A Volatility-Implied Factor Structure on Equity Option Prices

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Background

- The factor structure of equity returns is standard (e.g., CAPM, APT).
- But - there is no standard factor structure for equity options.
- This is a hole in the literature that needs to be filled.
- This paper will try to fill this gap.

Main Idea

- We will show how a factor structure for stock returns translates naturally into a volatility-implied factor structure for option prices.
- The key - any factor structure for stock returns implies a linear relation between stock return volatility and the variances and covariances of the factors and idiosyncratic risk.
- Since option prices increase with return volatility, we obtain a natural factor structure for options:
    - Option factors are implied variances/covariances of return factors.
    - Option factor loadings are combinations of return factor loadings.

Literature

- Traditional option pricing models do not accommodate a factor structure.
- Serban, Lehoczky, and Seppi (working paper, 2008) and Christoffersen, Fournier, and Jacobs (working paper, 2015) develop option pricing models with a common market factor.
    - They do not show how to incorporate a general factor structure for stock returns into option prices.

Contributions

- First paper to link general factor structures for equity with a factor structure for options.
- Present empirical evidence that the volatility-implied factor structure indeed provides a good explanation of option prices in the cross section and time series.
- Methodological contribution: Develop a modified Fama-MacBeth approach to estimate the implied variances and covariances of return factors.

Setup

- Assume a factor structure for equity returns:

$$r = \alpha + \sum_{k=1}^{K} \beta_k f_k + \varepsilon,$$

  where $\mathrm{E}(\varepsilon) = \mathrm{E}(f_k \varepsilon) = 0$.
- Taking variance on both sides yields:

$$\sigma^2 = \sum_{k,l=1}^{K} \beta_k \beta_l \sigma_{kl} + \sigma_\varepsilon^2$$

- Equity return variances are linearly related to variances and covariances of the return factors and the idiosyncratic variance.

A Factor Structure with Option Prices

- Option prices reflect the return volatility of the underlying assets.
- Taking option-implied variance on both sides of the return factor structure yields:

$$IV^2 = \sum_{k,l=1}^{K} \beta_k \beta_l IV_{kl} + IV_\varepsilon^2$$

- This provides a linear factor structure of option prices through option-implied volatility.
  - The "new" factors are variances/covariances of the return factors.
  - The "new" factor loadings are combinations of the return factor loadings.

CAPM 1-Factor Model

- Equity returns have a common market factor:

$$r = r_f + \beta_{mktrf} mktrf + \varepsilon$$

- Option-implied variances have 1-factor structure:

$$IV^2 = \beta^2_{mktrf} IV^2_{mktrf} + IV^2_{\varepsilon}.$$

Fama-French-Carhart 4-Factor Model

- Equity returns have four commons factors (market, size, value, momentum):

$$r = \alpha + \beta_{mktrf} mktrf + \beta_{smb} smb + \beta_{hml} hml + \beta_{umd} umd + \varepsilon,$$

- Option-implied variances have 1-factor structure (four variances and six covariances):

$$\begin{aligned} IV^2 \quad = \quad & \beta^2_{mktrf} IV^2_{mktrf} + \beta^2_{smb} IV^2_{smb} + \beta^2_{hml} IV^2_{hml} + \beta^2_{umd} IV^2_{umd} \\ & + 2\beta_{mktrf}\beta_{smb} IV_{mktrf,smb} + 2\beta_{mktrf}\beta_{hml} IV_{mktrf,hml} \\ & + 2\beta_{mktrf}\beta_{umd} IV_{mktrf,umd} + 2\beta_{smb}\beta_{hml} IV_{smb,hml} \\ & + 2\beta_{smb}\beta_{umd} IV_{smb,umd} + 2\beta_{hml}\beta_{umd} IV_{hml,umd} + IV^2_{\varepsilon}. \end{aligned}$$

A Technical Challenge

- We need to estimate

$$IV^2 = \sum_{k,l=1}^{K} \beta_k \beta_l IV_{kl} + IV_\varepsilon^2,$$

- But, typical factors such as *SMB*, *HML* and *UMD* do not have traded options.
- How can we estimate them?

A Modified Fama-MacBeth Approach

- Standard Fama-MacBeth:
  - Regress equity returns on return factors over time to estimate return factor loadings β_k.
  - Cross-sectionally regress equity returns on β_k to estimate factor premium.
- Modified Fama-MacBeth:
  - First stage is the same.
  - Cross-sectionally regress IV^2 on $\beta_k \beta_l$ to estimate implied variances and covariances of the return factors.
    - The regression constant is the cross-sectional average implied idiosyncratic variance.

