						
P^{FH}	0.0347	0.0449	0.0800	0.0288	-0.0101	-0.0452	0.0059	-0.0351	0.0160	0.0512
	(2.55)**	(1.98)**	(14.81)***	(597.26)***	(-0.45)	(-2.86)***	(0.43)	(-1.42)	(0.71)	(9.46)***
η_1^{FH}	0.7510	0.8508		0.0022						
ς_2^{FH}	1.2827	1.6264		0.0016						
ς_3^{FH}	0.1094	0.1206		0.0003						
ς_4^{FH}	0.1538	0.2733		0.0003						

Application #2: Momentum vs. Simulated Momentum



Momentum vs. Simulated Momentum

Columns (1) and (2) report GMM estimates for various performance indices of different portfolios, and column (3) reports difference test results. The t-statistics are included in the parentheses below the corresponding estimates. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (***) levels.

Performance Measures			
	(1) *umd*	(2) Simulated	(3) *umd*-Simulated
S	0.1675 (3.49)***	0.1673 (166.21)***	0.0001 (0.003)
P^{AS}	0.0630 (3.15)***	0.0772 (162.70)***	-0.0142 (-0.71)
P^{FH}	0.0288 (597.26)***	0.0494 (75925)***	-0.0206 (-426.04)***

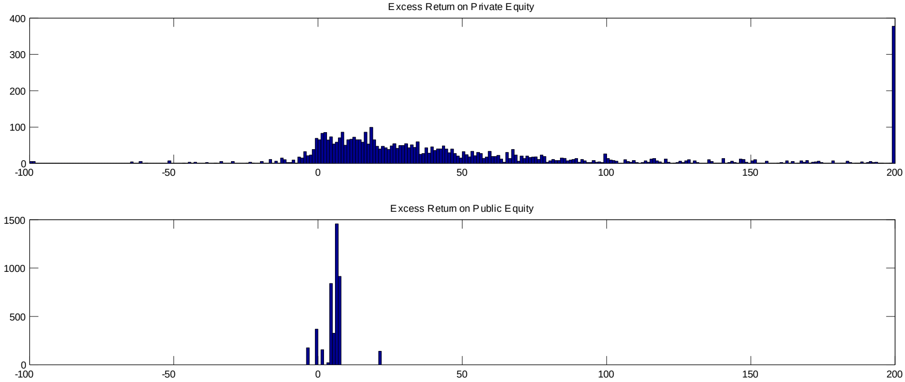
Application #3: Private vs. Public Equity

- Moskowitz and Vissing-Jorgensen (2002) provide a comparison of the performance of public vs. private equity investment from the point of view of individual investors.
- They find that the returns to private equity are not higher than those of public equity.
- This result is puzzling since private equity investments expose investors to a high level of idiosyncratic risk.
- They observe that private equity investment is right skewed.
- They conjecture that preference for skewness may be one reason for the tendency of individuals to invest in private equity.
- Is that the case?

Private vs. Public - Data and Methodology

- Data on individual household investment in private equity from the 2004 Survey of Consumer Finance (SCF).
- Private equity returns per household (conditional on survival):
  - Estimate excess returns obtain by households since the founding or the acquisition of a private firm.
  - Treat each household as an observation and estimate the average annual holding period return.
  - The average annual holding return is calculated as the sum of the geometric average annual capital gain and the current dividend return, (assumed to be representative) less geometric average risk free rate.
- Public equity alternative per household:
  - The geometric average annual excess return it would obtain by investing in the CRSP value-weighted market index for the same time period as its private equity holdings.

Private vs. Public



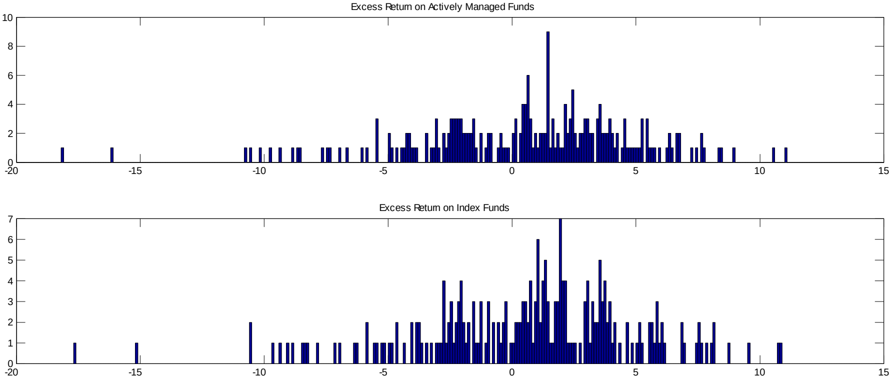
Private vs. Public Equity

Columns (1) and (2) report GMM estimates for various moments and performance indices of different portfolios, and column (3) reports difference test results. The t-statistics are included in the parentheses below the corresponding estimates. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (***) levels.

