Accounting Manipulation, Peer Pressure, and Internal Control*

Pingyang Gao
Booth School of Business
The University of Chicago
pingyang.gao@chicagobooth.edu

Gaoqing Zhang
Carlson School of Management
University of Minnesota
zhangg@umn.edu

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Abstract

We study firms' investment in internal control to reduce accounting manipulation. We first show the peer pressure for manipulation: one manager manipulates more if he suspects reports of peer firms are more likely to be manipulated. As a result, one firm's investment in internal control has a positive externality on peer firms. It reduces its own manager's manipulation, which in turn mitigates the manipulation pressure on managers in peer firms. Firms don't internalize this positive externality and thus under-invest in their internal control over financial reporting. The under-investment problem provides one justification for regulatory intervention in firms' internal control choices.

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1 Introduction

The wave of accounting frauds and restatements in early 2000 (e.g., Enron, WorldCom) have exposed the staggering failure of internal control over financial reporting in many firms (**?**). Until then a firm's internal control decisions had long been deemed as its private domain and outside the purview of the securities regulations that had traditionally focused mainly on disclosure of those decisions (**?**, **?**). However, the prevalence and magnitude of the internal control failures eroded the support for such practice and eventually led to the Sarbanes–Oxley Act of 2002 (SOX) in United States and similar legislatures in other countries. In addition to the enhanced disclosure requirements SOX has also mandated substantive measures to deter and detect accounting frauds[1]. Their mandatory nature has made these measures controversial (e.g., **?**, **?**). Even for those who felt that something had to be done with the firms' internal control over financial reporting, it may not be clear why it should be done through regulations. Is there a case for regulation that intervenes in firms' internal control decisions? Why don't firms have right incentives to choose the optimal level of internal control to assure the veracity of their financial statements? In fact, **?**, in an influential critique of SOX, argues that "The central policy recommendation of this Article is that the corporate governance provisions of SOX should be stripped of their mandatory force and rendered optional for registrants."

We construct a model to study firms' investment in internal control over financial reporting. In the model firms can invest in costly internal control to detect and deter its manager's accounting manipulation. We show that such investment by one firm has a positive externality on its peers. At the core of the channel for this externality is the peer pressure for accounting manipulation among firms: one manager's incentive to manipulate is increasing in his expectation that reports from peer firms are manipulated. As a result, a firm's investment in internal control reduces its own manager's manipulation, which, in turn, mitigates the pressure for manipulation on managers in peer firms. Since the firm doesn't internalize this externality, it under-invests in the internal control over financial reporting. Regulatory

[1] These mandates range from independent audit committee, auditor partner rotations, prohibition of non-audit service provided by auditors, and executives' certification and auditors' attestation to the internal control system.

interventions can improve the value of all firms by mandating a floor of internal control investment for all firms.

In our model, there are two firms with correlated fundamentals, indexed by A and B. Each manager's payoff is a weighted average of the current stock price and the fundamental value of *his own firm*. Investors rely on accounting reports to set stock prices. Accounting manipulation boosts accounting reports and allows the bad manager with successful manipulation to be pooled with the truly good ones. Investors rationally conjecture this pooling result and discount the pool accordingly to break even. In the equilibrium, the bad manager with successful manipulation receives an inflated stock price at the expense of the truly good manager. Accounting manipulation is detrimental to firm value and firms do have private incentives to invest in their internal control over financial reporting.

In such a setting peer pressure for accounting manipulation arises. By peer pressure we mean that one manager manipulates more if he expects that the other firm's report is more likely to be manipulated in equilibrium. In other words, the two managers' manipulation decisions are strategic complements in the sense of **?**. The mechanism works as follows. Consider manager A's manipulation decision. Rational investors utilize reports from both firms in setting the stock price of firm A due to their correlated fundamentals. Investors compare report A with report B to distinguish between the truly good firm A and the bad firm with successful manipulation. Manipulation of report B reduces its informativeness and makes it less useful for investors of firm A to cull out the bad one with successful manipulation. Anticipating that his fraudulent report is less likely to be confronted by report B, manager A expects a higher benefit from manipulation and thus increases his manipulation. The manipulation of report B creates a "pressure" on manager A to manipulate because the opportunity cost for manager A not to manipulate is higher.

To further see this intuition, consider a special case in which two firms' fundamentals are perfectly correlated and manager B doesn't manipulate. As a result, manager A will not manipulate either because any successful manipulation will be confronted by the report from manager B. If manager B is expected to manipulate a little bit, manager A now anticipates that his fraudulent report is sometimes camouflaged and thus has an incentive to manipulate.

The peer pressure for manipulation creates a positive externality of the firm's costly investment in internal control. Firm B's investment in internal control reduces its own manager's manipulation. The reduction of manipulation in firm B mitigates the pressure for manipulation on manager A, resulting in lower manipulation by manger A. However, firm B doesn't internalize this externality and under-invests in the internal control from the perspective of maximizing the value of two firms combined. This under-investment in internal control by individual firms suggests a rationale for regulatory intervention that imposes some floor of internal control over financial reporting.

The peer pressure for manipulation is often alleged in practice. One of the best known and most extreme example is the telecommunications industry around the turn of the new millennium (see **?** Footnote 1 for detailed references to such allegations). When WorldCom turned to aggressive and eventually illegal reporting practices to boost its performance, peers firms were under enormous pressure to perform. **?** claims: "Once WorldCom started committing accounting fraud to prop up their numbers, all of the other telecoms had to either (a) commit accounting fraud to keep pace with WorldCom's blistering growth rate, or (b) be viewed as losers with severe consequences." Qwest and Global Crossing ended up with accounting frauds while AT&T and Sprint took a series of actions that aimed to shore up their short-term performance at the expense of long-term viability. While these companies had plenty of their own problems, the relentless capital market pressure undoubtedly made matters worse (see **?**).

The peer pressure mechanism also generates new empirical predictions. The central prediction is that peer firms' manipulation decisions are correlated, even after controlling for their own fundamentals and characteristics. An exogenous increase in one firm's manipulation incentive also elevates the manipulation incentives in peer firms. For example, if one firm's manager is given a stronger incentive pay, the model predicts that not only its own manager but also the managers from peer firms are more likely to engage in manipulation. For another example, one bank's loan loss provisioning is increasing in the peer average even after controlling for the bank and its peers' fundamentals. Some recent papers have examined how one firm's fraudulent accounting affects investment decisions in peer firms (e.g., **?**, **?**).

Our model suggests an additional effect that one firm's accounting manipulation and internal control imposes on its peer firms.

1.1 Contributions and the literature review

We make two contributions. First, we provide a rational explanation of the peer pressure for manipulation arising from the capital market. Peer pressure for manipulation has been studied in other contexts. Both **?** and **?** study the interactions of firms' reporting choices and product market competition in a Cournot oligopoly setting. One result in **?** (Result 5) shows numerically that two firms' misreporting can be strategic complements when two firms misreporting cost difference is sufficiently large. **?** present an interesting "cross-firm earnings management" mechanism: firm A attempts to influence its investors' belief by changing its production decision that alters firm B's manipulation that in turn affects investors' use of report B in assessing firm A. They show that such cross-firm earnings management could serve as a commitment device for the oligopoly to reduce production and improve profitability.

The peer pressure for manipulation, in the form of collusion of actions, can also be induced by contractual payoff links among agents within the same firm (e.g., **?**, **?**, **?**). **?** show that firms' disclosure decisions interact with each other when there is an exogenous tournament structure of payoffs to firms. There are also behavioral explanations for the peer pressure for manipulation that one manager's unethical behavior diminishes the moral sanction for others to engage in the same behavior (e.g., **?**, **?** **?**). This explanation is often labeled as reporting culture, code of ethics, or social norms.

Our paper complements these explanations from a capital market pressure perspective that managers intervene in the reporting process to influence capital markets' inferences about their firms. Capital market pressure is often viewed as a major motivation for accounting manipulation (**?**). We assume neither the contractual links among managers nor the complementarity of manipulation costs.

? study a model of "enforcement thinning" in which the manipulation decisions among firms are also strategic complements. The regulator's budget of enforcement against frauds is fixed. As the number of firms engaging in accounting manipulation increases, the probability that each firm will be subject to the regulator's investigation becomes smaller and thus more firms engage in manipulation.

Our second contribution is to provide one rationale for regulating firms' internal control over financial reporting. As **?** have pointed out, understanding the positive externalities of regulations are crucial for their justification in the first place. Our model suggests that the proposal in **?** that the internal control mandates in SOX should be made optional is flawed. Competition among firms (or state laws) doesn't lead to socially optimal investment in internal control.

This relates our paper to the literature on the externalities of disclosure and corporate governance. **?** provide an excellent summary of various potential rationales for disclosure regulation. **?**, **?** and **?** study how truthful disclosure by one firm can affect the decisions of investors of other firms. **?** study a model of voluntary disclosure with verifiable messages in which firms' information receipt is uncertain but correlated with each other. They show that one firm's disclosure threshold depends on the number of peer firms and the nature of the private information. **?** also study a voluntary disclosure model with verifiable messages in which two agents forecast the same fundamental. They show that the agents' disclosure strategies are strategic complements when concealing information is costly, but strategic substitutes when disclosing information is costly. They also examine a model of costly signaling similar to that in **?** and show that the sender's incentive to misreport decreases as public information quality improves.

There have also been a series of recent papers that study the externality of managerial compensation and corporate governance in monitoring managerial consumption of private benefit. The key channel is through the imperfection in the labor market for managers (e.g., **?**, **?**, **?**).

Our model is concerned with accounting manipulation because of our focus on the externality of internal control. Disclosure alone in our model doesn't solve the under-investment problem. In our model the firms' internal control decisions are perfectly observed by investors and peer managers. Yet, the under-investment problem still arises.