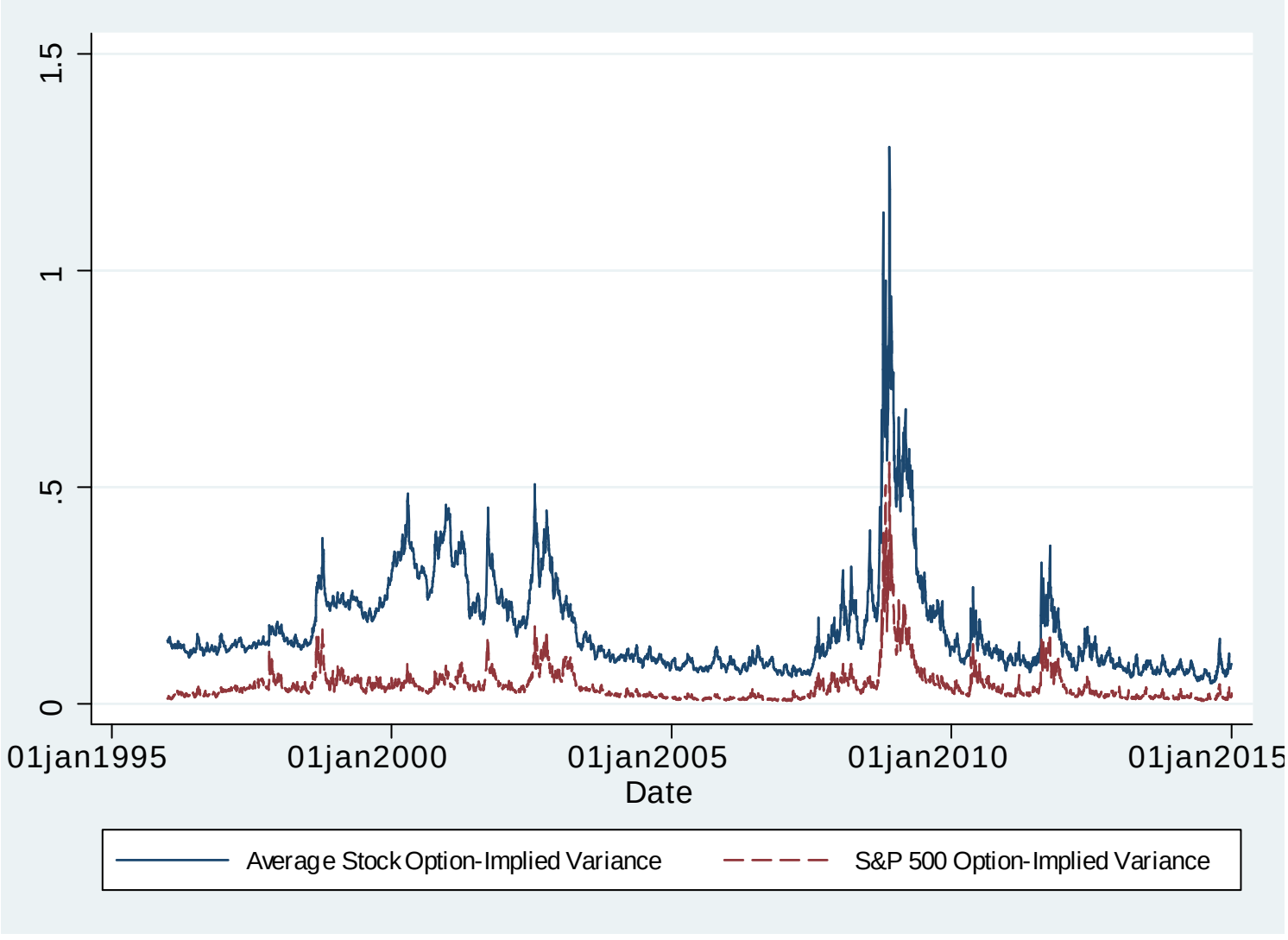
Data and Sample

- Data sources: OptionMetrics, CRSP, FF factors, Compustat
- Sample period: January 1996 to december 2014
- Restrict attention to S&P 500 constituents during the sample period as smaller stocks tend not to have liquid option trading.

Empirical Estimation

- Use the square of the Black-Scholes implied volatility provided by OptionMetrics as the option-implied variance of equity returns.
  - Take the average implied volatility of the at-the-money call and put options that mature in 30 days.
- Focus on three factor models:
  - CAPM 1-factor model
  - Fama-French-Carhart 4-factor model
  - Hou-Xue-Zhang (2015) 4-factor model (market, size, investment, profitability)

Option-Implied Variance



Option-Implied Variance

Year	Number of Stocks	Mean	Median	S.D.	Min	Max
1996	685	0.1320	0.0871	0.1181	0.0237	0.7841
1997	722	0.1503	0.0989	0.1858	0.0236	3.0220
1998	745	0.2034	0.1445	0.2817	0.0285	6.1896
1999	737	0.2343	0.1777	0.1836	0.0361	1.9525
2000	708	0.3386	0.2441	0.2568	0.0202	1.5579
2001	675	0.2951	0.1922	0.2721	0.0418	1.7791
2002	694	0.2778	0.1791	0.2663	0.0380	1.6042
2003	692	0.1559	0.1085	0.1432	0.0245	1.4683
2004	698	0.1064	0.0740	0.1056	0.0144	1.1918
2005	695	0.0932	0.0674	0.0926	0.0120	1.1272
2006	695	0.0925	0.0709	0.0724	0.0111	0.6782
2007	685	0.1059	0.0885	0.0698	0.0019	0.6004
2008	658	0.3619	0.2930	0.2761	0.0318	2.4714
2009	644	0.3364	0.2465	0.2997	0.0008	2.2244
2010	640	0.1347	0.1062	0.1229	0.0072	1.5028
2011	629	0.1472	0.1206	0.1336	0.0049	1.4820
2012	623	0.1148	0.0805	0.1530	0.0044	2.6541
2013	618	0.0809	0.0579	0.0834	0.0118	1.2612
2014	610	0.0755	0.0535	0.0831	0.0040	1.3157

Factor Loadings - CAPM

	Mean	Median	S.D.	Min	Max
β_{mktrf}	1.0292	0.9926	0.3590	-0.0768	2.3396
β^2_{mktrf}	1.3074	1.0977	0.9065	0.0155	5.8187

Factor Loadings - FFC

	Mean	Median	S.D.	Min	Max
β_{mktrf}	1.0764	1.0748	0.2543	0.0265	2.0662
β_{smb}	0.1918	0.1670	0.3445	-0.8199	1.7685
β_{hml}	0.1729	0.2529	0.6120	-3.6693	2.7848
β_{umd}	-0.0836	-0.0688	0.2408	-1.4730	1.1197
β^2_{mktrf}	1.3186	1.2513	0.5980	0.0770	4.2698
β^2_{smb}	0.3189	0.1907	0.3713	0.0098	3.1955
β^2_{hml}	0.8052	0.4948	1.0645	0.0144	14.4982
β^2_{umd}	0.3528	0.2049	0.3873	0.0078	3.5822
$2\beta_{mktrf}\beta_{smb}$	0.5580	0.3755	0.9056	-1.4827	5.2218
$2\beta_{mktrf}\beta_{hml}$	0.5546	0.5857	1.3755	-4.4739	10.2483
$2\beta_{mktrf}\beta_{umd}$	-0.2253	-0.1750	0.6025	-3.9971	3.0061
$2\beta_{smb}\beta_{hml}$	0.2019	0.1168	0.6057	-2.8109	5.0804
$2\beta_{smb}\beta_{umd}$	-0.0696	-0.0222	0.3001	-2.7415	0.9653
$2\beta_{hml}\beta_{umd}$	-0.0043	-0.0274	0.6592	-9.2461	6.4819

