Does Speculative Activity Have Real Effects?

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Introduction

- **Does speculation have real effects on the economy?**
  - consumption and investment
  - impact on asset prices
- **Huge literature on heterogeneous beliefs in pure exchange economy**
  - different beliefs influence asset prices through discount rate
  - consumption or dividend is fixed
- **Small literature on speculation in production economy**
  - consumption or dividend adjust endogenously
  - implications of speculation on asset prices
  - Detemple and Murthy (1994), Panageas (2005), Baker, Hollifield and Osambela (2015)

Overview

A production economy

- power utility
- different beliefs
  - on fundamental risk
  - on extraneous risk, monetary policy
- adjustment costs

Speculation over fundamental risk, extraneous risk or both

- CRRA > 1,
  - increases consumption and decreases investment
  - may decrease asset value
- CRRA < 1,
  - decreases consumption and increases investment
  - may increase asset value
  - may generate boom-and-bust cycle in asset price and investment
    - such cycles happened in many asset markets, housing and stock markets

Model

- **Two Investors with power utility,** $\frac{c_{i,t}^{1-\gamma}}{1-\gamma}$
  - different time preferences: ρ_i
  - aggregate consumption: $c_t = c_{1,t} + c_{2,t}$

- **Linear production, output rate:** $\alpha A_t K_t$ **in consumption good, numeraire**
  - $\alpha > 0$, e.g., 5%
  - capital stock K_t

$$dK_t = \Big(-\delta K_t + \frac{1}{A_t}\{ I_t - a\, I_t 1_{\{I_t>0\}} + b\, I_t 1_{\{I_t<0\}} \}\Big) dt$$

  - I_t – investment (measured in consumption good)

$$I_t + c_t \leq \alpha A_t K_t$$

  - adjustment costs: $a, b > 0$, e.g., 10%, 20%

  - A_t – productivity (conversion between capital and consumption)

- **Different beliefs**
  - on growth rate of productivity A_t (fundamental)

$$dA_t = \mu A_t dt + \sigma A_t dB_t = (\mu - \eta\beta\sigma)A_t dt + \sigma A_t dB_t^2$$

  - on some extraneous event, e.g., monetary policy
  - beliefs of investor 2 described by N_t relative to investor 1's

$$dN_t = -\beta N_t d\tilde{B}_t, \quad \tilde{B}_t = \eta B_t + \sqrt{1-\eta^2}\hat{B}_t$$

  - $\eta = 1$: only disagree on fundamental risk
  - $\eta = 0$: only disagree on extraneous risk

- **Dynamics of AK under investor 1's probability measure**

$$\begin{aligned} dA_t K_t &= A_t dK_t + K_t dA_t \\ &= \Big[A_t K_t(\mu - \delta) + \big(\alpha A_t K_t - c_t\big)\big(1 - a1_{\{\alpha A_t K_t > c_t\}} \\ &\qquad + b1_{\{\alpha A_t K_t < c_t\}}\big)\Big] dt + A_t K_t \sigma \, dB_t. \end{aligned}$$

  - the dynamics of AK has 3 distinctive regions: capital accumulation, no adjustment, and capital depletion

Model: Financial Markets

- **Production runs by a competitive firm**
  - each investor endows θ_i shares of the firm

- **Investors can trade in the fundamental risk, extraneous risk, or both**
  - enough financial assets to span all traded risk
  - markets are complete
  - investor do not need to trade shares of the firm

- **Investor chooses consumption and shares of financial asset to maximize utility**

$$E^i \left[\int_0^\infty e^{-\rho_i t} \frac{c_{i,t}^{1-\gamma}}{1-\gamma} \right]$$

subject to $W_t^i \geq -\theta_i S_t$
  - W_t^i – financial wealth
  - S_t – value of the firm

- **Both investors agree on how to run the firm**
  - firm chooses a production plan to maximize firm value

$$E^1\left[\int_0^\infty \xi_t c_t \, dt\right]$$

  - ξ_t – state price density
  - c_t – aggregate consumption or dividend of the firm