The manipulation component of our model belongs to the class of signal jamming models (e.g., **?**) with one important departure. Like in **?**, investors in our model have rational expectations and are not fooled by manipulation *on average*. Unlike in **?**, manipulation allows the bad managers with successful manipulation to be pooled with the truly good ones and receive an inflated price. This allocational consequence of manipulation is a key element in generating the strategic complementarity between firms' manipulation choices. We implement this departure by constraining the managers' message space, an approach adopted in **?**, **?**, **?** and **?**. Our model differs from **?**. In his model the two managers' manipulation decisions are correlated because their manipulation varies across the state. Conditional on the state, the first manager's manipulation is *decreasing* in his expectation of the second manager's manipulation. There are other modeling devices that break the fully separating equilibrium in **?**. For example, **?** and **?** introduce investors' uncertainty about the manager's objective functions. For another example, **?** and **?** rely on investors' uncertainty about the manager's cost of manipulation. Yet another example is **?** who adopt an equilibrium selection criterion that favors the manager. More broadly, the signal jamming model has been widely used to study economic consequences of earnings management. We refer readers to some recent surveys, including **?**, **?**, **?**, **?**, and **?**, due to the size of the literature (as partially evidenced by the number of surveys).

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 solves the equilibria and examine the strategic relation among firms' manipulation and internal control investment. Section 4 discusses some extensions and Section 5 concludes.

2 The model

The economy consists of two firms, indexed by A and B. There are four dates, $t = 0$, 1, 2, and 3. All parties are risk neutral and the risk free rate is normalized to 1.

Each firm has a project that pays out *gross* cash flow s_i, $i \in \{A, B\}$, at $t = 3$. s_i is either high ($s_i = 1$) or low ($s_i = 0$). The prior probability that $s_i = 1$ is θ_i. The firm's *net* cash flow at $t = 3$, denoted as V_i, differs from the gross cash flow s_i for two reasons explained below. We refer to the net cash flow V_i as the firm's long-term value and the gross cash flow s_i as the firm's fundamental or type.

The payoff function of manager i is

$$U_i = \delta_i P_i + (1 - \delta_i) V_i,\ i \in \{A, B\}. \tag{1}$$

The manager's interests are not fully aligned with the firm's long-term value V_i. Instead, the manager cares about both the long-term firm value at $t = 3$ and the short-term stock price P_i at $t = 2$. $\delta_i \in (0, 1)$ measures the manager's relative focus on the two.

Managers' concern for short-term stock price performance is empirically descriptive. For example, **?** argues that take-overs would force managers to tender their shares at the market prices even if they would like to hold the stocks for the longer term. For another example, **?** and **?** contend that managers' reputation concern could lead them to focus on the short-term stock prices at the expense of the firm's long-term value. Alternatively, managers' stock-based compensation or equity funding for new projects can also induce them to focus on their firms' short-term stock price performance.

The stock price P_i at $t = 2$ is influenced by both firms' accounting reports. Each firm's financial reporting process is as follows. At $t = 1$, each manager privately observes the fundamental s_i. After observing his type s_i, each manager issues an accounting report $r_i \in \{0, 1\}$. The good manager always reports truthfully in equilibrium, *i.e.*, $r_i(s_i = 1) = 1$. The bad manager with $s_i = 0$ may manipulate the report. The probability that the bad manger issues a good report, *i.e.*, $r_i(s_i = 0) = 1$, is

$$\mu_i \equiv \Pr(r_i = 1 | s_i = 0, m_i, q_i) = m_i(1 - q_i).$$

μ_i is the probability that the bad firm successfully issues a fraudulent report. This probability is determined jointly by the manager's manipulation decision m_i and the firm's internal control choice q_i. $m_i \in [0, 1]$ is the bad manager's efforts to overstate the report. To economize on notations, we often use m_i to denote the bad manager's manipulation $m_i(s_i = 0)$ and omit the argument $s_i = 0$ whenever no confusion arises. Manipulation effort m_i is the manager's choice at $t = 1$ after he has observed s_i. m_i reduces the firm's long-term value by $C_i(m_i)$. $C_i(m_i)$ has the standard properties of a cost function (similar to the Inada conditions): $C_i(0) = 0$, $C_i'(0) > 0$, $C_i'(1) = \infty$ and $C_i'' > c$. c is a constant sufficiently large to guarantee that the manager's equilibrium manipulation choice is unique.

$q_i \in [0, 1]$ denotes the quality of the firm's internal control over financial reporting. It is interpreted as the probability that the manager's overstatement is detected and prevented by the internal control system. q_i is the firm's choice at $t = 0$ and reduces the firm's cash flow by $K_i(q_i)$. $K_i(q_i)$ has the standard properties of a cost function as well: $K_i(0) = 0$, $K_i'(0) > 0$, $K_i'(1) = \infty$ and $K_i'' > k$. k is a constant sufficiently large to guarantee that the firms' equilibrium internal control choice is unique. Unlike m_i, the firm's choice of q_i is publicly observable.

Overall, the bad manager can take actions to inflate the report, but his attempt is checked by the internal control system. We model the cost of manipulation as a reduction in the firm's long-term value. Both accrual manipulation and real earnings management are eventually costly to the firm (e.g., **?**). When the manager engages in accrual manipulation, the cost includes not only the direct cost of searching for opportunities, but also the indirect cost of the distraction of the manager's focus and the associated actions to cover up the manipulation. Real earnings management directly distorts the firm's decisions and decreases the firm's cash flows. Our results are also robust to the alternative interpretation that C_i or part of C_i is the manager's private cost, such as the psychic suffering, the potential reputation loss, and the possible legal consequences (e.g., **?**).

The firm's net cash flow (*i.e.*, the long-term firm value) can now be written as

$$V_i = s_i - C_i(m_i(s_i)) - K_i(q_i).$$

V_i is lower than the gross cash flow s_i by two terms, the cost of manipulation C_i and the cost of internal control K_i.

Finally, the two firms are symmetric, that is, $\theta_A = \theta_B = \theta$, $C_A(\cdot) = C_B(\cdot) = C(\cdot)$, $K_A(\cdot) = K_B = K(\cdot)$, and $\delta_A = \delta_B = \delta$. We keep the subscription i in the text to highlight the derivation of the equilibria. The only connection between the two firms is that their gross cash flows or types s_i are correlated.[2] The correlation coefficient ρ can be either positive or negative. For example, if s_i is related to customers' preferences for American cars, then the gross cash flows for GM and Ford are positively correlated. However, if s_i refers to a firm's market share, then a higher s_A for GM is likely to indicate a lower s_B for Ford. For simplicity we assume away the trivial case of $\rho = 0$. In addition, $\rho \in [\underline{\rho}, 1]$ is bounded from below by $\underline{\rho} \equiv \max\{-\frac{\theta}{1-\theta}, -\frac{1-\theta}{\theta}\}$, instead of -1. $\underline{\rho} < 0$ and approaches -1 when $\theta = \frac{1}{2}$. This is because two Bernoulli variables are perfectly negatively correlated $(i.e., \rho = -1)$ only if their marginal probabilities satisfy $\theta_A = 1 - \theta_B$, which holds for two symmetric firms only if $\theta_A = \theta_B = \frac{1}{2}$.

In sum, the timeline of the model is summarized as follows.

1. $t = 0$, firm i publicly chooses its internal control quality q_i;
2. $t = 1$, manager i privately chooses manipulation m_i after privately observing s_i;
3. $t = 2$, investors set stock price P_i after observing both report r_A and r_B;
4. $t = 3$, cash flows are realized and paid out.

The equilibrium solution concept is Perfect Bayesian Equilibrium (PBE). A PBE is characterized by the set of decisions and prices, $\{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_A, r_B)\}$, such that

1. $q_i^* = \arg\max_{q_i} E_0[V_i]$ maximizes the long-term firm value expected at $t = 0$;
2. $m_i^*(s_i) = \arg\max_{m_i(s_i)} E_1[U_i|s_i]$ maximizes the manager's payoff expected at $t = 1$ after observing s_i;
3. $P_i^*(r_A, r_B) = E_2[V_i|r_A, r_B]$ is set to be equal to investors' expectation of the long-term firm value, conditional on both firms' reports (r_A, r_B);
4. The players have rational expectations in each stage. In particular, both the manager's and investors' beliefs about the other manager's manipulation are consistent with Bayes rules, if possible.

[2] There is an empirical literature documenting the intra-industry information transfer, e.g., **?**, **?**, and **?**.

3 The analysis

In this section we analyze the model in sequence. We first examine how one manager's manipulation decision is influenced by his expectation of the manipulation of his peer firm's report and then study the design of internal control.

3.1 Equilibrium manipulation decisions

3.1.1 Equilibrium manipulation with only one firm

To highlight the driving forces behind the peer pressure for manipulation, we start with a benchmark with only firm A (or equivalently the two firms' fundamentals are not correlated). We solve the benchmark case of a single firm by backward induction. Investors at $t = 2$ set the stock price $P_A^*(r_A)$ to be equal to their expectation of the firm value V_A upon observing report r_A. Since they don't observe the manager's actual choice of manipulation m_A, investors conjecture that the manager has chosen m_A^* in equilibrium if his type is bad and 0 otherwise. Recall that the probability that the bad type succeeds in manipulation is $\mu_A = m_A(1 - q_A)$. Since q_A is observable to investors at $t = 2$, investors thus conjecture $\mu_A^* = m_A^*(1 - q_A)$. Expecting this data generating process, investors use the Bayes rule to update their belief about the firm type. Define $\theta_A(r_A) \equiv \Pr(s_A = 1|r_A)$ as investors' posterior belief about the firm being a good type conditional on report r_A.

We first have $\theta_A(0) = 0$. Since the good firm always issues the favorable report $r_A = 1$ and only the bad firm may issue the unfavorable report $r_A = 0$, investors learn that the firm issuing $r_A = 0$ is a bad type for sure.

Upon observing the favorable report $r_A = 1$, investors are uncertain about the firm type. The favorable report $r_A = 1$ can be issued by either the truly good firm ($s_A = 1$) or the bad firm with successful manipulation ($s_A = 0$). The population of the former is θ_A and of the latter is $(1 - \theta_A)\mu_A^*$. Using this knowledge, investors update their belief about the firm type as follows:

$$\theta_A(1) \equiv \Pr(s_A = 1|r_A = 1) = \frac{\theta_A}{\theta_A + (1 - \theta_A)\mu_A^*}. \tag{2}$$

Investors become more optimistic upon observing the favorable report $r_A = 1$ because the probability of issuing the favorable report is higher for the good firm than for the bad firm, *i.e.*, $1 \geq \mu_A^*$. However, investors do discount the favorable report to reflect the possibility that it is manipulated. If the bad firm cannot manipulate (*i, e.*, $\mu_A^* = 0$), then $\theta_A(1) = 1$. Investors take the favorable report at face value and don't discount it at all. If the bad firm always succeeds in manipulation (*i.e.*, $\mu_A^* = 1$), then $\theta_A(1) = \theta_A$. Investors completely discard the favorable report. If the probability of manipulation is in between, $\theta_A(1) \in (\theta_A, 1)$.