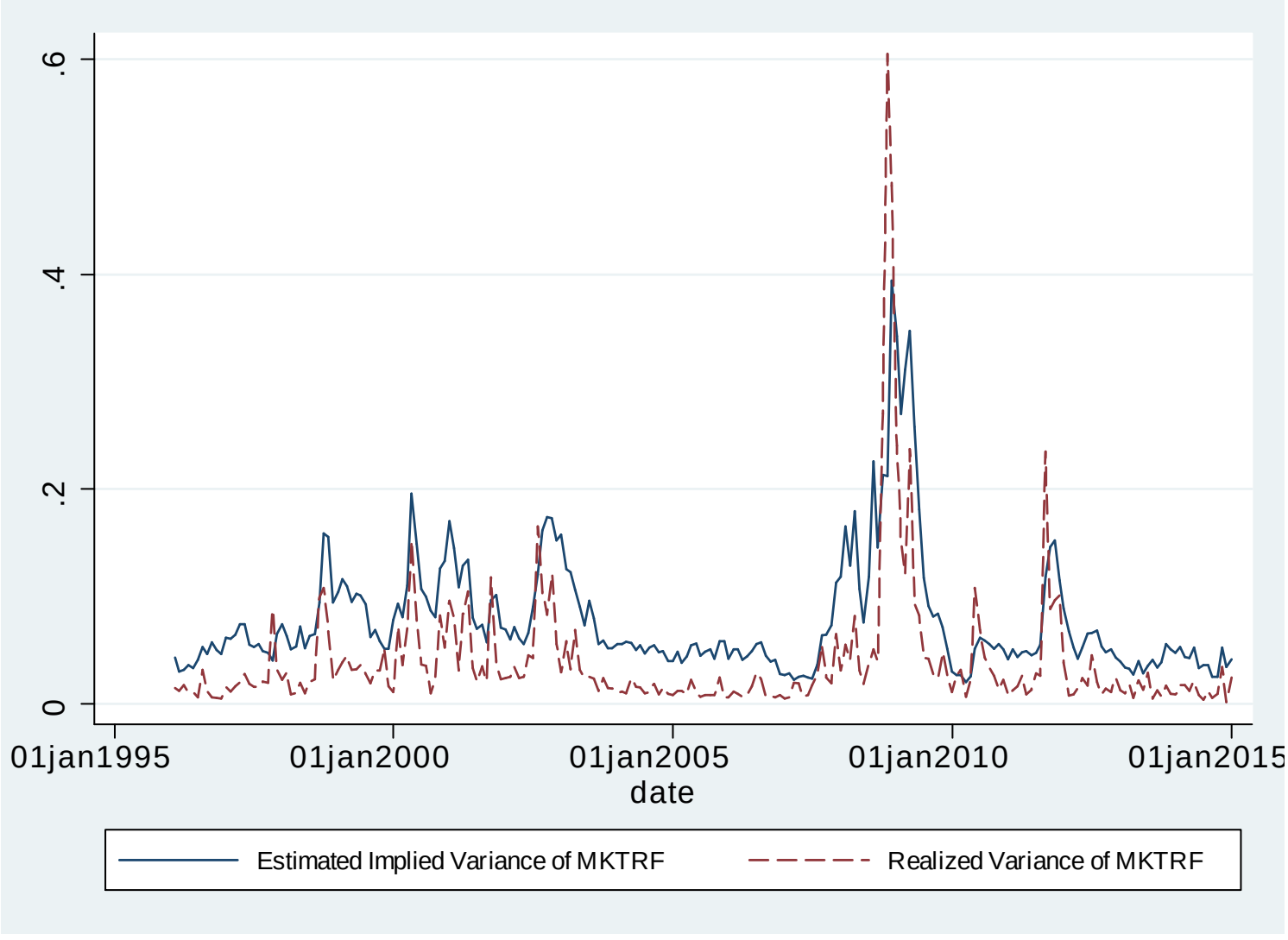
Factor Loadings - HXZ

	Mean	Median	S.D.	Min	Max
β_{mktrf}	1.0212	1.0049	0.2661	-0.1155	1.9631
β_{me}	-0.0752	-0.0765	0.2595	-1.2401	2.1210
$\beta_{i/a}$	0.0283	0.1007	0.2979	-2.0916	1.2108
β_{roe}	-0.1419	-0.1064	0.2443	-2.6577	0.7595
β^2_{mktrf}	1.1986	1.0889	0.6020	0.0476	4.0582
β^2_{me}	0.3762	0.2481	0.3795	0.0110	4.5331
$\beta^2_{i/a}$	0.2731	0.1310	0.4154	0.0002	4.7896
β^2_{roe}	0.2591	0.1387	0.3707	0.0071	7.0702
$2\beta_{mktrf}\beta_{me}$	-0.1250	-0.1334	0.6281	-3.5401	2.9984
$2\beta_{mktrf}\beta_{i/a}$	0.0091	0.1815	0.7306	-5.1698	3.7210
$2\beta_{mktrf}\beta_{roe}$	-0.3548	-0.2208	0.5994	-3.5448	1.7747
$2\beta_{me}\beta_{i/a}$	0.0339	0.0065	0.3791	-2.6678	4.3277
$2\beta_{me}\beta_{roe}$	-0.1719	-0.0758	0.5194	-11.2720	2.9492
$2\beta_{i/a}\beta_{roe}$	-0.0461	-0.0179	0.3318	-3.5334	2.5432