- **Equilibrium production and consumption is equivalent to the solution to following planner's problem**
  - choose $c_{1,t}$, $c_{2,t}$ with $c_{1,t} + c_{2,t} = c_t$ to maximize

$$E^1\left[\int_0^\infty e^{-\rho_1 t}\left(\lambda \frac{c_{1,t}^{1-\gamma}}{1-\gamma} + (1-\lambda) M_t \frac{c_{2,t}^{1-\gamma}}{1-\gamma}\right) dt\right]$$

  subject to the dynamics of $M_t = e^{(\rho_1-\rho_2)t} N_t$ and $A_t K_t$

Model Solution

- **Properties of value function**
  - the value function has the form

$$V(AK, M) = \frac{(AK)^{1-\gamma}}{1-\gamma} h(M)$$

  - $h(M)/(1-\gamma)$ is convex, $h(M)$ is increasing, and $h(M)/M$ is decreasing

$$0 \leq \frac{Mh'(M)}{h(M)} \leq 1$$

- **Homogeneity reduces the Bellman equation into ODE of $h(M)$**

- **As the dynamics of AK has 3 distinctive forms**
  - $h(M)$ satisfies 3 distinctive ODEs
  - feasibility determines **free boundaries** and which **ODE** applies at particular M

- **Capital accumulation region**

$$\left(\frac{-\Gamma_1}{1-\gamma}+\alpha(1-a)\right)h(M)+\frac{\beta^2M^2}{2(1-\gamma)}h''(M)+\left(\frac{\rho_1-\rho_2}{1-\gamma}-\eta\beta\sigma\right)Mh'(M)$$
$$+\frac{\gamma}{1-\gamma}\left[\lambda^{\frac{1}{\gamma}}+((1-\lambda)M)^{\frac{1}{\gamma}}\right][(1-a)h(M)]^{1-\frac{1}{\gamma}}=0$$

  - capital depletion is similar by replacing a with $-b$

- **No-adjustment region**

$$\frac{-\Gamma_1}{1-\gamma}h(M)+\frac{\beta^2M^2}{2(1-\gamma)}h''(M)+\left(\frac{\rho_1-\rho_2}{1-\gamma}-\eta\beta\sigma\right)Mh'(M)$$
$$+\frac{\alpha^{1-\gamma}}{1-\gamma}\left[\lambda^{\frac{1}{\gamma}}+((1-\lambda)M)^{\frac{1}{\gamma}}\right]^{\gamma}=0$$

  - with conditions

$$(1-a)h(M)\leq\alpha^{-\gamma}\left[\lambda^{\frac{1}{\gamma}}+((1-\lambda)M)^{\frac{1}{\gamma}}\right]^{\gamma}\leq(1+b)h(M)$$

- Boundary conditions

$$\lim_{M \to 0} h(M) = \lambda h_1, \quad \lim_{M \to \infty} \frac{h(M)}{M} = (1 - \lambda) h_2$$

where

$$h_i = \begin{cases} \frac{\bar{c}_{ia}^{-\gamma}}{1-a}, & (1-\gamma)(1-a) < \frac{\Gamma_i}{\alpha} < 1 - a \\ \frac{\alpha^{1-\gamma}}{\Gamma_i}, & 1 - a \le \frac{\Gamma_i}{\alpha} \le 1 + b \quad \text{or} \quad \frac{\Gamma_i}{\alpha} \le (1-\gamma)(1-a) \\ \frac{\bar{c}_{ib}^{-\gamma}}{1+b}, & \frac{\Gamma_i}{\alpha} > 1 + b \end{cases}$$

  - **optimal consumption/production to AK ratio at autarky**

$$\bar{c}_{ia} = \frac{\Gamma_i - (1-\gamma)\alpha(1-a)}{(1-a)\gamma}, \quad \bar{c}_{ib} = \frac{\Gamma_i - (1-\gamma)\alpha(1+b)}{(1+b)\gamma}$$

  - **output-price (firm value) ratios at autarky**

$$\Gamma_1 = \rho_1 - (1-\gamma)\left(\mu - \delta - \frac{\gamma\sigma^2}{2}\right), \Gamma_2 = \rho_2 - (1-\gamma)\left(\mu - \delta - \eta\beta\sigma - \frac{\gamma\sigma^2}{2}\right)$$