The stock price $P_A^*(r_A)$ is then set to be equal to investors' expectation of the firm value V_i :

$$P_A^*(r_A) = \theta_A(r_A) + (1 - \theta_A(r_A))(0 - C_A^*) - K_A(q_A). \tag{3}$$

The good firm generates gross cash flow of 1. The bad firm generates gross cash flow of 0 and incurs the manipulation cost of $C_A^* = C_A(m_A^*)$. Both types pay the internal control cost $K_A(q_A)$.[3] It is obvious that $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*) > 0$. Investors pay a higher price for the favorable report, despite the manipulation. As a result the manager who cares about short-term stock price prefers report $r_A = 1$ to $r_A = 0$.

Anticipating the investors' pricing response to report $P_A^*(r_A)$, the bad manager chooses m_A to maximize his expected utility $E_1[U_A(m_A)|s_A = 0]$ defined in equation 1. We denote the manager's best response to the investors' conjecture m_A^* as $\tilde{m}_A^*(m_A^*)$ or simply $\tilde{m}_A^*$. Its first-order condition is

$$H(m_A)|_{m_A = \tilde{m}_A^*(m_A^*)} \equiv \delta_A \frac{\partial \mu_A}{\partial m_A} \theta_A(1)(1 + C_A^*) - (1 - \delta_A)\, C_A'(m_A) = 0. \tag{4}$$

Equation 4 describes the trade-off of the manipulation decision. The first term is the marginal benefit of manipulation. It increases the firm's chance of issuing the favorable report $r_A = 1$, at a marginal rate of $\frac{\partial \mu_A}{\partial m_A} = 1 - q_A$. The favorable report, in turn, increases its stock price by $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*)$. The second term is the marginal cost. Manipulation reduces the firm's future cash flow by $C_A(m_A)$. Since the manager has a stake of $1 - \delta_A$

[3] We normalize the gross cash flow of the bad type to be 0. As a result, we have negative net cash flow for the bad firm. This can be easily fixed by introducing a positive baseline gross cash flow that is large enough to cover the cost of manipulation and internal control. Such a setting complicates the notations but would affect none of our formal results.

in the long-term firm value, he bears part of the manipulation cost as well. The manager thus chooses the optimal manipulation level such that the marginal benefit is equated to the marginal cost. We use $H(m_A)$ defined in equation 4 to denote the difference of the marginal benefit and marginal cost for an arbitrary manipulation m_A. Therefore, $\tilde{m}_A(m_A^*)$, defined by $H(\tilde{m}_A(m_A^*)) = 0$, characterizes the manager's best response to investors' conjecture m_A^*. In equilibrium, the investors' conjecture is consistent with the manager's optimal choice, that is, $\tilde{m}_A(m_A^*) = m_A^*$. This rational expectations requirement implies that the optimal choice m_A^* is defined by $H(m_A^*) = 0$. This manipulation game in general could have multiple equilibria, that is, the equation $H(m_A^*) = 0$ can have multiple solutions. Since the multiplicity is not our focus, we obtain the unique equilibrium by the assuming that both the manipulation cost function and later the internal control cost functions are sufficiently convex (see **?**). With the unique equilibrium determined, we conduct comparative statics to understand the determinants of the manager's optimal manipulation choice.

Lemma 1 *When there is only firm A, m_A^* is increasing in θ_A and δ_A, and decreasing in q_A.*

These properties of the equilibrium manipulation decisions are standard. First, m_A^* is increasing in the investors' prior belief about the firm type (θ_A) before observing report r_A. When θ_A is higher, investors expect that report $r_A = 1$ is more likely to come from the good firm and thus attach a higher valuation to it. The bad manager takes advantage of this optimism by manipulating more. Second, the manager manipulates more if he cares about the short-term stock price more.

Third, internal control reduces manipulation. All else being equal, an improvement in internal control quality detects manipulation more often and reduces the probability of fraudulent reports. This direct effect deters the manager's manipulation. However, there is also an indirect effect. Investors also anticipate the reduced manipulation in equilibrium and thus become more generous in their valuation of the favorable report, *i.e.*, $\frac{\partial}{\partial q_A}\left(P_A^*(1) - P_A^*(0)\right) > 0$. This entices the manager to manipulate more. Overall, the direct effect dominates the indirect effect for the following reason. An improvement in the internal control system affects only the bad type (since the good type doesn't manipulate). From the perspective of investors who observe only $r_A = 1$, the probability that the firm is a bad type and thus affected by the improvement is $1 - \theta_A(1)$. Based on this belief they increase their valuation for $r_A = 1$. In contrast, the bad manager understands that the probability his report will be affected is 1, higher than what investors expect. Therefore, from the bad manager's perspective, the improvement in internal control will reduce his probability of receiving the favorable report $r_A = 1$ more than being compensated by investors' increased valuation for report $r_A = 1$. As a result, he manipulates less in equilibrium.

Our single-firm model is a variant of the signal jamming model (e.g., **?**) with one important difference. It has the defining feature that even though the manager attempts to influence investors' belief through unobservable and costly manipulation, investors with rational expectations are not systematically misled. *On average* they see through the manipulation and break even. The manipulation eventually hurts the firm value through the distorted decisions.

Our model differs from **?** in that information asymmetry between investors and the manager persists in equilibrium. The manager knows his type while investors observe only report $r_A = 1$ that is a noisy signal of the manager's type s_A. This information asymmetry is consequential for investors' pricing. Since investors condition the pricing decision only on report r_A, the same stock price $P_A^*(1)$ is paid to both the truly good firm ($s_A = 1$) and the bad firm ($s_A = 0$) with successful manipulation. Investors rationally anticipate this information asymmetry and price protect themselves by discounting both types of firms. However, the non-discriminatory discounting implies that the stock price $P_A^*(1)$ is too low from the perspective of the truly good manager but too high from the perspective of the bad manager with successful manipulation. Even though manipulation doesn't systematically mislead investors, it does reduce the report's informativeness to investors.

3.1.2 Equilibrium manipulation with two firms

When there are two firms with correlated fundamentals, investors can also use the peer firm's report to improve their pricing decisions. We show that the informational spillover creates a peer pressure for manipulation: manager A's incentive to manipulate is increasing in his expectation that manager B has successfully manipulated report r_B.

We redefine the notations to accommodate the addition of firm B. Investors now use both report r_A and r_B to update their belief and set the stock price. Denote $\theta_A(r_A, r_B) \equiv \Pr(s_A = 1|r_A, r_B)$ as the investors' posterior about firm A being a good type after observing both reports r_A and r_B. We also use ϕ to denote a null signal. For example, $\theta_A(r_A, \phi)$ is the investors' posterior after observing r_A but before observing r_B. Thus, $\theta_A(r_A, \phi) = \theta_A(r_A)$ defined in the single-firm case. Similarly, we denote $P_A^*(r_A, r_B)$ as the stock price conditional on r_A and r_B. In addition, both investors and manager A observe firm B's internal control q_B and report r_B, but neither observes manager B's actual choice of manipulation m_B. Thus, both investors and manager A have to conjecture manager $B's$ equilibrium manipulation choices. Rational expectations require that in equilibrium the conjectures by both investors and manager A are the same as manager $B's$ equilibrium choice m_B^*. Moreover, since q_B is observable, investors and manager A conjecture that the probability that manager B successfully issues a fraudulent report is $\mu_B^* = m_B^*(1 - q_B)$.

Investors use report r_A and r_B to update their belief about s_A. Note that r_A and r_B are independent conditional on s_A. Thus, investors' belief can be updated in two steps. First, investors use r_A to update their prior from θ_A to the posterior $\theta_A(r_A, \phi)$. Second, treating $\theta_A(r_A, \phi)$ as a new prior for s_A, investors then use report r_B to update their belief just like in a single firm case. The conditional independence assumption allows us to convert the two-firm case into two iterations of the single-firm case.[4]

The equilibrium stock price $P_A^*(r_A, r_B)$ is now set to be equal to investors' expectation of the firm value V_i conditional on r_A and r_B :

$$P_A^*(r_A, r_B) = \theta_A(r_A, r_B) + (1 - \theta_A(r_A, r_B))(0 - C_A^*) - K_A(q_A). \tag{5}$$

[4] The claim is proved as follows. We can rewrite the conditional probability

$$\Pr(s_A|r_A, r_B) = \frac{\Pr(s_A|r_B)\Pr(r_A|s_A, r_B)}{\sum_{s_A} \Pr(s_A|r_B)\Pr(r_A|s_A, r_B)} = \frac{\Pr(s_A|r_B)\Pr(r_A|s_A)}{\sum_{s_A} \Pr(s_A|r_B)\Pr(r_A|s_A)}.$$

The first step uses the definition of conditional probability function and the Bayes rule, while the second step utilizies the conditional independence result that $\Pr(r_A|s_A, r_B) = \Pr(r_A|s_A)$. The rewriting makes it clear that adding report r_B to investors' information set is equivalent to replacing investors' prior of $\Pr(s_A)$ with a new one $\Pr(s_A|r_B)$. Similarly, we can change the order of r_A and r_B :

$$\Pr(s_A|r_A, r_B) = \frac{\Pr(s_A|r_A)\Pr(r_B|s_A, r_A)}{\sum_{s_A} \Pr(s_A|r_A)\Pr(r_B|s_A, r_A)} = \frac{\Pr(s_A|r_A)\Pr(r_B|s_A)}{\sum_{s_A} \Pr(s_A|r_A)\Pr(r_B|s_A)}.$$

Like in the single-firm case (eqn 3), the unfavorable report $r_A = 0$ reveals $s_A = 0$ perfectly. As a result, the additional report r_B doesn't affect investors' belief, *i.e.*, $\theta_A(0, \phi) = \theta_A(0, 1) = \theta_A(0, 0)$. Thus we focus on the favorable report $r_A = 1$.