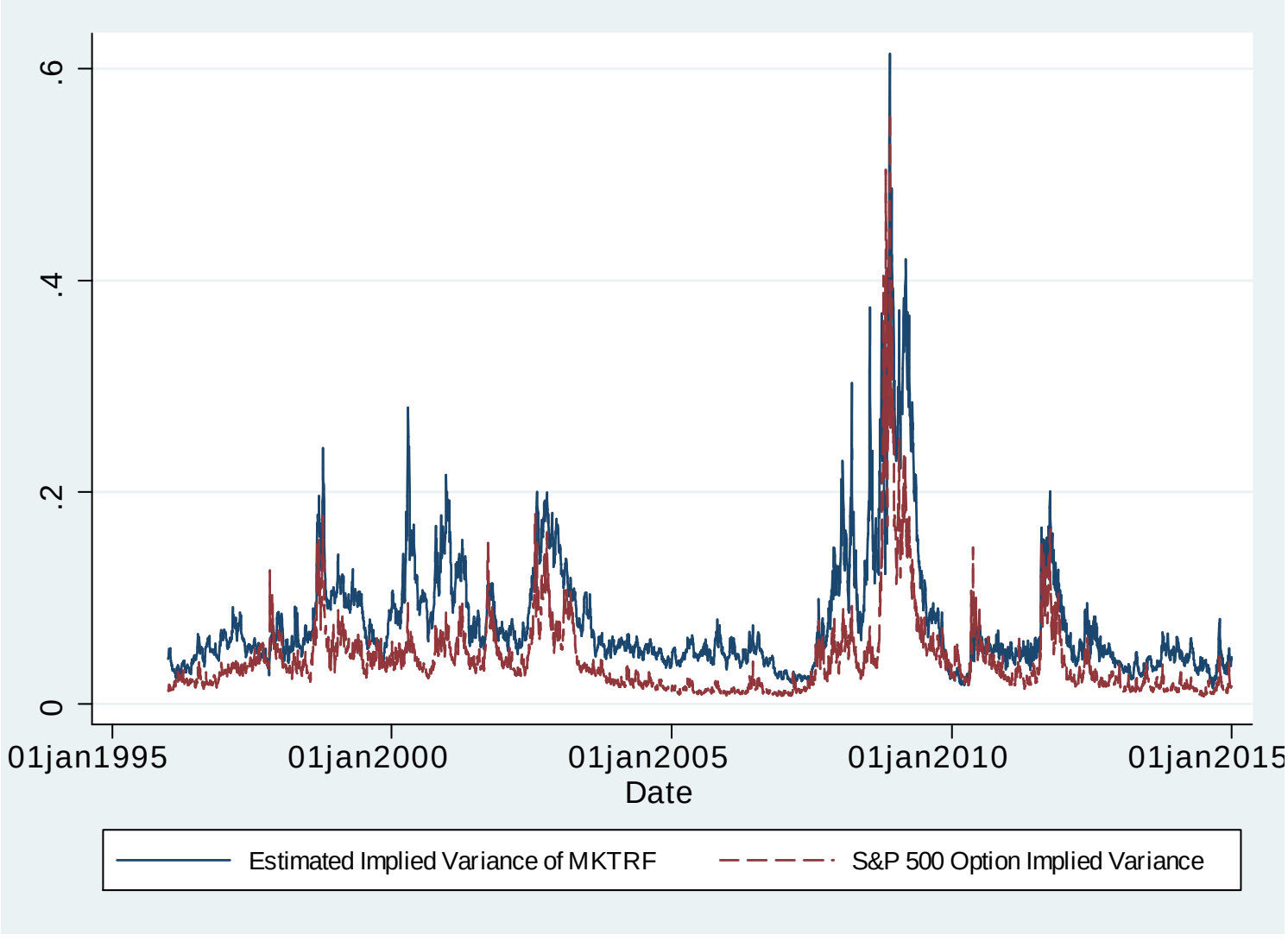
Modified Fama-Macbeth Estimation

	CAPM		FFC		HXZ
IV^2_{mktrf}	0.0778***	IV^2_{mktrf}	0.0402***	IV^2_{mktrf}	0.0445***
		IV^2_{smb}	0.0386***	IV^2_{me}	0.0291***
		IV^2_{hml}	0.0191***	$IV^2_{i/a}$	0.0252***
		IV^2_{umd}	0.0688***	IV^2_{roe}	0.0715***
		$IV_{mktrf,smb}$	0.0205***	$IV_{mktrf,me}$	0.0016*
		$IV_{mktrf,hml}$	-0.0105***	$IV_{mktrf,i/a}$	-0.0205***
		$IV_{mktrf,umd}$	-0.0061***	$IV_{mktrf,roe}$	-0.0232***
		$IV_{smb,hml}$	-0.0020**	$IV_{me,i/a}$	0.0012
		$IV_{smb,umd}$	-0.0052***	$IV_{me,roe}$	0.0120***
		$IV_{hml,umd}$	-0.0037***	$IV_{i/a,roe}$	-0.0069***
Cons	0.0740***	*Cons*	0.0809***	*Cons*	0.0816***
Adj R^2	0.3144	*Adj* R^2	0.4915	*Adj* R^2	0.4536