Optimal Consumption to AK ratio

- in capital accummulation, no adjustment, and depletion

$$\frac{c_t^*}{A_t K_t} = \begin{cases} \left(1 - \frac{M_t h'(M_t)}{h(M_t)}\right) \bar{c}_{1a} + \frac{M_t h'(M_t)}{h(M_t)} \bar{c}_{2a} - \frac{\beta^2 M_t^2 h''(M_t)}{2\gamma(1-a)h(M_t)} \\ \alpha \\ \left(1 - \frac{M_t h'(M_t)}{h(M_t)}\right) \bar{c}_{1b} + \frac{M_t h'(M_t)}{h(M_t)} \bar{c}_{2b} - \frac{\beta^2 M_t^2 h''(M_t)}{2\gamma(1+b)h(M_t)} \end{cases}$$

- $\gamma > 1$, $h''(M) < 0$ ($h(M)/(1-\gamma)$ is convex)
  - speculation increases aggregate consumption, decreases investment
  - if $\bar{c}_{1b}, \bar{c}_{2b} > \alpha$, deplete capital for all M
- $\gamma < 1$, $h''(M) > 0$
  - speculation decreases aggregate consumption, increases investment
  - if $\bar{c}_{1a}, \bar{c}_{2a} < \alpha$, accumulate capital for all M

Interest Rate

Interest rate in **Accumulation**, **No-Adjustment**, and **Depletion**

$$r_t = \begin{cases} \alpha(1-a) + \mu - \delta - \gamma\sigma^2 - \eta\beta\sigma\frac{M_t h'(M_t)}{h(M_t)} \\ \frac{\lambda^{\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}} + ((1-\lambda)M_t)^{\frac{1}{\gamma}}}\bar{r}_1 + \frac{((1-\lambda)M_t)^{\frac{1}{\gamma}}}{\lambda^{\frac{1}{\gamma}} + ((1-\lambda)M_t)^{\frac{1}{\gamma}}}\bar{r}_2 + \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\frac{(\lambda(1-\lambda)M_t)^{\frac{1}{\gamma}}}{\left(\lambda^{\frac{1}{\gamma}} + ((1-\lambda)M_t)^{\frac{1}{\gamma}}\right)^2}\beta^2 \\ \alpha(1+b) + \mu - \delta - \gamma\sigma^2 - \eta\beta\sigma\frac{M_t h'(M_t)}{h(M_t)} \end{cases}$$

- weighted average of autarky rates in adjustment regions
  - e.g., for capital accumulation

$$\alpha(1-a) + \mu - \delta - \gamma\sigma^2, \quad \alpha(1-a) + (\mu - \eta\beta\sigma) - \delta - \gamma\sigma^2$$

- consumption-weighted average plus an adjustment term in no-adjustment regions
  - r_1 and r_2 are autarky interest rates in pure exchange phase

Value of Firm (aggregate consumption)

Output-price ratio in **Accumulation**, **No-Adjustment**, and **Depletion**

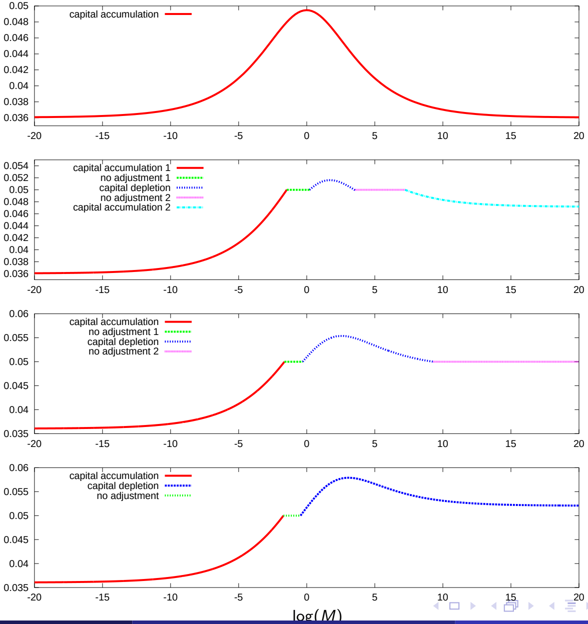
$$\frac{\alpha A_t K_t}{S_t} = \begin{cases} (1-a)\alpha \\ \left(1 - \frac{M_t h'(M_t)}{h(M_t)}\right)\Gamma_1 + \frac{M_t h'(M_t)}{h(M_t)}\Gamma_2 - \frac{\beta^2}{2}\frac{M_t^2 h''(M_t)}{h(M_t)} \\ (1+b)\alpha \end{cases}$$