Conditional on $r_A = 1$, report r_B will also affect investors' belief about s_A and the effect can go in either directions. Consider first the case of positive correlation $\rho > 0$. If firm B issues report $r_B = 1$, it improves investors' belief that $s_B = 1$. Because s_B is positively correlated with s_A, investors also believe that $s_A = 1$ is more likely. Hence, $\theta_A(1, 1) - \theta_A(1, \phi) > 0$. Report $r_B = 1$ induces investors to discount $r_A = 1$ less and enhances the credibility of $r_A = 1$. In this sense, report $r_B = 1$ provides camouflage for manager A's fraudulent report. On the other hand, if firm B issues an unfavorable report $r_B = 0$, then investors revise their belief about both firms' types and we have $\theta_A(1, 0) - \theta_A(1, \phi) < 0$. Report $r_B = 0$ reduces the credibility of report $r_A = 1$. In this sense, report $r_B = 0$ confronts the fraudulent report $r_A = 1$. Thus, manager A prefers $r_B = 1$ to $r_B = 0$.

Consider the other case of $\rho < 0$. Now if firm B issues a favorable report $r_B = 1$, investors are more pessimistic about firm A's type, *i.e.*, $\theta_A(1, 1) - \theta_A(1, \phi) < 0$. Otherwise, if firm B issues an unfavorable report $r_B = 0$, investors revise upward their belief about $s_A = 1$ and pay firm A a higher price. From the perspective of the bad manager A, report $r_B = 1$ confronts his fraudulent report $r_A = 1$ while report $r_B = 0$ provides cover for him. Therefore, manager A prefers $r_B = 0$ to $r_B = 1$.

The magnitude by which investors revise their belief upon observing r_B depends on the report B's informativeness about s_B and the correlation ρ between s_A and s_B. As we have discussed in the single firm case, the report B's informativeness about s_B is decreasing in μ_B^*. Investors attach less credibility to $r_B = 1$ as they suspect that it is more likely from a bad manager. We summarize these discussions in the next lemma.

Lemma 2 *When there are two firms,*

1. *if* $\rho > 0$, $\theta_A(1, 1) - \theta_A(1, \phi)$ *and* $\theta_A(1, 1) - \theta_A(1, 0)$ *are positive and decreasing in* μ_B^*;
2. *if* $\rho < 0$, $\theta_A(1, 1) - \theta_A(1, \phi)$ *and* $\theta_A(1, 1) - \theta_A(1, 0)$ *are negative and increasing in* μ_B^*.

With investors' equilibrium response to the reports, we can verify that, like in the single firm case, manager A has incentive to manipulate because $P_A^*(1, r_B) - P_A^*(0, r_B) = \theta_A(1, r_B)(1 + C_A^*) > 0$ for any r_B.

Anticipating the investors' pricing response, manager A chooses manipulation at $t = 1$. At the time manager A chooses his manipulation, he doesn't observe report r_B or manager B's actual choice of manipulation m_B. Instead, he conjectures that manager B will choose manipulation m_B^* and thus succeed in issuing a fraudulent report with probability $\mu_B^* = m_B^*(1 - q_B)$. His best response to μ_B^*, denoted as $\tilde{m}_A^*(\mu_B^*)$, is determined by the following first-order condition:

$$H^A(m_A; m_B^*)|_{m_A = \tilde{m}_A^*} \equiv \delta_A \frac{\partial \mu_A}{\partial m_A} E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A^*) - (1 - \delta_A) C_A'(m_A) = 0. \quad (6)$$

Equation 6 is similar to equation 4 in the single firm case except that $\theta_A(1)$ is replaced by $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ (and that we have imposed investors' rational expectations about m_A^*). Manager A bases his manipulation on his expectation about investors' belief about his firm averaged over report r_B. We call $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ the manager's expected payoff (from successful manipulation) for ease of notation. Since μ_B^* is not affected by manager A's actual choice of m_A, we can treat μ_B^* as a parameter in equation 6 and examine the manager's best (manipulation) response to parameter μ_B^*. Since μ_B^* and m_B^* differ only by an observable constant, we use $\tilde{m}_A^*(.)$ as the manager's best response to both m_B^* and μ_B^* to save on notations.

Proposition 1 *For any interior μ_B^*, $\tilde{m}_A^*(\mu_B^*)$ is increasing in μ_B^*. In particular, $\tilde{m}_A^*(\mu_B^*)$ is increasing in m_B^* and decreasing in q_B.*

Proposition 1 states that manager A's incentive to manipulate is increasing in his *suspicion* that manager B will succeed in manipulating report r_B. The suspicion arises from either his conjecture of manager B's manipulation m_B^* or his observation of firm B's internal control choice q_B. This resembles some aspects of the peer pressure for manipulation we have discussed in the Introduction. When manager A expects manager B to manipulate more, the opportunity cost for manager A to refrain from manipulating is higher.

We discuss the intuition behind Proposition 1. To fix the idea, suppose the new manager to firm B has a higher δ_B. Both investors and manager A observe this change and conjecture that the equilibrium m_B^* and μ_B^* will be higher (according to Lemma 1). We need to explain why manager A responds to this expected increase in μ_B^* by increasing his own manipulation.

We can write out the bad manager's expected payoff or his expectation of the investors' belief about his firm's type as

$$E_{r_B}[\theta_A(1, r_B)|s_A = 0] = \theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)[\theta_A(1, 1) - \theta_A(1, 0)] \tag{7}$$

We start with a benchmark in which the investors are assumed to be naive. They don't anticipate the change in the distribution of report r_B induced by an expected increase in μ_B^*. In other words, we impose $\frac{\partial[\theta_A(1,1)-\theta_A(1,0)]}{\partial \mu_B^*} = 0$. Equation 7 then suggests that the effect of μ_B^* on manager A's expectation is determined solely by its effect on $\Pr(r_B = 1|s_A = 0)$. Again we consider the case of $\rho > 0$ first. Manager A expects that an increase in μ_B^* will increase the frequency of favorable report $r_B = 1$. Since report $r_B = 1$ provides camouflage for manager A's manipulation (part 1 of Lemma 2), manager A now is tempted to manipulate more. Consider the other case of $\rho < 0$. Manager A still expects that $r_B = 1$ becomes more often as μ_B^* increases. However, report $r_B = 1$ now contradicts firm A's fraudulent report (part 2 of Lemma 2). As a result, manager A manipulates less.

In sum, when investors are naive, the manager's response to his peer firm' manipulation depends on the sign of the correlation. As he expects an increase in the manipulation likelihood of his peer firm's report, he manipulates more when two firms' fundamentals are positively correlated but less when they are negatively correlated.

Investors, however, are rational in our model. They understand the inflation of the distribution of report r_B induced by an increase in μ_B^* and discount report $r_B = 1$ accordingly. The next lemma summarizes how investors' rationality works in our context.

Lemma 3 $E_{r_B}[\theta_A(1, r_B)] = \theta_A(1, \phi)$ *and is independent of* μ_B^*.

Lemma 3 is an immediate consequence of the investors' rationality or the law of iterated expectations. It merely states that the arrival of report r_B will not affect investors' belief on average. Since the average belief is not influenced by the presence of report r_B, it is independent of μ_B^* that influences the distribution of r_B. Combining Lemma 3 with Lemma 2 that describes how μ_B^* affects investors' belief ex post (after observing r_B), we can describe how rational investors respond to an expected increase in μ_B^*:[5]

$$\theta_A(1,1) - \theta_A(1,0) = \frac{\theta_A(1,\phi) - \theta_A(1,0)}{\Pr(r_B = 1 | r_A = 1; \mu_B^*)}. \tag{8}$$

Equation 8 states that investors discount report $r_B = 1$ to fully offset their expectations of the inflation in the distribution of report r_B. Since the numerator $\theta_A(1,\phi) - \theta_A(1,0)$ is not a function of μ_B^*, μ_B^* affects the investors' expectation through two channels. First, as μ_B^* increase, investors expect to receive $r_B = 1$ more often. That is, the denominator $\Pr(r_B = 1|r_A = 1)$ is increasing in μ_B^*. We call this the probability effect of μ_B^*. Second, rationally anticipating the inflation, investors discount report $r_B = 1$ accordingly. That is, $\theta_A(1,1) - \theta_A(1,0)$, the belief difference investors attach to report $r_B = 1$ relative to $r_B = 0$, moves towards 0 as μ_B^* increases, as can be seen from Lemma 2. We term this effect the discounting effect of μ_B^*. Lemma 3 claims that the probability and the discounting effects have to cancel out each other perfectly so that μ_B^* doesn't affect the investors' belief about s_A averaged over r_B. Hence equation 8.

We can rewrite the bad manager's expected payoff (equation 7) by plugging in equation 8:

$$E_{r_B}[\theta_A(1, r_B)|s_A = 0] = \theta_A(1,0) + \frac{\Pr(r_B = 1|r_A = 1)}{\Pr(r_B = 1|s_A = 0)}[\theta_A(1,\phi) - \theta_A(1,0)]. \tag{9}$$

Since μ_B^* affects neither $\theta_A(1,0)$ nor $\theta_A(1,\phi) - \theta_A(1,0)$, equation 9 shows that μ_B^* affects the bad manager's expected payoff only through its differential effects on the manager and investors' beliefs about the probability effect. Consider the case of $\rho > 0$ first. An increase in μ_B^* makes $r_B = 1$ more frequent if firm B is a bad type. Investors observe $r_A = 1$ and believe that firm B is a bad type with probability $\Pr(s_B = 0|r_A = 1)$. In contrast, the bad manager with private information $s_A = 0$ knows that firm B is more likely to be the bad type than

[5] This equation is obtained as follows. We write out investors' expectation of $s_A = 1$ as $E_{r_B}[\theta_A(1, r_B)] = \theta_A(1,0) + \Pr(r_B = 1|r_A = 1)[\theta_A(1,1) - \theta_A(1,0)] = \theta_A(1,\phi)$. The second equality is given by Lemma 3. Rearranging the terms leads to equation 8.

investors believe, *i.e.*, $\Pr(s_B = 0|s_A = 0) > \Pr(s_B = 0|r_A = 1)$ for $\rho > 0$. Thus, from the perspective of the bad manager A investors under-estimate the probability effect and don't discount $r_B = 1$ sufficiently when they use report $r_B = 1$ to evaluate $r_A = 1$. Since report $r_B = 1$ provides camouflage for manager A's manipulation (part 1 of Lemma 2), investors' insufficient discounting of $r_B = 1$ increases manager A's payoff from delivering $r_A = 1$ and thus induces manager A to manipulate more.