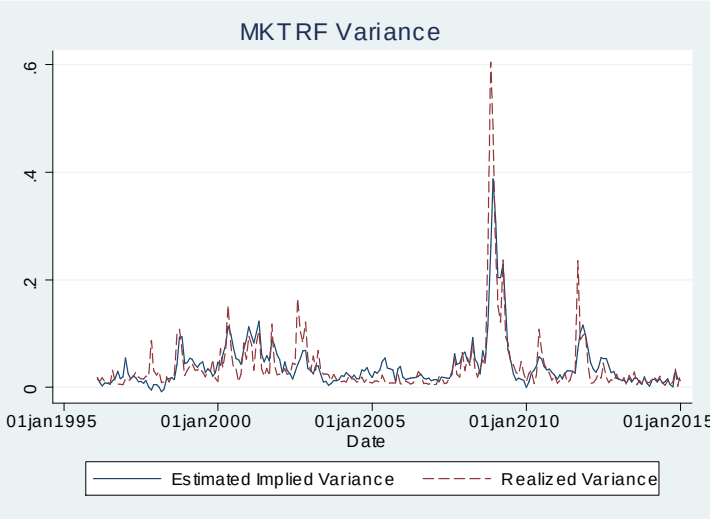
Modified Fama-MacBeth Estimation - CAPM

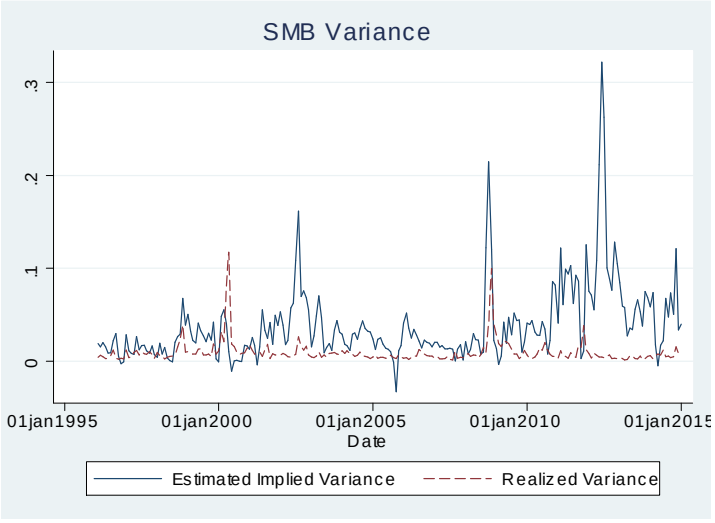


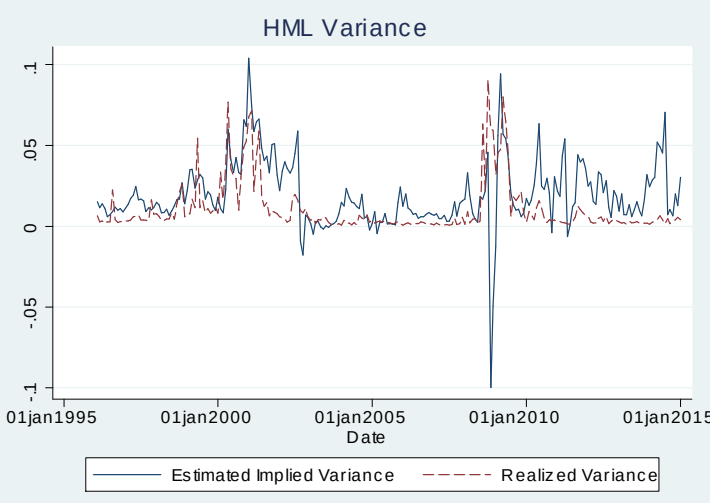
Modified Fama-MacBeth Estimation - CAPM

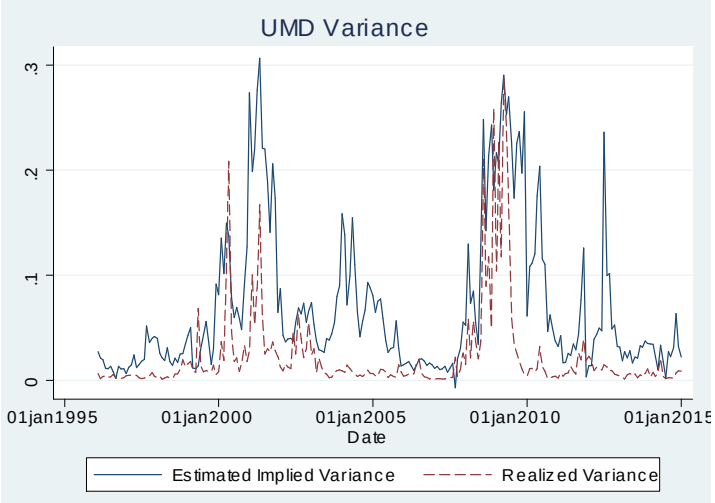


Modified Fama-MacBeth Estimation - FFC

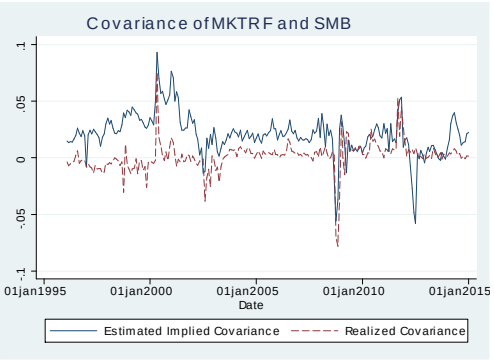


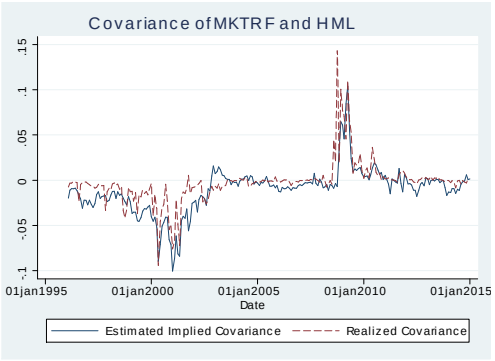


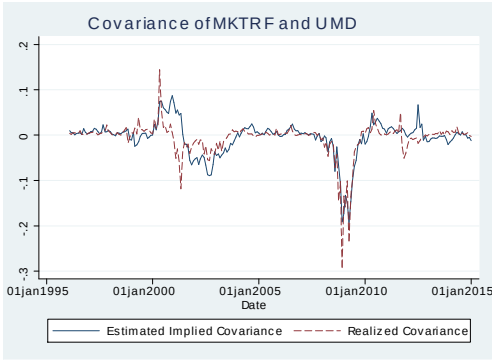


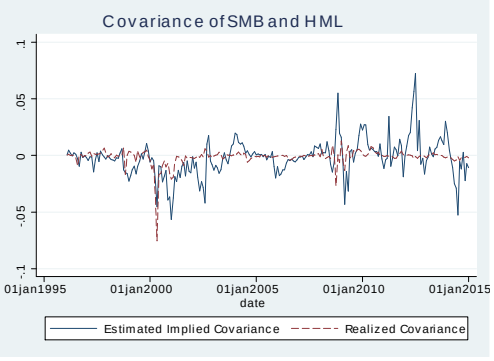


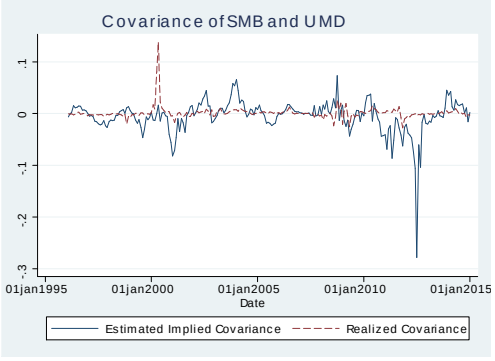
Modified Fama-MacBeth Estimation - FFC

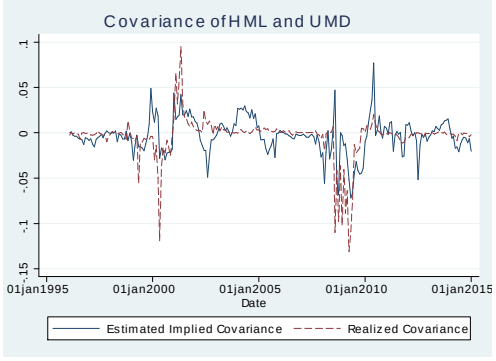












Implied vs. Realized - CAPM

	Implied	Realized	Correlation
σ^2_{mktrf}	0.0779	0.0393	0.7359***

Implied vs. Realized - FFC

	Implied	Realized	Correlation
σ^2_{mktrf}	0.0402	0.0393	0.8248***
σ^2_{smb}	0.0385	0.0098	0.0813
σ^2_{hml}	0.0192	0.0105	0.3897***
σ^2_{umd}	0.0687	0.0227	0.6438***
$\sigma_{mktrf,smb}$	0.0206	0.0012	0.3370***
$\sigma_{mktrf,hml}$	-0.0106	-0.0025	0.7840***
$\sigma_{mktrf,umd}$	-0.0062	-0.0077	0.7432***
$\sigma_{smb,hml}$	-0.0021	-0.0014	0.2998***
$\sigma_{smb,umd}$	-0.0052	0.0012	0.1585**
$\sigma_{hml,umd}$	-0.0037	-0.0045	0.4185***