- adjustment regions: constants
- no-adjustment (pure exchange)
  - $\gamma > 1$ ($h'' < 0$)
    - speculation ↑ output-price ratio → ↓ price-output ratio
  - $\gamma < 1$ ($h'' > 0$)
    - speculation ↓ output-price ratio → ↑ price-output ratio

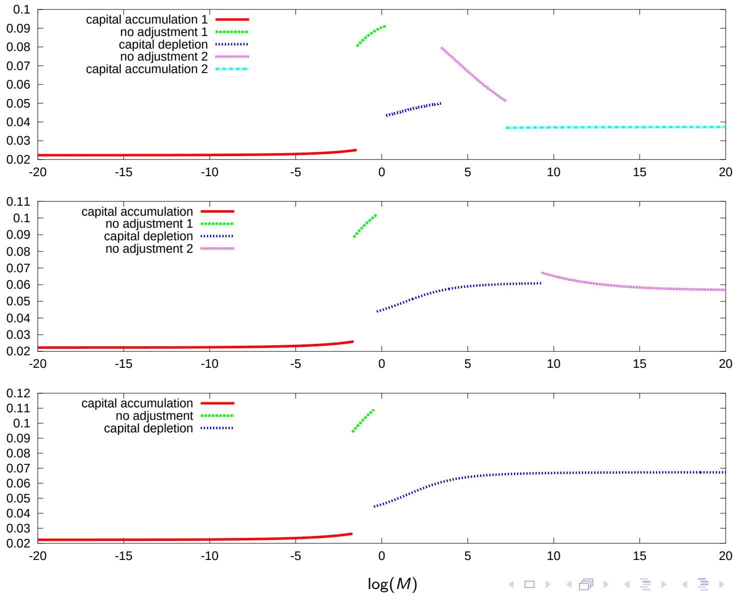
Comparative Statics

- Shooting method
  - It is a standard method to solve boundary value ODEs
    - adjust the slope at one end such that the solution reaches the other side on the target
    - a minor complication is that we need to determine the free boundaries along the way
- Parameter values
  - preferences: $\gamma = 0.3, 3$, $\rho_1 = \rho_2 = 0.05$
  - production: $\alpha = 0.05$, $\mu - \delta = -0.02$, $\sigma = 0.03$
  - beliefs: $\beta = -1$, $\eta \sim$
  - adjustment costs: $a = 0.1$, $b = 0.2$
    - irreversible capital: $b = \infty$

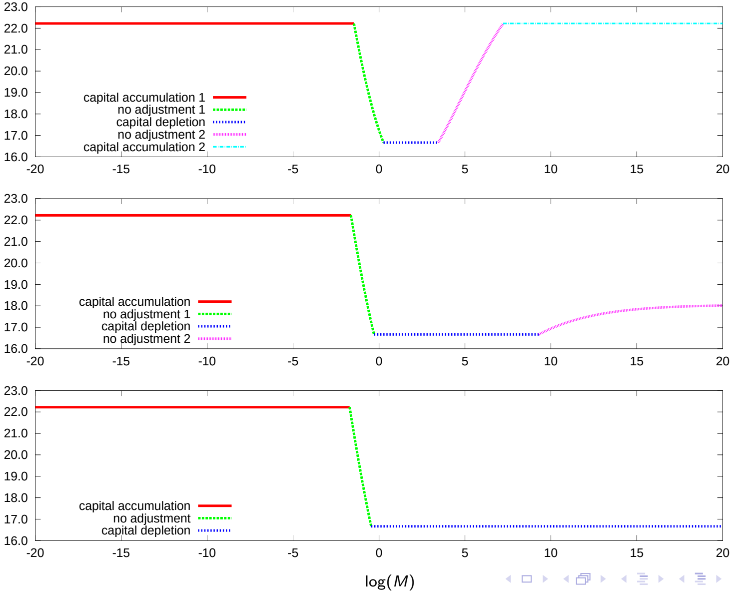
Effects on Consumption: $\gamma > 1$, $\eta = 0, 0.5, 0.8, 1$