Now consider the case of $\rho < 0$. An increase in μ_B^* still makes $r_B = 1$ more frequent if firm B is a bad type. Investors use report $r_A = 1$ to forecast the type of firm B while manager A uses $s_A = 0$. With $\rho < 0$, the bad manager knows that firm B is less likely to be a bad type than investors believe, *i.e.*, $\Pr(s_B = 0|s_A = 0) < \Pr(s_B = 0|r_A = 1)$ for $\rho < 0$. As a result, from the bad manager's perspective investors over-estimate the probability effect and discount $r_B = 1$ excessively. Since report $r_B = 1$ confronts the fraudulent report $r_A = 1$ (part 2 of Lemma 2), investors' excessive discounting of report $r_B = 1$ reduces the threat that manager A's fraudulent report is confronted. As a result, he manipulates more.

Overall, regardless of the sign of ρ, manager A manipulates more when he suspects that manager B is more likely to manipulate successfully. This peer pressure for manipulation is driven by two intertwined forces. First, manipulation by manager A leads to information asymmetry about firm A's fundamental in equilibrium between manager A and investors, as we have discussed in the single-firm case. This information asymmetry enables the bad manager with successful manipulation to be pooled with the truly good ones and benefit from manipulation. Second, the information asymmetry also enables manager A to forecast the impact of μ_B^* better than investors. Proposition 1 emphasizes that, conditional on the availability of report r_B, an increase in the manipulation of report r_B exacerbates the information asymmetry and benefits the fraudulent manager A. In other words, more manipulation of report B makes it easier for the fraudulent report from firm A to be camouflaged by $r_B = 1$ (when $\rho > 0$) and harder to be confronted by $r_B = 1$ (when $\rho < 0$). As a result, the bad manager A increases his manipulation.

An extreme case can illustrate the intuition further. Consider a special case $\rho = 1$ so that s_A and s_B are perfectly correlated. Suppose we start with $m_B^* = 0$ and thus $\mu_B^* = 0$. Since manager B never manipulates, investors don't discount report $r_B = 1$. However, the bad manager A will not manipulate either because he fears that there will be no camouflage for his fraudulent report from report $r_B = 1$. The perfect correlation between s_A and s_B means that he privately knows that $s_B = 0$ for sure and thus $r_B = 1$ with probability 0. Now suppose m_B^* increases by a small amount ε so that $\mu_B^* > 0$. The bad manager now will engage in a positive amount of manipulation. He anticipates that investors will discount $r_B = 1$ a little bit by narrowing the price difference $\theta_A(1,1) - \theta_A(1,0)$ slightly. However, he also expects that there is a positive probability of receiving $r_B = 1$, resulting in a positive expected payoff of manipulation. Thus, as μ_B^* moves away from 0, manager A starts to manipulate as well.

? have proven a general statistical result that, loosely speaking, a more informative experiment will on average bring the posteriors of two agents with different priors closer to their respective priors. They call it information-validates-the-prior (IVP) theorem. The IVP theorem provides another way to see the intuition of Proposition 1. As we have discussed at the end of the previous section, information asymmetry occurs in equilibrium between manager A and the investors in our model. The bad manager A's belief about s_A is lower than that of the investors. Firm B's report can be viewed as an informative (though endogenous) experiment about s_A. As manager B increases manipulation in equilibrium, the informativeness of the experiment becomes lower. According to the IVP theorem, the disagreement or information asymmetry between manager A and his investors increases. As a result, manager A manipulates more. Therefore, Proposition 1 provides another application of the IVP theorem.

To fully pin down the effect of the internal control on manipulation, we have to endogenize firm B's manipulation decision as well. $H^A(\tilde{m}_A^*; m_B^*) = 0$ defined by equation 6 characterizes manager A's best response to manager B's equilibrium choice. Using the same procedure, we can solve manager B's best response to manager A's equilibrium choice, $\tilde{m}_B^*\left(m_A^*\right)$. It is characterized by the following equation:

$$H^B(m_B; m_A^*)|_{m_B=\tilde{m}_B^*} \equiv \delta_B\,(1-q_B)\,E_{r_A}[\theta_B(1,r_A)|s_B=0](1+C_B^*) - (1-\delta_B)\,C_B'(m_B) = 0. \tag{10}$$

Equation 6 and equation 10 jointly determine the managers' optimal choices of manipulation (m_A^*, m_B^*) through $m_A^* = \tilde{m}_B^*(m_A^*)$ and $m_B^* = \tilde{m}_A^*(m_B^*)$. Again the equilibrium is unique under our assumption that the manipulation cost functions are sufficiently convex. After we solve for the unique equilibrium, the optimal manipulation decisions (m_A^*, m_B^*) can be expressed as functions of the firms' choices of internal control (q_A, q_B) and we can examine the equilibrium effect of internal control on manipulation.

Proposition 2 $\frac{\partial m_A^*(q_A,q_B)}{\partial q_A} < 0$ *and* $\frac{\partial m_B^*(q_B,q_A)}{\partial q_A} < 0$.

Proposition 2 confirms that a firm's internal control has an externality on its peer firm. An improvement in one firm's internal control quality reduces not only its own manager's manipulation (*i.e.*$\frac{\partial m_A^*(q_A,q_B)}{\partial q_A} < 0$) but also the peer manager's manipulation (*i.e.*$\frac{\partial m_B^*(q_B,q_A)}{\partial q_A} < 0$) via the mitigation of the peer pressure for manipulation. Lemma 1 shows that a firm's internal control directly deters its own manager's manipulation and reduces μ_A^* (the probability that report A is manipulated). Proposition 1 suggests that, through symmetry between firm A and B, a lower μ_A^* results in a lower m_B^*. In other words, firm A's internal control indirectly mitigates the peer pressure for manipulation on manager B and reduces manager B's manipulation. Of course, the reduction of manipulation by manager B alleviates the pressure on manager A as well, setting into motion a loop of feedbacks. Through this loop, the effect of a firm's internal control on its own manager's manipulation is amplified.

3.2 Equilibrium internal control decisions

In the previous section, we characterized the managers' manipulation decisions at $t = 1$, taking the firms' internal control choices q_A and q_B at $t = 0$ as given. In this section we endogenize the firms' internal control decisions. We show that even though firms do have private incentive to invest in internal control, they under-invest in internal control due to the externality described in Proposition 2.

After understanding managers' manipulation decisions, we fold back to $t = 0$ and consider firm A's private incentive to invest in costly internal control over financial reporting. Since firm A doesn't observe firm B's choice of internal control at the time of choosing q_A, it conjectures that firm B will choose q_B^* in equilibrium. Moreover, firm A anticipates that managers at $t = 1$ respond to its actual choice of q_A through $m_A^*(q_A, q_B^*)$ and $m_B^*(q_B^*, q_A)$. Based on these expectations, the firm value at $t = 0$ as a function of its internal control choice q_A is

$$V_{A0}(q_A; q_B^*) \equiv E_0[V_A(q_A, q_B^*)] = \theta - \Pr(s_A = 0) C_A(m_A^*(q_A, q_B^*)) - K_A(q_A). \qquad (11)$$

The firm value at $t = 0$ has three components. The first is the expected gross cash flow $E_{v_A}[s_A] = \theta$ in absence of manipulation and internal control investment. Second, manipulation generates the expected deadweight loss $\Pr(s_A = 0) C_A(m_A^*(q_A, q_B^*))$. The existence of this deadweight loss means that the firm has private incentive to invest in internal control to prevent manipulation. Finally, the internal control investment itself consumes resources and reduces the firm value by $K_A(q_A)$.

Firm A at $t = 0$ chooses q_A to maximizes $V_{A0}(q_A, q_B^*)$ subject to the managers' subsequent equilibrium manipulation responses $m_A^*(q_A, q_B^*)$ and $m_B^*(q_B^*, q_A)$. Differentiating $V_{A0}(q_A, q_B^*)$ in equation 11 with respect to q_A, the first-order condition is

$$\frac{\partial V_{A0}}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A} - K'(q_A) = 0. \qquad (12)$$

The first term $\frac{\partial V_{A0}}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A}$ captures the benefit of internal control q_A in reducing manipulation. It is balanced by the marginal cost $K'(q_A)$. Note that $\tilde{q}_A^*(q_B^*)$ implied by the first order condition is firm A's best response to its conjecture of firm B's equilibrium internal control choice q_B^*. We can solve firm B's internal control decision using the same procedure. Taking q_A^* as given, firm B's best response to q_A^*, $\tilde{q}_B^*(q_A^*)$, is characterized by a similar first order condition. Intercepting the two best responses $q_A^* = \tilde{q}_A^*(q_B^*)$ and $q_B^* = \tilde{q}_B^*(q_A^*)$, we prove that there exists a unique equilibrium when the cost function of internal control investment is sufficiently convex. This complete the characterization of the entire equilibrium.

Proposition 3 *The unique equilibrium* $\{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_B, r_A)\}$ *is collectively characterized by equation 12, equation 6,* $m_A^*(s_A = 1) = 0$, *equation 5 and their* *counterparts for firm B.*

After we have solved for the entire equilibrium, we can compare the private and social incentives for internal control investment. So far in the model, each firm chooses its internal control investment independently to maximize its own firm value. Now consider a hypothetical case in which a social planner chooses the internal control decisions for both firms to maximize the value of the two firms combined. Denote the social planer's choices as (q_A^{SP}, q_B^{SP}). To avoid the distributional issue between the two firms, we use the assumption that the two firms are symmetric and denote $q^* = q_A^* = q_B^*$ and $q^{SP} = q_A^{SP} = q_B^{SP}$.

Proposition 4 *Firms under-invest in their internal control relative to the social planer's choice, i.e.,* $q^* < q^{SP}$.