Implied vs. Realized - HXZ

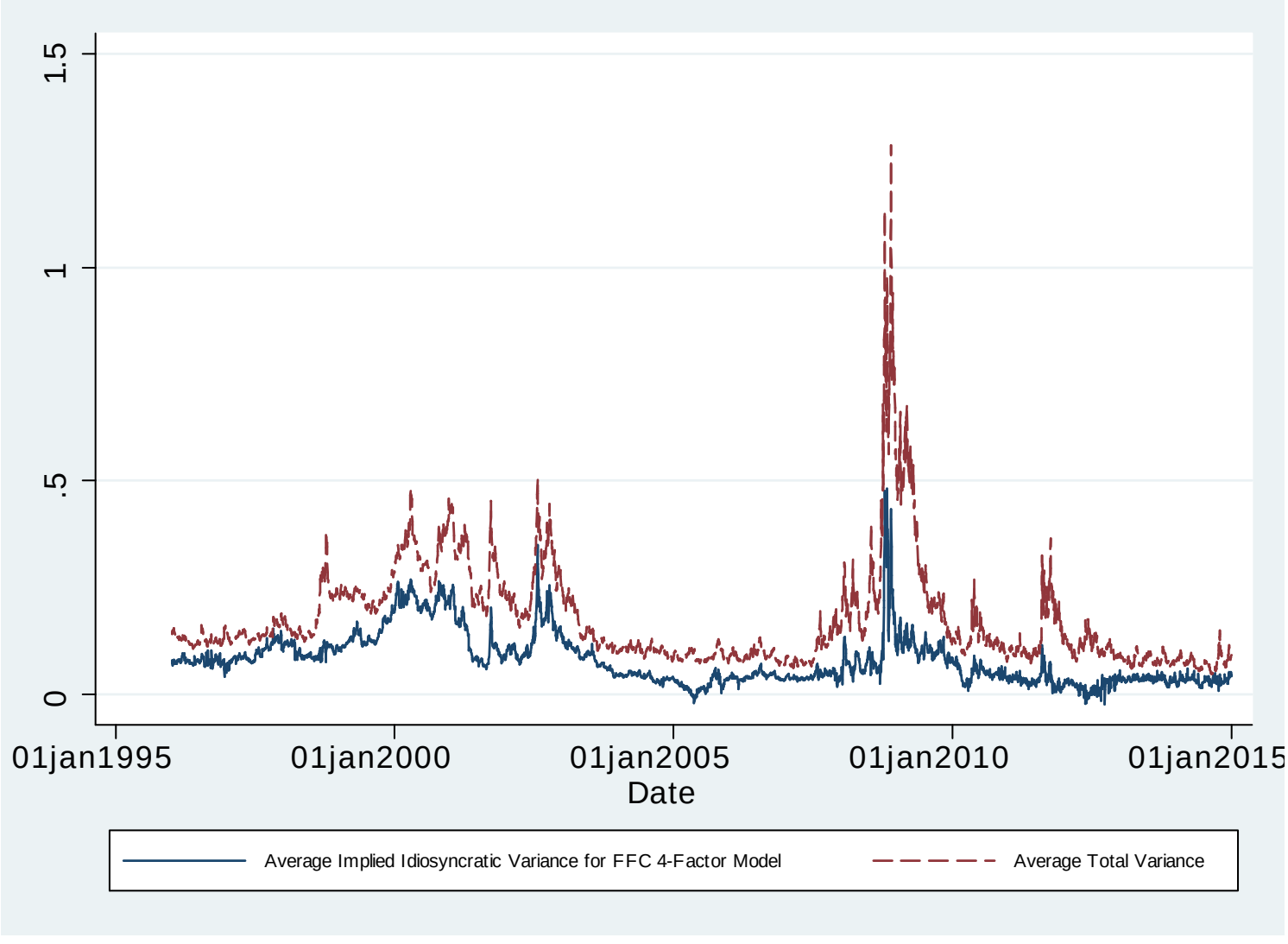
	Implied	Realized	Correlation
σ^2_{mktrf}	0.0444	0.0393	0.7869***
σ^2_{me}	0.0291	0.0039	0.1788***
$\sigma^2_{i/a}$	0.0253	0.0123	0.2123***
σ^2_{roe}	0.0712	0.0140	0.3518***
$\sigma_{mktrf,me}$	0.0015	0.0007	0.5638***
$\sigma_{mktrf,i/a}$	-0.0205	-0.0066	0.557***
$\sigma_{mktrf,roe}$	-0.0064	-0.0234	0.6765***
$\sigma_{me,i/a}$	0.0011	-0.0012	-0.2795***
$\sigma_{me,roe}$	0.0117	0.0007	0.1948***
$\sigma_{i/a,roe}$	-0.0071	0.0032	0.0268

Modified Fama-Macbeth Estimation

Are the covariance terms important?

	FFC			HXZ	
IV^2_{mktrf}	0.0402***	0.0313***	IV^2_{mktrf}	0.0445***	0.0590***
IV^2_{smb}	0.0386***	0.0742***	IV^2_{me}	0.0291***	0.0258***
IV^2_{hml}	0.0191***	0.0203***	$IV^2_{i/a}$	0.0252***	0.0466***
IV^2_{umd}	0.0688***	0.0829***	IV^2_{roe}	0.0715***	0.0899***
$IV_{mktrf,smb}$	0.0205***		$IV_{mktrf,me}$	0.0016*	
$IV_{mktrf,hml}$	-0.0105***		$IV_{mktrf,i/a}$	-0.0205***	
$IV_{mktrf,umd}$	-0.0061***		$IV_{mktrf,roe}$	-0.0232***	
$IV_{smb,hml}$	-0.0020**		$IV_{me,i/a}$	0.0012	
$IV_{smb,umd}$	-0.0052***		$IV_{me,roe}$	0.0120***	
$IV_{hml,umd}$	-0.0037***		$IV_{i/a,roe}$	-0.0069***	
Cons	0.0809***	0.0757***	*Cons*	0.0816***	0.0615***
Adj R^2	0.4915	0.3870	*Adj* R^2	0.4536	0.3932

Implied Idiosyncratic Variance - FFC



Time Series Tests

Can the option factors explain time series variations in option prices?