Panel A: Various Moment			
	(1) Private	(2) Public	(3) Private-Public
μ_1	196.5432	5.5895	190.9537
	(5.26)***	(92.12)***	(5.11)***
m_2	6.1506×10^6	16.2216	6.1505×10^6
	(1.90)*	(23.29)***	(1.90)*
m_3	5.1344×10^{11}	73.0088	5.1344×10^{11}
	(1.75)*	(8.58)***	(1.75)*
m_4	4.6177×10^{16}	2400.33	4.6177×10^{16}
	(1.74)*	(15.69)***	(1.74)*

Panel B: Performance Measures			
	(1) Private	(2) Public	(3) Private-Public
S	0.0793	1.3878	-1.3086
	(10.57)***	(50.26)***	(-45.96)***
P^{AS}	0.0549	0.7971	-0.7422
	(18.55)***	(41.59)***	(-38.18)***
P^{FH}	0.0100	0.2583	-0.2483
	(174466)***	(47694232)***	(-4293137)***

Application #4: Active vs. Passive Mutual Funds (1991-2009)



Mutual Fund Selection

- Question: Do mutual funds selected based on the two indices possess favorable moment properties?
- In each month between 1967-2009 and for each actively managed equity mutual fund we calculate the P^{AS} and P^{FH} measures based on the most recent 60 monthly excess returns.
- We then rank all mutual funds in each month based on their indices (separately for P^{AS} and P^{FH}).
- Two portfolios of "selected" mutual funds by equal-weighting the top decile mutual funds.
- Rebalance monthly.
- For the purpose of comparison, we construct an investment strategy based on the Sharpe ratio

Portfolios of Selected Mutual Funds

Columns (1)-(4) report GMM estimates for various moments and performance indices of different portfolios, and columns (5)-(10) report difference test results. The t-statistics are included in the parentheses below the corresponding estimates. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (***) levels.

Panel A: Various Moments and Performance Measures										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	AS	FH	S	MKT	AS-FH	AS-S	AS-MKT	FH-S	FH-MKT	S-MKT
μ_1	0.4961	0.4671	0.5383	0.4269	0.0290	-0.0422	0.0692	-0.0712	0.0402	0.1114
	(2.80)***	(2.68)***	(2.66)***	(2.09)**	(1.89)*	(-0.96)	(1.13)	(-1.32)	(0.62)	(1.77)*
m_2	16.1544	15.6887	21.2108	21.4319	0.4657	-5.0564	-5.2774	-5.5221	-5.7432	-0.2211
	(11.31)***	(11.06)***	(11.89)***	(11.46)***	(3.55)***	(-9.92)***	(-6.82)***	(-9.67)***	(-7.49)***	(-0.27)
m_3	-37.2520	-36.9873	-59.1041	-55.5719	-0.2647	21.8521	18.3199	22.1168	18.5846	-3.5321
	(-2.23)**	(-2.17)**	(-2.62)***	(-1.99)**	(-0.18)	(2.85)***	(1.29)	(2.76)***	(1.38)	(-0.25)
m_4	1312.89	1284.98	2090.75	2264.29	27.91	-777.86	-951.40	-805.77	-979.31	-173.55
	(4.12)***	(3.92)***	(4.43)***	(3.35)***	(0.96)	(-4.29)***	(-2.34)**	(-4.36)***	(-2.53)**	(-0.48)
S	0.1234	0.1179	0.1169	0.0922	0.0055	0.0066	0.0312	0.0011	0.0257	0.0247
	(2.69)***	(2.57)**	(2.55)**	(2.03)**	(1.42)	(0.77)	(2.48)**	(0.10)	(1.92)*	(1.81)*
P^{AS}	0.0582	0.0565	0.0482	0.0384	0.0018	0.0100	0.0199	0.0082	0.0181	0.0099
	(2.66)***	(2.55)**	(2.54)**	(2.03)**	(0.97)	(2.12)**	(3.16)***	(1.43)	(2.69)***	(1.74)*
P^{FH}	0.0497	0.0484	0.0424	0.0347	0.0013	0.0074	0.0150	0.0061	0.0137	0.0076
	(3.87)***	(3.65)***	(3.30)***	(2.49)**	(1.17)	(2.95)***	(3.89)***	(1.91)*	(3.47)***	(1.78)*

Certainty Equivalent from Portfolios of Selected Mutual Funds

Columns (1)-(4) report certainty equivalent from investing in different portfolios, and columns (5)-(10) report difference test results. The t-statistics are included in the parentheses below the corresponding estimates. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (***) levels.

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
γ	AS	FH	S	MKT	AS-FH	AS-S	AS-MKT	FH-S	FH-MKT	S-MKT
3	0.2409	0.2207	0.2107	0.0901	0.0201	0.0302	0.1507	0.0101	0.1306	0.1205
5	0.0701	0.0501	-0.0300	-0.1494	0.0201	0.1001	0.2196	0.0800	0.1995	0.1195
10	-0.4295	-0.4295	-0.7035	-0.8423	0	0.2740	0.4128	0.2740	0.4128	0.1388
15	-1.0154	-1.0154	-1.5270	-1.7456	0	0.5116	0.7302	0.5116	0.7302	0.2186
20	-1.7494	-1.7494	-2.5513	-3.0010	0	0.8019	1.2516	0.8019	1.2516	0.4497

Conclusion

- If we want to take high moments and rare disasters seriously, then we should account for them in our performance evaluation indices.
- We propose one way to do this, which is consistent with a plausible theory of AS and FH.
- The indices we consider encapsulate all distribution moments in a manner consistent with asset pricing.
- High moments and rare disasters appear to have a material effect on performance, and should not be neglected.