To see the under-investment problem, consider the social planner's internal control decision of q_A, given her choice q_B^{SP} for firm B. Anticipating the equilibrium manipulation $m_A^*\left(q_A, q_B^{SP}\right)$ and $m_B^*\left(q_B^{SP}, q_A\right)$, the social planner chooses q_A to maximize the value of both firms $V_{A0}\left(q_A, q_B^{SP}\right) + V_{B0}\left(q_B^{SP}, q_A\right)$ and the first-order condition becomes

$$\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} + \frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial q_A} - K_A'(q_A) = 0.$$

The first term $\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A}$ is the benefit of internal control for firm A. The second term $\frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial q_A}$ captures the positive externality firm A's internal control exerts on firm B. Proposition 2 suggests that $\frac{\partial m_B^*}{\partial q_A} < 0$. The reduction in m_B^* improves the firm value of firm B. Since firm A ignores this benefit when choosing q_A, q_A^* is lower than the level the social planner chooses.

Proposition 4 suggests that there could be a coordination failure among firms' individual choices of internal control over financial reporting. It provides one rationale for intervention in firms' internal control investment. In the presence of peer pressure for manipulation, one firm's internal control investment has positive externality on other firms. A floor of internal control investment could improve the firm value of all firms. Looking through the lens of Proposition 4, the proposal in **?** that the internal control mandates in SOX should be made optional is flawed. Competition among firms (or state laws) doesn't lead to socially optimal investment in internal control.

We have highlighted one potential benefit of regulating internal control over financial reporting. However, regulations come with their own costs. Since **?**, we know that regulators' incentives may be aligned with the regulated firms and organized vested interest groups more than with the social welfare. Moreover, even well-intended regulators may not be able to achieve the full potential of regulations due to the information disadvantage in designing and implementing regulations. Thus, while our model provides one possible benefit of regulations like SOX, a comprehensive evaluation of SOX is a complicated enterprise and beyond the scope of our paper. We refer the readers to the recent surveys like **?** and **?**.

4 Extensions

The core of our model is the peer pressure for manipulation. A manager manipulates more when he expects that the reports from his peer managers are more likely to be manipulated. This peer pressure for manipulation leads to the externality of internal control investment over financial reporting and ultimately the under-investment in internal control investment. We have made a number of simplifying assumptions in the baseline model in order to highlight the economic forces behind the peer pressure for manipulation. In this section, we discuss two extensions to the baseline model.

4.1 N Firms

We have focused on a two-firm economy in the baseline model. It is straightforward to extend the model to $N > 2$ firms. Suppose the pair-wise correlation coefficient between any two firms i and j is $\rho_{ij} \neq 0$. Consider manager i's manipulation decision m_i^* as a best response to $\mu_j^* \equiv m_j^*(1 - q_j)$. μ_j^* is manager i's conjectured probability that a bad firm j has successfully manipulated its report. The following results can be obtained using a similar proof of Proposition 1.

Proposition 5 *$m_i^*(\mu_j^*)$ is increasing in μ_j^* for any $j \neq i$, regardless of the sign of ρ_{ij}.*

A manager manipulates more if he suspects that any of his peer firms' reports is more likely to be manipulated. We omit the proof of this Proposition because it is an immediate result of the conditional independence property of the reports and Proposition 1. Conditional on the fundamental s_i, report r_i and r_j are independent of each other. As a result, the presence of any additional firms $k \in N \backslash \{i, j\}$ doesn't affect the interaction between firm i and j. In other words, investors can first use all firms' reports r_k, other than r_i and r_j, to update their belief about s_i. Treating this posterior as a prior, investors continue to use report r_i and r_j, like in our baseline model of two firms.[6] The peer pressure holds for any pair of firms within in the N firms as long as their fundamentals are correlated.

4.2 Imperfect information

Another simplification in the baseline model is that we assume that the managers know their firms' fundamentals perfectly. If at the time managers receive only noisy private information about their firms' fundamentals there would be measurement errors in the reports in the absence of manipulation. Manipulation then may help correct the measurement errors.

Now we assume that the fundamental or the gross cash flow is $v_i \in \{0, 1\}$. Each manager receives a noisy signal $s_i \in \{0, 1\}$ about $v_i : \Pr(s_i = 1|v_i = 1) = \Pr(s_i = 0|v_i = 0) = \tau \in [\frac{1}{2}, 1]$. τ measures the quality of managers' signals and our baseline model is a special case of $\tau = 1$. With this specification, we can replicate Proposition 1 that manager A with $s_i = 0$ manipulates more if he expects that report B is more likely to be manipulated. The proof goes through essentially by redefining the fundamental s_i. Even though managers receive noisy signals about the fundamentals, they still know more than the investors and thus the information asymmetry persists in equilibrium.

5 Discussions and the conclusion

We have presented a model to show that a firm's investment in internal control has a positive externality on peer firms. The core of the mechanism is the strategic complementarity of

[6] See footnote 4 for the discussion of the implications of the conditional independence properties.

manipulation. A manager manipulates more if he expects that the reports from peer firms are more likely to be manipulated. As a result, a firm's investment in internal control benefits not only itself by reducing its own manager's manipulation but also the peer firms by mitigating the manipulation pressure on peer managers. Without internalizing this positive externality, firms under-invest in internal control over financial reporting. Regulations that provide a floor of internal control investment can mitigate the under-investment problem.

We have presented a stylized model to deliver the intuition for the strategic complementarity of manipulation and the externality of a firm's internal control. In particular, the binary structure has dramatically simplified the exposition. However, we believe that the economic forces behind the peer pressure for manipulation are more general. The strategic complementarity between the two managers' manipulation decisions is driven by two features of the model. First, manager A can forecast manager B's manipulation better than the investors. Second, manipulation reduces the report's informativeness. The first feature arises naturally in a setting when the managers have private information about their own fundamentals that are correlated with each other. Thus, as long as the second feature is preserved in a richer model in which manipulation leads to information degradation, the strategic complementarity between two managers' decisions is expected to be intact.

We have focused on the capital market pressure as the driver for accounting manipulation, which appears empirically important (e.g., **?**). As a result, we have assumed that the two firms are independent except for the correlation of their fundamentals. In practice, peer firms are likely to interact with each other in other areas (such as product markets, labor markets, performance benchmarking, and regulation) and those interactions may also lead to interactions of their accounting decisions. As we have discussed in the literature review, these other interactions are promising venue for future research.

6 Appendix

Proof. of Lemma 1: For notational ease, we use $C_A^* = C_A(m_A^*)$ whenever no confusion arises. For a given $q_A \in (0,1)$, after we impose the rational expectations requirement, m_A^* is determined by the first-order condition:

$$H(m_A^*) = \delta_A (1 - q_A) \theta_A (1) (1 + C_A^*) - (1 - \delta_A) C_A'(m_A^*) = 0.$$

We first verify that the equilibrium is unique, *i.e.*, $H(m_A^*) = 0$ has a unique solution. First, under our assumption that C_A is sufficiently convex, we have $\frac{\partial H(m_A^*)}{\partial m_A^*} < 0$. This is because

$$\begin{aligned}
\frac{\partial H(m_A^*)}{\partial m_A^*} &= \delta_A (1 - q_A) \frac{\partial}{\partial m_A^*} [(1 + C_A^*) \theta_A (1)] - (1 - \delta_A) C_A''(m_A^*) \\
&= (\delta_A (1 - q_A) \theta_A (1))^2 \frac{1 + C_A^*}{1 - \delta_A} + \delta_A (1 - q_A) (1 + C_A^*) \frac{\partial \theta_A (1)}{\partial m_A^*} - (1 - \delta_A) C_A''(m_A^*).
\end{aligned}$$

The sign of $\frac{\partial H(m_A^*)}{\partial m_A^*}$ is dominated by the sign of $C_A''(m_A^*)$. Second, at $m_A^* = 0$, $H(0) = \delta_A (1 - q_A) \theta_A (1; m_A^* = 0) > 0$. Finally, at $m_A^* = 1$, $H(1) = \delta_A (1 - q_A) \theta_A (1; m_A^* = 1) (1 + C_A (1)) - (1 - \delta_A) C_A'(1) = -\infty < 0$. Therefore, by the intermediate value theorem, the equilibrium m_A^* that satisfies $H(m_A^*) = 0$ is unique.

For any parameter $x \in \{\theta_A, \delta_A, q_A\}$, the application of the implicit function theorem generates

$$\frac{\partial m_A^*}{\partial x} = -\frac{\frac{\partial H(m_A^*; x)}{\partial x}}{\frac{\partial H(m_A^*)}{\partial m_A^*}}.$$

The denominator $\frac{\partial H(m_A^*)}{\partial m_A^*} < 0$. As a result, the sign of $\frac{\partial m_A^*}{\partial x}$ is the same as that of $\frac{\partial H(m_A^*; x)}{\partial x}$. In particular,

$$\frac{\partial H(m_A^*; \delta_A)}{\partial \delta_A} = (1 - q_A) (1 + C_A^*) \theta_A (1) + C_A'(m_A^*) > 0,$$

$$\begin{aligned}
\frac{\partial H(m_A^*; \theta_A)}{\partial \theta_A} &= \delta_A (1 - q_A) (1 + C_A^*) \frac{\partial \theta_A (1)}{\partial \theta_A} \\
&= \delta_A (1 - q_A) (1 + C_A^*) \frac{\mu_A^*}{\left[\theta_A + (1 - \theta_A) \mu_A^*\right]^2} > 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H(m_A^*; q_A)}{\partial q_A} &= -\delta_A (1 + C_A^*) \theta_A (1) + \delta_A (1 - q_A) (1 + C_A^*) \frac{\partial \theta_A (1)}{\partial q_A} \\
&= \delta_A (1 + C_A^*) (-\theta_A (1) + \theta_A (1) (1 - \theta_A(1))) \\
&= -\delta_A (1 + C_A^*) \theta_A^2 (1) < 0.
\end{aligned}$$

Notice that for the last result $\frac{\partial H(m_A^*; q_A)}{\partial q_A} < 0$, there are two components. The first component $-\delta_A (1 + C_A^*) \theta_A(1) < 0$ is the direct effect and the second component $\delta_A (1 - q_A) (1 + C_A^*) \frac{\partial \theta_A(1)}{\partial q_A} > 0$ is the indirect effect. As we discussed in the text, the direct effect dominates the indirect effect. ■

Proof. of Lemma 2: We first use the Bayes rule to write out

$$\begin{aligned}\theta_A(1,1)-\theta_A(1,\phi) &= \frac{\theta_A(1,\phi)}{\theta_A(1,\phi)+(1-\theta_A(1,\phi))\frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}}-\theta_A(1,\phi)\\ &= \frac{\theta_A(1,\phi)(1-\theta_A(1,\phi))\left[1-\frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}\right]}{\theta_A(1,\phi)+(1-\theta_A(1,\phi))\frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}}.\end{aligned}$$

$\theta_A(1,\phi)=\theta_A(1)$ is expressed in equation 2.