- SPDR sector ETFs
- Dow Jones Industrial Average (DJIA) stocks

Time Series Return Factor Regression Adjusted R-Squared - SPDR Sector ETFs

	CAPM	FFC	HXZ
XLB	0.5816	0.6192	0.6068
XLV	0.6157	0.6366	0.6185
XLP	0.4264	0.4809	0.5507
XLY	0.7212	0.7310	0.7350
XLE	0.4650	0.5364	0.5014
XLF	0.6972	0.8261	0.7158
XLI	0.7924	0.8016	0.8023
XLK	0.7488	0.8319	0.8047
XLU	0.3863	0.4620	0.4177
Average	0.6038	0.6584	0.6392

Time Series Return Factor Regression Adjusted R-Squared - DJIA Stocks

	CAPM	FFC	HXZ
AXP	0.5359	0.5856	0.5444
AAPL	0.2243	0.2514	0.2462
BA	0.3317	0.3374	0.3393
CAT	0.3993	0.4215	0.4104
CHV	0.3575	0.4266	0.4039
CSCO	0.4255	0.5108	0.4890
KO	0.2147	0.2619	0.2848
DIS	0.4202	0.4248	0.4303
DD	0.4111	0.4564	0.4361
XOM	0.3663	0.4333	0.4302
GE	0.5481	0.5750	0.5521
GS	0.5193	0.5401	0.5295
HD	0.4041	0.4081	0.4063
INTC	0.1398	0.1802	0.1536
IBM	0.3723	0.4028	0.3852
⋮	⋮	⋮	⋮
Average	0.3548	0.3926	0.3811

Time Series Option Factor Regression Adjusted R-Squared - SPDR Sector ETFs

Panel A: Using Estimated Implied Market Variance							
	CAPM	FFC			HXZ		
	Full	Full	Dropping Cov		Full	Dropping Cov	
XLB	0.5531	0.6763	0.6657	0.5437	0.6712	0.6084	0.5830
XLV	0.4142	0.5223	0.4796	0.3686	0.5122	0.3744	0.4466
XLP	0.6245	0.7059	0.6549	0.3887	0.7409	0.5095	0.6682
XLY	0.7216	0.8465	0.8077	0.5945	0.7993	0.6376	0.7368
XLE	0.5422	0.8154	0.7896	0.6092	0.7870	0.6544	0.6461
XLF	0.7559	0.8866	0.7635	0.6118	0.7788	0.6879	0.8291
XLI	0.6180	0.7747	0.7194	0.6321	0.7362	0.6034	0.6684
XLK	0.3881	0.6309	0.4055	0.3121	0.7131	0.4532	0.6844
XLU	0.3865	0.4211	0.3884	0.2560	0.4579	0.3436	0.3813
Average	0.5560	0.6977	0.6305	0.4796	0.6885	0.5414	0.6271

Time Series Option Factor Regression Adjusted R-Squared - SPDR Sector ETFs

Panel B: Using Option-Implied Variance of S&P 500 Index							
	CAPM	FFC			HXZ		
	Full	Full	Dropping Cov		Full	Dropping Cov	
XLB	0.7122	0.7425	0.7292	0.7180	0.7301	0.7095	0.7219
XLV	0.6168	0.6283	0.6159	0.6169	0.6295	0.6188	0.6275
XLP	0.7539	0.8442	0.7696	0.7565	0.8401	0.7606	0.8206
XLY	0.9456	0.9525	0.9465	0.9462	0.9509	0.9460	0.9459
XLE	0.8110	0.8439	0.8284	0.8192	0.8649	0.8323	0.8491
XLF	0.7387	0.8480	0.7665	0.7802	0.8229	0.7711	0.7720
XLI	0.8996	0.9081	0.9043	0.9060	0.9045	0.9002	0.9015
XLK	0.4160	0.7311	0.4892	0.4758	0.7329	0.5437	0.7267
XLU	0.5231	0.5492	0.5242	0.5259	0.5505	0.5271	0.5380
Average	0.7130	0.7831	0.7304	0.7272	0.7807	0.7344	0.7670

Time Series Option Factor Regression Adjusted R-Squared - DJIA Stocks

Panel A: Using Estimated Implied Market Variance							
	CAPM	FFC			HXZ		
	Full	Full	Dropping Cov		Full	Dropping Cov	
AXP	0.7607	0.8806	0.7976	0.5793	0.7843	0.6254	0.8030
AAPL	0.2398	0.6641	0.2813	0.1354	0.6765	0.2353	0.5572
BA	0.6348	0.7628	0.6786	0.4992	0.7997	0.5577	0.7355
CAT	0.6517	0.7617	0.7278	0.5578	0.7021	0.5420	0.6915
CHV	0.5784	0.7896	0.7768	0.5032	0.7213	0.5591	0.6157
CSCO	0.2552	0.6879	0.3261	0.2978	0.7972	0.4199	0.7550
KO	0.4932	0.6495	0.4386	0.1939	0.7953	0.3321	0.6456
DIS	0.6239	0.7569	0.6146	0.4522	0.7990	0.5224	0.7170
DD	0.6908	0.7755	0.7186	0.4822	0.7551	0.5224	0.7227
XOM	0.5707	0.7241	0.6871	0.4449	0.7322	0.5242	0.5867
GE	0.6613	0.7812	0.6615	0.5844	0.6880	0.5177	0.7085
GS	0.6899	0.8420	0.8282	0.5167	0.7927	0.7140	0.7601
HD	0.6763	0.7516	0.6347	0.4067	0.8275	0.5929	0.7434
INTC	0.4237	0.5949	0.4344	0.4656	0.6510	0.2627	0.5504
IBM	0.4442	0.6767	0.4029	0.2172	0.7774	0.3900	0.6930
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Average	0.5529	0.7143	0.5605	0.3914	0.7511	0.4681	0.6703

Time Series Option Factor Regression Adjusted R-Squared - DJIA Stocks

Panel B: Using Option-Implied Variance of S&P 500 Index							
	CAPM	FFC			HXZ		
	Full	Full	Dropping Cov		Full	Dropping Cov	
AXP	0.7881	0.8716	0.8253	0.8315	0.8458	0.7962	0.7980
AAPL	0.1867	0.6956	0.2832	0.2178	0.6427	0.2621	0.5355
BA	0.7954	0.8890	0.8291	0.8262	0.8863	0.8266	0.8753
CAT	0.8093	0.8584	0.8459	0.8431	0.8427	0.8198	0.8251
CHV	0.8325	0.8588	0.8465	0.8487	0.8551	0.8424	0.8390
CSCO	0.2263	0.7522	0.3762	0.3555	0.7765	0.4450	0.7407
KO	0.5369	0.8238	0.5770	0.5347	0.8517	0.5793	0.7453
DIS	0.7722	0.8804	0.7889	0.7929	0.8940	0.8163	0.8689
DD	0.8161	0.8893	0.8547	0.8391	0.8862	0.8279	0.8552
XOM	0.8228	0.8615	0.8324	0.8373	0.8420	0.8272	0.8253
GE	0.6577	0.8112	0.7353	0.7758	0.7625	0.6621	0.6733
GS	0.7256	0.7702	0.7522	0.7473	0.7922	0.7396	0.7354
HD	0.7246	0.8652	0.7464	0.7381	0.8606	0.7729	0.8489
INTC	0.2708	0.6515	0.5153	0.4434	0.6566	0.3078	0.5417
IBM	0.4548	0.8140	0.5161	0.4708	0.8095	0.5421	0.7467
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Average	0.6023	0.8000	0.6535	0.6399	0.8022	0.6489	0.7449