Effects on Interest Rate: $\gamma > 1$, $\eta = 0.5, 0.8, 1$



Effects on Firm Value $\frac{S_t}{\alpha A_t K_t}$: $\gamma > 1$, $\eta = 0.5, 0.8, 1$



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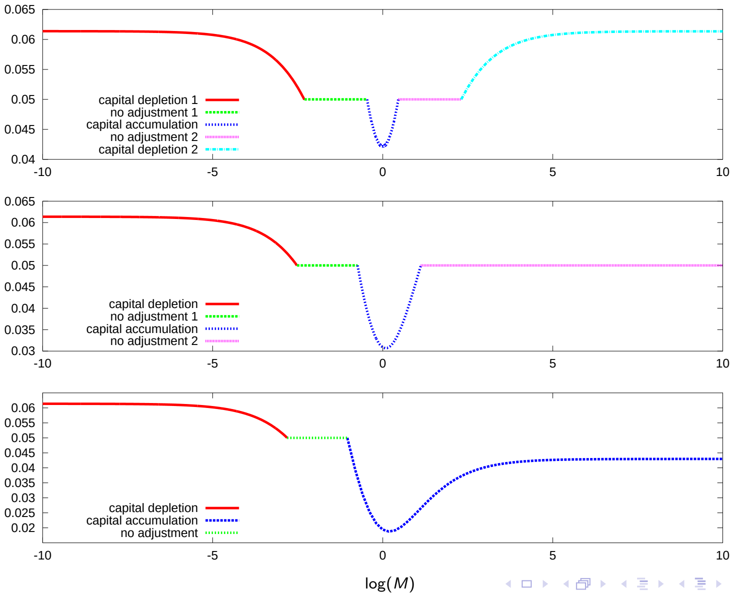

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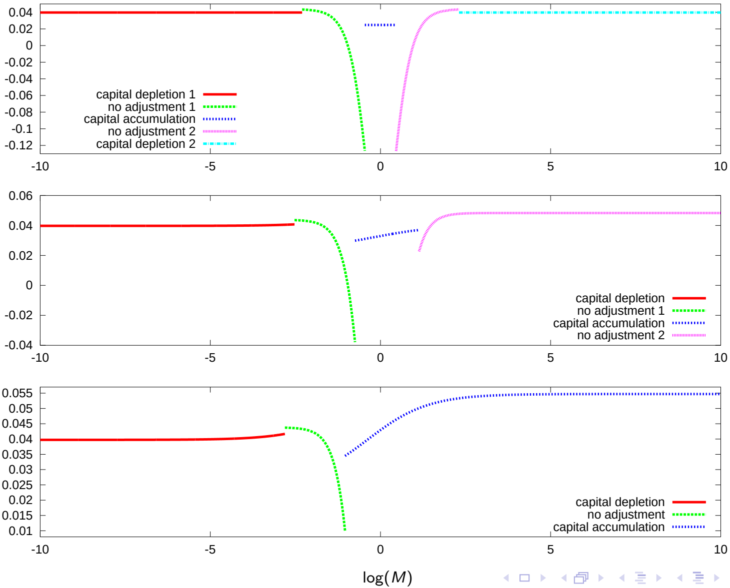