Therefore, $\theta_A(1,1)-\theta_A(1,\phi)>0$ if and only if $\frac{\Pr(r_B=1|s_A=0)}{\Pr(r_B=1|s_A=1)}<1$ or equivalently $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}>1$. Moreover, since $\theta_A(1,\phi)$ is independent of μ_B^*, $\theta_A(1,1)-\theta_A(1,\phi)$ is increasing in μ_B^* if and only if $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ is increasing in μ_B.

We then write out

$$\begin{aligned}&\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}-1\\ =&\ \frac{\Pr(r_B=1|s_B=1,s_A=1)\Pr(s_B=1|s_A=1)+\Pr(r_B=1|s_B=0,s_A=1)\Pr(s_B=0|s_A=1)}{\Pr(r_B=1|s_B=1,s_A=0)\Pr(s_B=1|s_A=0)+\Pr(r_B=1|s_B=0,s_A=0)\Pr(s_B=0|s_A=0)}-1\\ =&\ \frac{\Pr(r_B=1|s_B=1)\Pr(s_B=1|s_A=1)+\Pr(r_B=1|s_B=0)\Pr(s_B=0|s_A=1)}{\Pr(r_B=1|s_B=1)\Pr(s_B=1|s_A=0)+\Pr(r_B=1|s_B=0)\Pr(s_B=0|s_A=0)}-1\\ =&\ \frac{\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}\Pr(s_B=1|s_A=1)+\Pr(s_B=0|s_A=1)}{\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}\Pr(s_B=1|s_A=0)+\Pr(s_B=0|s_A=0)}-1\\ =&\ \frac{\left(\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}-1\right)\Pr(s_B=1|s_A=1)+1}{\left(\frac{\Pr(r_B=1|s_B=1)}{\Pr(r_B=1|s_B=0)}-1\right)\Pr(s_B=1|s_A=0)+1}-1\\ =&\ \frac{\left(\frac{1}{\mu_B^*}-1\right)}{\left(\frac{1}{\mu_B^*}-1\right)\Pr(s_B=1|s_A=0)+1}\left(\Pr(s_B=1|s_A=1)-\Pr(s_B=1|s_A=0)\right)\\ =&\ \frac{\left(\frac{1}{\mu_B^*}-1\right)}{\left(\frac{1}{\mu_B^*}-1\right)\Pr(s_B=1|s_A=0)+1}\rho.\\ \propto&\ \rho\end{aligned}$$

"$\propto$" reads as "have the same sign as." The second last equality holds because

$$\begin{aligned}\Pr(s_B=1|s_A=1)-\Pr(s_B=1|s_A=0) &= \frac{\Pr(s_B=1,s_A=1)}{\Pr(s_A=1)}-\frac{\Pr(s_B=1,s_A=0)}{\Pr(s_A=0)}\\ &= \frac{\Pr(s_B=1,s_A=1)}{\Pr(s_A=1)}-\frac{\Pr(s_B=1)-\Pr(s_B=1,s_A=1)}{\Pr(s_A=0)}\\ &= \frac{\theta^2+\rho\theta(1-\theta)}{\theta}-\frac{\theta-\left(\theta^2+\rho\theta(1-\theta)\right)}{1-\theta}\\ &= \rho.\end{aligned}$$

The last step holds because $\mu_B^*\in(0,1)$ and thus $\frac{1}{\mu_B^*}>1$. Thus, $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}>1$ if and only if $\rho > 0$.

Moreover, we can show that $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ is increasing in μ_B^* if and only if $\rho < 0$, because

$$\begin{aligned} \frac{\partial}{\partial \mu_B^*} \frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)} &\propto -\left(\Pr\left(s_B = 1|s_A = 1\right) - \Pr\left(s_B = 1|s_A = 0\right)\right) \\ &\propto -\rho. \end{aligned} \tag{13}$$

The proof for the properties of $\theta_A(1,1) - \theta_A(1,0)$ is similar and hence omitted. Therefore, we have proved Lemma 2. ■

Proof. of Proposition 1 and Lemma 3:

The proof of Lemma 3 is straightforward. $E_{r_B}[\theta_A(1, r_B)] = E[\theta_A(1, \phi)] = \theta_A(1)$. The first step is by the reverse use of the law of iterated expectation and the second step is by definition. From equation 2, we know that $\theta_A(1)$ is independent of μ_B^*. Therefore, $E_{r_B}[\theta_A(1, r_B)]$ is independent of μ_B^*.

Now we prove Proposition 1. For given interior q_A and μ_B^* and investors' conjecture m_A^*, manager A's best response $\tilde{m}_A^*(\mu_B^*)$ is determined by the first-order condition:

$$H^A(\tilde{m}_A^*; \mu_B^*) \equiv \delta_A (1 - q_A) E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A(\tilde{m}_A^*(\mu_B^*))) - (1 - \delta_A) C_A'(\tilde{m}_A^*) = 0.$$

The application of the implicit function theorem generates

$$\frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*} = -\frac{\frac{\partial H^A(\tilde{m}_A^*;\mu_B^*)}{\partial \mu_B^*}}{\frac{\partial H^A}{\partial \tilde{m}_A^*}}.$$

The denominator $\frac{\partial H^A}{\partial \tilde{m}_A^*}$ is negative following a similar proof in Lemma 1. Thus, the sign of $\frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*}$ is the same as that of $\frac{\partial H^A(\tilde{m}_A^*;\mu_B^*)}{\partial \mu_B^*}$. Note that μ_B^* shows up in $H^A(\tilde{m}_A^*; \mu_B^*)$ only through $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. Therefore, $\frac{\partial H^A(\tilde{m}_A^*;\mu_B^*)}{\partial \mu_B^*}$ has the same sign as $\frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial \mu_B^*}$. We now prove that $\frac{\partial E_{r_B}[\theta_A(1,r_B)|s_A=0]}{\partial \mu_B^*} > 0$. In words, the bad manager becomes more optimistic about investors' belief as μ_B^* increases.

We first write out the investors' belief about $s_A = 1$ before they observe r_B but after they observe $r_A = 1$:

$$\begin{aligned} \theta_A(1, \phi) &\equiv \Pr\left(s_A = 1|r_A = 1\right) \\ &= \Pr(r_B = 0|r_A = 1)\theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)\theta_A(1, 1) \\ &= \theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)\left(\theta_A(1, 1) - \theta_A(1, 0)\right). \end{aligned}$$

The second step writes out the expectation and the third regroups the terms. This gives us equation 8 in the text which we reproduce here:

$$\theta_A(1, 1) - \theta_A(1, 0) = \frac{\theta_A(1, \phi) - \theta_A(1, 0)}{\Pr(r_B = 1|r_A = 1)}. \tag{14}$$

We can similarly write out the bad manager's expectation about the investors' expectation of $s_A = 1$ as

$$\begin{aligned}
& E_{r_B}\left[\theta_A(1, r_B)|s_A = 0\right] \\
= \; & \Pr(r_B = 0|s_A = 0)\theta_A(1,0) + \Pr(r_B = 1|s_A = 0)\theta_A(1,1) \\
= \; & \theta_A(1,0) + \Pr(r_B = 1|s_A = 0)\left(\theta_A(1,1) - \theta_A(1,0)\right) \\
= \; & \theta_A(1,0) + \frac{\Pr(r_B = 1|s_A = 0)}{\Pr(r_B = 1|r_A = 1)}\left(\theta_A(1,\phi) - \theta_A(1,0)\right) \\
= \; & \theta_A(1,0) + \frac{\Pr(r_B = 1|s_A = 0)\left(\theta_A(1,\phi) - \theta_A(1,0)\right)}{\Pr(r_B = 1|s_A = 0)\Pr(s_A = 0|r_A = 1) + \Pr(r_B = 1|s_A = 1)\Pr(s_A = 1|r_A = 1)} \\
= \; & \theta_A(1,0) + \frac{1}{\Pr(s_A = 0|r_A = 1) + \frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}\Pr(s_A = 1|r_A = 1)}\left(\theta_A(1,\phi) - \theta_A(1,0)\right).
\end{aligned}$$

Again the first step writes out the expectation and the second regroups the terms. The third step plugs in equation 14. The last step writes out the total probability of $\Pr(r_B = 1|r_A = 1)$ and reorganize the terms. Note that μ_B^* only affects the likelihood ratio $\frac{\Pr(r_B=1|s_A=1)}{\Pr(r_B=1|s_A=0)}$ in the last equality. Thus, we have

$$\begin{aligned}
\frac{\partial E_{r_B}\left[\theta_A\left(1, r_B\right)|s_A = 0\right]}{\partial \mu_B^*} & \propto -\left(\theta_A(1,\phi) - \theta_A(1,0)\right)\frac{\partial}{\partial \mu_B^*}\frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)} \\
& \propto \left(\theta_A(1,\phi) - \theta_A(1,0)\right)\rho \\
& > 0.
\end{aligned}$$

The second step relies on expression 13, the result from the proof in Lemma 1. Therefore, regardless of ρ, $E_{r_B}\left[\theta_A(1, r_B)|s_A = 0\right]$ is increasing in μ_B^*. Lastly, since $\mu_B^* = m_B^*\left(1 - q_B\right)$ is increasing in m_B^* and decreasing in q_B, $\tilde{m}_A^*\left(\mu_B^*\right)$ is increasing in m_B^* and decreasing in q_B. ■

Proof. of Proposition 2: We first prove that the equilibrium $\left(m_A^*\left(q_A, q_B\right), m_B^*\left(q_B, q_A\right)\right)$ is unique in two steps. First, we solve for manager A's unique best response $\tilde{m}_A^*(m_B^*)$. This part is similar to the proof in Lemma 1 because manager A's best response problem (after imposing the investors' rational expectations) is essentially a single firm problem with given m_B^* and q_B.