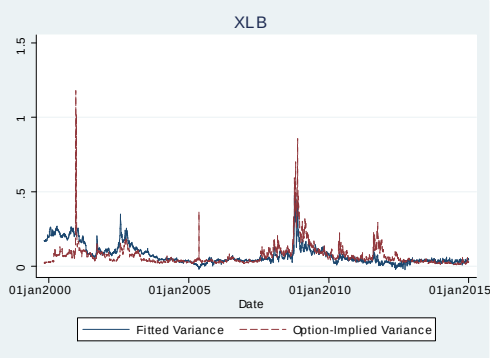
Recovering Implied Variance for Large Portfolios

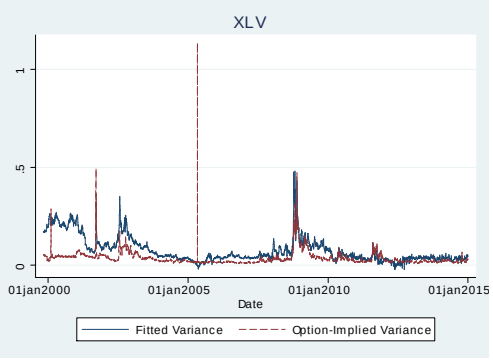
- Large portfolios are well-diversified, containing little idiosyncratic risk.
- Assuming implied idiosyncratic variance to be zero:

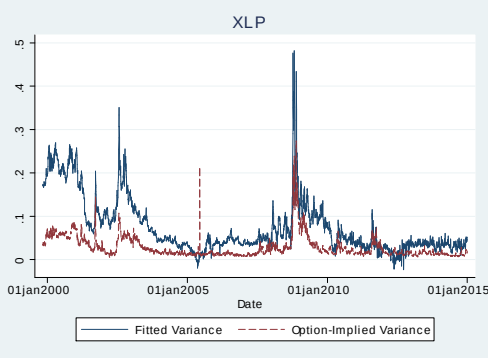
$$\widehat{IV}^2 = \sum_{k,l=1}^{K} \widehat{\beta}_k \widehat{\beta}_l \widehat{IV}_{kl},$$

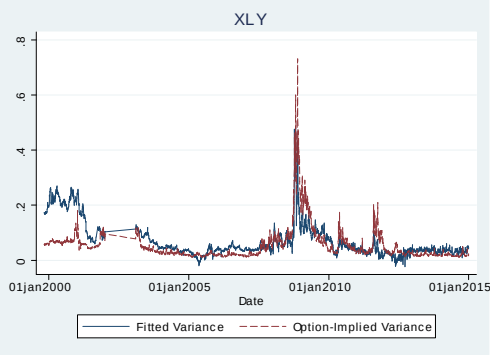
- Obtain $\widehat{\beta}_k$ from the first stage of Fama-MacBeth regression.
- Obtain $\widehat{IV}_{kl}$ from the second stage of modified Fama-MacBeth procedure.

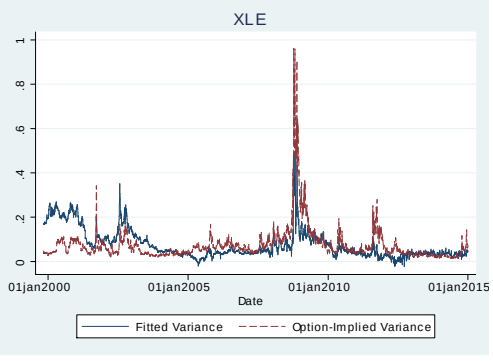
Implied Variance of SPDR Sector ETFs - FFC

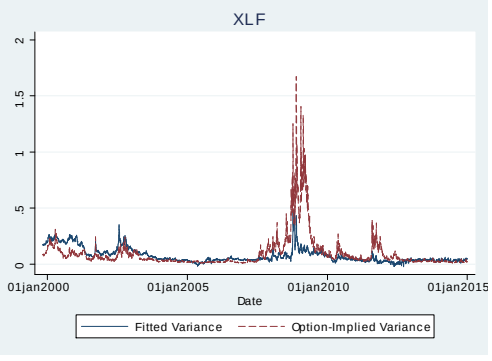


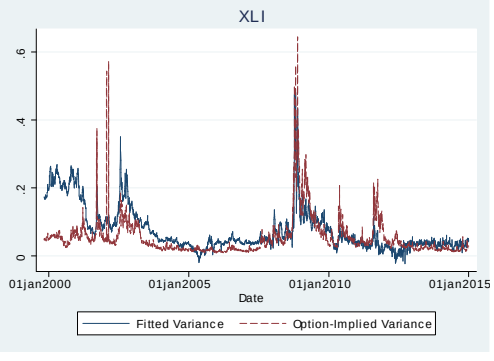


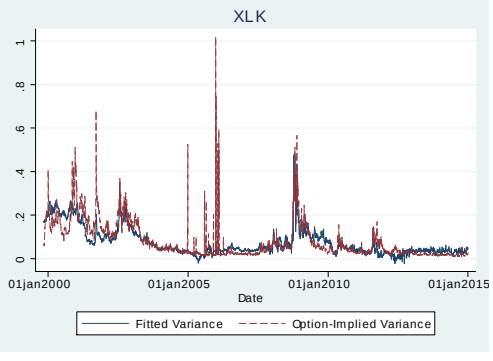


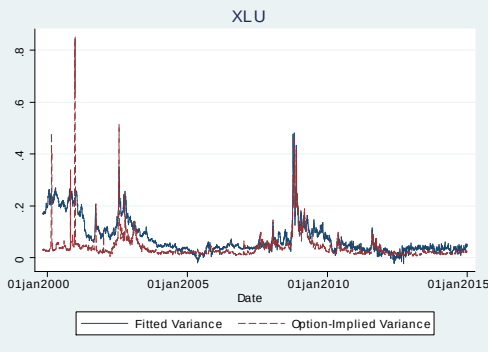












Conclusion

- A factor structure of equity returns translates naturally into a factor structure of option prices through option-implied volatility:
  - Option factors are implied variances/covariances of return factors.
  - Option factor loadings are combinations of return factor loadings.
- Develop a modified Fama-MacBeth approach to estimate the option-implied variances and covariances of return factors.
- The volatility-implied factor structure provides a good explanation of option prices in the cross section and time series.