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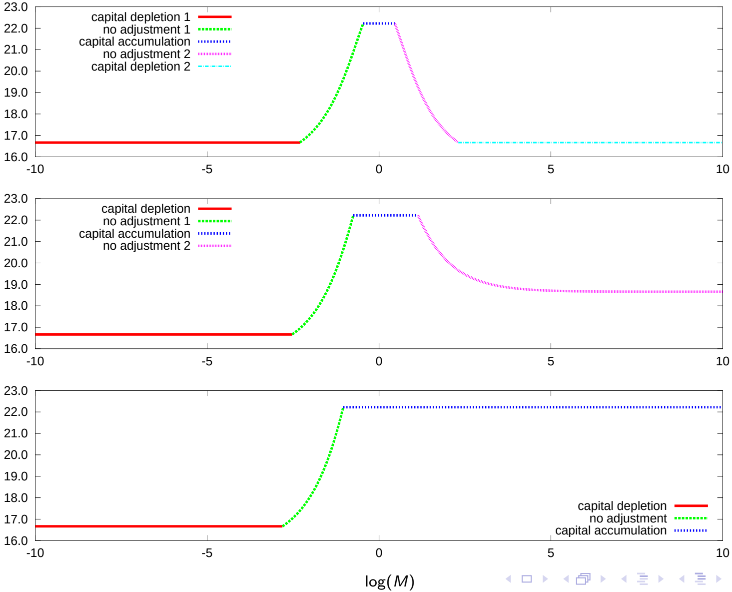
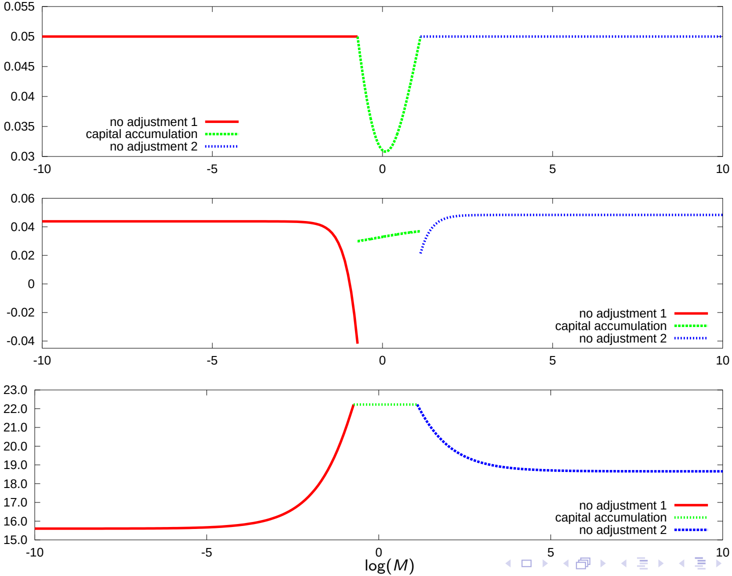
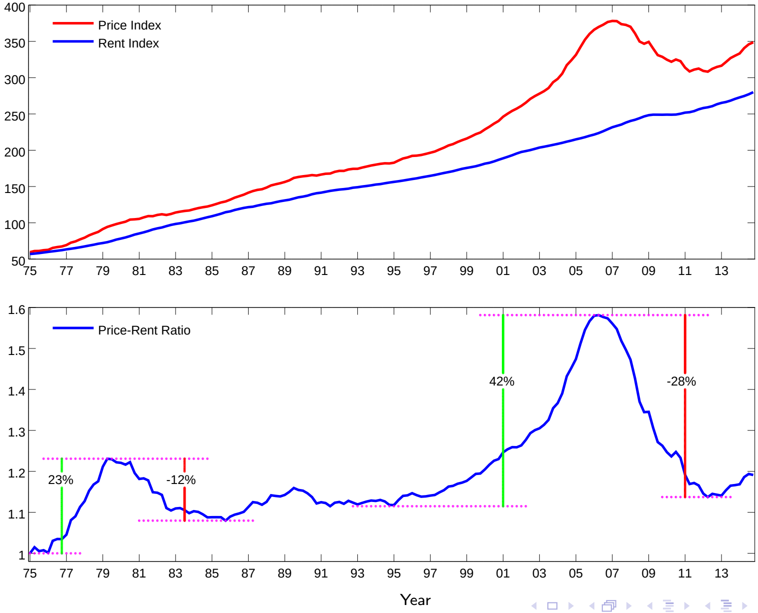


Irreversibility ($\gamma < 1$): Consumption, Interest Rate, Price-output Ratio



US Housing Booms and Busts: 1975-2014



Conclusion

- Speculation on fundamental and/or extraneous risk has profound impacts in a production economy
  - consumption and investment
  - asset prices
  - in conjunction with adjustment costs, the model can generate observed asset boom-and-bust cycles
- Welfare analysis
  - Is opening financial markets Pareto optimal?