Second, we plug manager A's best response into manager B's first order condition and show that manager B's optimization has a unique solution as well. By substituting the best response $\tilde{m}_A^*\left(m_B^*\right)$ into $H^B(\tilde{m}_B^*; m_A^*) = 0$ and obtain

$$H^B(\tilde{m}_B^*; \tilde{m}_A^*\left(\tilde{m}_B^*\right)) = 0.$$

We show that this equation has a unique solution when the cost functions are sufficiently convex. At $\tilde{m}_B^* = 0$, $H^B(0; \tilde{m}_A^*\left(0\right)) = \delta_B\left(1 - q_B\right)\left(1 + C_B^*\left(0\right)\right) > 0$. At $\tilde{m}_B^* = 1$, $H^B(1; \tilde{m}_A^*\left(1\right)) = \delta_B\left(1 - q_B\right)\left(1 + C_B^*\left(1\right)\right)E_{r_A}\left[\theta_B\left(1, r_A; \tilde{m}_B^* = 1, \tilde{m}_A^* = \tilde{m}_A^*\left(1\right)\right)|s_A = 0\right] - C_B'(1) = -\infty < 0$.

In addition, we verify that $H^B(\tilde{m}_B^*; \tilde{m}_A^*(\tilde{m}_B^*))$ is strictly decreasing in $\tilde{m}_B^*$.

$$\begin{aligned}
& \frac{dH^B(\tilde{m}_B^*; \tilde{m}_A^*(\tilde{m}_B^*))}{d\tilde{m}_B^*} \\
= \; & \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} + \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\partial \tilde{m}_A^*(\tilde{m}_B^*)}{\partial \tilde{m}_B^*} \\
= \; & \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} - \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}} \\
= \; & \frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*} \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_B^*} - \frac{\partial H^B(\tilde{m}_B^*; \tilde{m}_A^*)}{\partial \tilde{m}_A^*} \frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}},
\end{aligned}$$

where the second step is from $\frac{\partial \tilde{m}_A^*(\tilde{m}_B^*)}{\partial \tilde{m}_B^*} = -\frac{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_B^*}}{\frac{\partial H^A(\tilde{m}_A^*; \tilde{m}_B^*)}{\partial \tilde{m}_A^*}}$. When C_A and C_B are sufficiently convex, it is easy (but tedious) to verify that the numerator is positive (the Hessian matrix of the objective function is negative definitive). The denominator is negative from the first step. Thus, $\frac{dH^B(\tilde{m}_B^*; \tilde{m}_A^*(\tilde{m}_B^*))}{d\tilde{m}_B^*} < 0$. Therefore, $H^B(\tilde{m}_B^*; \tilde{m}_A^*(\tilde{m}_B^*))$ is decreasing in $\tilde{m}_B^*$, and by the intermediate value theorem, $H^B(\tilde{m}_B^*; \tilde{m}_A^*(\tilde{m}_B^*)) = 0$ has a unique solution $m_B^*(q_B, q_A)$. In addition, $m_A^*(q_A, q_B) = \tilde{m}_A^*(m_B^*(q_B, q_A))$ is also unique. Now we can write the first order condition of manager A as $H^A(m_A^*(q_A, q_B); m_B^*(q_B, q_A))$.

To derive $\frac{\partial m_A^*}{\partial q_A}$ and $\frac{\partial m_B^*}{\partial q_A}$, the application of the multivariate implicit function theorem generates

$$\begin{aligned}
\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial \mu_B^*}{\partial m_B^*} \frac{\partial m_B^*}{\partial q_A} &= 0, \\
\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} \frac{\partial m_B^*}{\partial q_A} + \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \left(\frac{\partial \mu_A^*}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A} + \frac{\partial \mu_A^*}{\partial q_A} \right) &= 0,
\end{aligned}$$

which can be simplified into

$$\begin{aligned}
\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{\partial m_A^*}{\partial q_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} + (1 - q_B) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial m_B^*}{\partial q_A} &= 0, \\
\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} \frac{\partial m_B^*}{\partial q_A} + \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \left((1 - q_A) \frac{\partial m_A^*}{\partial q_A} - m_A^* \right) &= 0.
\end{aligned}$$

Solving the two equations gives

$$\begin{aligned}
\frac{\partial m_A^*}{\partial q_A} &= \frac{-\left[\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} \frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1 - q_B) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \right]}{\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} - (1 - q_B)(1 - q_A) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}}, \\
\frac{\partial m_B^*}{\partial q_A} &= \frac{\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \left(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} (1 - q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} m_A^* \right)}{\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} - (1 - q_B)(1 - q_A) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}}.
\end{aligned}$$

We have shown that in the unique equilibrium, the denominator is positive. Hence the signs of $\frac{\partial m_A^*}{\partial q_A}$ and $\frac{\partial m_A^*}{\partial q_B}$ are determined by their numerators, respectively. First,

$$\frac{\partial m_A^*}{\partial q_A} \propto -\left[\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1-q_B)\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*}\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\right].$$

From a proof similar to that of Lemma 1, we have $\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0$ and $\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} < 0$.Proposition 1 shows $\frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} > 0$ and $\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0$. As a result, $\frac{\partial m_A^*}{\partial q_A} < 0$.

Similarly,

$$\frac{\partial m_B^*}{\partial q_A} \propto \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*}\left(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A}(1-q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*}m_A^*\right),$$

where $\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0$, $\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0$, $\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} < 0$. As a result, $\frac{\partial m_B^*}{\partial q_A} < 0$. ■

Proof. of Proposition 3: The proof of the uniqueness of the internal control equilibrium is similar to that of the manipulation choice in Proposition 2. In short, when $K_A(q_A)$ is sufficiently convex, the LHS of the first-order condition of q_A, $\frac{\partial V_{A0}}{\partial m_A^*}\frac{\partial m_A^*}{\partial q_A} - K'(q_A)$, is decreasing in q_A, making the best response $\tilde{q}_A^*(q_B^*)$ unique. Substituting $\tilde{q}_A^*(q_B^*)$ into manager B's best response gives

$$\frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial \tilde{q}_B}|_{q_A^*=\tilde{q}_A^*(q_B^*)} - K_B'(\tilde{q}_B) = 0.$$

When $K_B(q_B)$ is sufficiently convex, the LHS $\frac{\partial V_{B0}}{\partial m_B^*}\frac{\partial m_B^*}{\partial \tilde{q}_B}|_{q_A^*=\tilde{q}_A^*(q_B^*)} - K_B'(\tilde{q}_B)$ is decreasing in $\tilde{q}_B$, making the solution of $q_B^* = \tilde{q}_B^*(q_B^*)$ unique. As a result, the equilibrium decisions $q_A^* = \tilde{q}_A^*(q_B^*)$, $m_A^*(q_A^*, q_B^*)$, and $m_B^*(q_B^*, q_A^*)$ are also unique. ■

Proof. of Proposition 4: From the main text, given firm B's individual internal control choice q_B^*, an individual firm A's internal control choice q_A^* is determined by

$$\frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial m_A^*}\frac{\partial m_A^*(q_A^*, q_B^*)}{\partial q_A} - K_A'(q_A^*) = 0.$$

Since the two firms are symmetric, we have $q_A^* = q_B^* = q^*$ and

$$\frac{\partial V_{A0}(q^*, q^*)}{\partial m_A^*}\frac{\partial m_A^*(q^*, q^*)}{\partial q_A} - K_A'(q^*) = 0.$$

Given her choice q_B^{SP}, the social planner's internal control decision q_A^{SP} is given by

$$\frac{\partial V_{A0}\left(q_A^{SP}, q_B^{SP}\right)}{\partial m_A^*}\frac{\partial m_A^*\left(q_A^{SP}, q_B^{SP}\right)}{\partial q_A} + \frac{\partial V_{B0}\left(q_A^{SP}, q_B^{SP}\right)}{\partial m_B^*}\frac{\partial m_B^*\left(q_A^{SP}, q_B^{SP}\right)}{\partial q_A} - K_A'(q_A^{SP}) = 0.$$

Similarly, since the two firms are symmetric, the social planner's choices of internal control for the two firms are also symmetric, i.e., $q_A^{SP} = q_B^{SP} = q^{SP}$, we have

$$\frac{\partial V_{A0}\left(q^{SP}, q^{SP}\right)}{\partial m_A^*}\frac{\partial m_A^*\left(q^{SP}, q^{SP}\right)}{\partial q_A} + \frac{\partial V_{B0}\left(q^{SP}, q^{SP}\right)}{\partial m_B^*}\frac{\partial m_B^*\left(q^{SP}, q^{SP}\right)}{\partial q_A} - K_A'(q^{SP}) = 0.$$

Notice that for k sufficiently large, the social planner's objective function is strictly concave in q^{SP} and the first-order condition is strictly decreasing in q^{SP}.

To compare q^* and q^{SP}, we define

$$f(q) = \frac{\partial V_{A0}(q,q)}{\partial m_A^*} \frac{\partial m_A^*(q,q)}{\partial q_A} + \frac{\partial V_{B0}(q,q)}{\partial m_B^*} \frac{\partial m_B^*(q,q)}{\partial q_A} - K_A'(q).$$

$f(q)$ is the social planer's first order condition and thus we have $f(q^{SP}) = 0$. Moreover, we have

$$\begin{aligned} f(q^*) &= \frac{\partial V_{A0}(q^*,q^*)}{\partial m_A^*} \frac{\partial m_A^*(q^*,q^*)}{\partial q_A} + \frac{\partial V_{B0}(q^*,q^*)}{\partial m_B^*} \frac{\partial m_B^*(q^*,q^*)}{\partial q_A} - K_A'(q^*) \\ &= \frac{\partial V_{B0}(q^*,q^*)}{\partial m_B^*} \frac{\partial m_B^*(q^*,q^*)}{\partial q_A} \end{aligned}$$

The second equality is due to the first order condition for $q^* : \frac{\partial V_{A0}(q^*,q^*)}{\partial m_A^*} \frac{\partial m_A^*(q^*,q^*)}{\partial q_A} - K_A'(q^*) = 0$. Since $\frac{\partial V_{B0}(q^*,q^*)}{\partial m_B^*} < 0$ and $\frac{\partial m_B^*(q^*,q^*)}{\partial q_A} < 0$, we have $f(q^*) > 0$. That is, evaluated at $q = q^*$, the first-order condition for the social planner is positive. Since the social planner's first-order condition is strictly decreasing in q, we have $q^* < q^{SP}$. Firms under-invest in their internal control relative to the socially optimal level. ■